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A Dynamic Winkler Model of Arbitrary Bonding Conditions Between Piled-Raft Foundations and the Soil

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Abstract: Models of piled-raft foundations are important in understanding dynamic soil-foundation interaction problems. These models usually consist of a model of surface plate and a model of embedded pile, which are coupled through direct kinematic compatibility and equilibrium at their interface. This results in a perfectly bonded contact condition at the plate-soil and pile-soil interfaces. However, a perfectly bonded contact is an inaccurate model for many practical applications. In this work, we present a model of arbitrary bonding conditions at the plate-soil interface. A Winkler layer is incorporated at that interface so that various continuity and equilibrium conditions can be incorporated. An Indirect-Boundary Element scheme is used to model the fully coupled raft-Winkler layer-pile-soil interaction, which accurately takes into account the energy transfer from and to each part of the system. The model enables the representation of arbitrary conditions between the bounds of perfectly bonded and perfectly uncoupled plate-soil interaction.

Keywords: dynamic soil-foundation interaction, Winkler layer, BEM-FEM coupling

INTRODUCTION

The authors of this work have recently presented a model of the dynamic interaction between a piled-raft foundation and its underlying soil (Barros, Labaki and Mesquita, 2018). Their work considered time-harmonic vertical excitations. The raft was modeled as a rigid, circular surface plate. The model was obtained by discretizing the unknown traction distribution at the plate-soil interface into concentric annular surfaces in which the tractions were assumed to be constant (Lysmer, 1956). The rigid plate condition that the vertical displacement of all annular surfaces was the same yielded an algebraic system of equations from which the contact tractions were obtained (Labaki, Mesquita and Rajapakse, 2014). The pile was modeled as a one-dimensional, elastic finite element body. Its bonding with the surrounding soil was obtained by establishing direct kinematic compatibility and equilibrium at discrete points of the pile-soil contact. Finally, the soil was modeled as a homogeneous, transversely isotropic, three-dimensional half-space (Rajapakse and Wang, 1993). Stress and displacement fields in the half-space were obtained through the Indirect Boundary Element Method (IBEM) (Vable and Ammons, 1995) for subsequent bonding with the plate and pile bodies. This fully coupled model accurately accounted for the energy transfer from the plate to the pile through the soil, the disregard of which has been shown to incur in significant physical inconsistencies (Barros et al., 2018).

Despite its contribution to the understanding of piled raft-soil interaction problems, the aforementioned model is limited in that it considers perfectly bonded contact between the pile and the soil. In practical applications, this is the unattainable limiting condition in which the construction of the system free of flaws. This article proposes an improvement in the model to consider intermediate bonding conditions. A Winkler layer is added at the plate-soil interface, in which arbitrary bonding conditions from perfectly bonded to perfectly uncoupled may be considered. The paper presents numerical results for different bonding conditions. The results provide significant insights on the effect of imperfect construction in the response of the system.

Statement of the problem

Consider a three-dimensional, transversely isotropic half-space described by constitutive parameters c_{11}, c_{12}, c_{13}, c_{33} and c_{44}, and mass density ρ. A pile of radius a_p, length l_p, Young's modulus E_p, and mass density ρ_p is buried in the half-space. The pile tip is connected to a rigid circular plate or radius a_b and mass m_b. An elastic Winkler layer of vertical reaction coefficient k_w connects the plate to the surface of the soil (Fig. 1). Throughout this article, sub-indices p, b, and w refer respectively to the plate, the pile, and the Winkler layer. The system is under vertical, time-harmonic excitations of frequency ω. The aim of this work is to determine the influence of k_w in the vertical dynamic response of the surface plate.

FORMULATION

Let $u_r(r, z)$ and $u_z(r, z)$ be the displacements of the half-space in the r and z directions, respectively. A solution for the equations of motion of the half-space were presented by Rajapakse and Wang (1993) in terms of Hankel transforms

– improper integrals of singular, oscillatory-decaying kernels. Computing these integrals is the most computationally demanding task in the present model. In the present implementation, these are obtained by incorporating a small damping factor η into the elastic constants in order to smooth out singularities (Christensen, 2010), plus a series extrapolation scheme to deal with the oscillatory-decaying terms (Wynn, 1956).

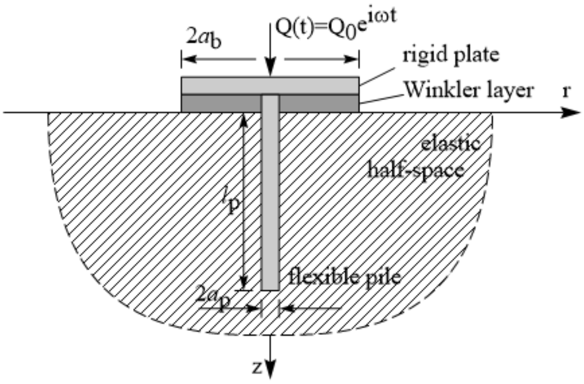


Figure 1 – Piled-raft and Winkler layer over the soil surface

Consider initially the perfectly bonded plate–pile–half-space problem. For this case, the equilibrium at the plate–half-space contact surface can be written as:

$$Q_0 - 2\pi\int_{a_p}^{a_b} t_b r dr - f_0 = -m_b\omega^2 u_0 \,, \tag{1}$$

in which Q_0 is the amplitude of the external force (Fig. 1), $t_b(r)$ is the contact traction at the interface, f_0 is the force at the plate-pile interface, and u_0 is the rigid-body displacement of the plate. The unknown traction field t_b can be discretized into n_b concentric disc areas of inner and outer radii r_{1k} and r_{2k}, $\mathbf{s_b}=\{s_{b1}\ s_{b2}\ \ldots\ s_{bnb}\}^T$ ($s_{bk}=\pi r^2_{2k}-\pi r^2_{1k}$) with uniform traction distribution t_{bi} (i=1,n_b), $\mathbf{t_b}=\{t_{b1}\ t_{b2}\ \ldots\ t_{bnb}\}^T$. Equation (1) for the discretized area is:

$$\mathbf{s_b}^T\mathbf{t_b} + f_0 - m_b\omega^2 u_0 = Q_0 \,. \tag{2}$$

On the other hand, the pile–half-space contact surface is discretized into n_p cylindrical segments of length l_e and radius a_p, plus one disc element of radius a_p at the pile tip. A total of $n_n=n_p+1$ elements are used to discretize the pile–half-space interaction. The dynamic equilibrium of this system is written as

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{u}_f + \mathbf{A}\mathbf{t_p} - f_0\mathbf{i_0} = \mathbf{0} \,, \tag{3}$$

in which $\mathbf{i_0}=\{1\ 0\ 0\ \ldots\ 0\}^T$, $\mathbf{t_p}=\{t_{p1}\ t_{p2}\ \ldots\ t_{pnp}\}^T$ is the vector of contact tractions at the discretized pile–half-space interface, $\mathbf{u_f}$ is the vector of nodal displacements of the discretized interface, and $\mathbf{A}$ is a transformation matrix given by $A_{i,i}= A_{i,i-1}=\pi a_p l_e^{(i)}$, i=1,…,$n_p$; $A_{np+1,np+1}=\pi a_p^2$, $A_{ij}=0$ otherwise.

In Eq. (3), **K** and **M** are global stiffness and mass matrices of classical one-dimensional, two-noded elastic finite elements which are used to model the pile in this formulation.

An Indirect Boundary Element framework is used to incorporate the response of the half-space into the formulation above. The values of constant tractions and displacements at the n_n+n_b elements due to arbitrary fictitious loads **q** can be expressed as:

$$\mathbf{u_p} = \mathbf{U_{pp}q_p} + \mathbf{U_{pb}q_b} \,, \tag{5}$$

$$\mathbf{u_b} = \mathbf{U_{bp}q_p} + \mathbf{U_{bb}q_b} \,, \tag{6}$$

$$\mathbf{t_p} = \mathbf{T_{pp}q_p} + \mathbf{T_{pb}q_b} \text{ , and} \tag{7}$$

$$\mathbf{t_b} = \mathbf{T_{pp}q_p} + \mathbf{T_{pb}q_b} \,, \tag{8}$$

in which $\mathbf{u_p}$ and $\mathbf{u_b}$ are the vertical displacements of the pile and plate elements, $\mathbf{t_p}$ and $\mathbf{t_b}$ are the tractions in those elements, and $\mathbf{q_p}$ and $\mathbf{q_b}$ are the fictitious loads applied in the pile and plate elements. The terms of matrices $\mathbf{U_{ij}}$ and $\mathbf{T_{ij}}$

refer to displacement and tractions of elements of i (i=b,p) due to unit fictitious loads applied in elements of j (j=b,p), in which b and p refer to the plate or the pile, respectively.

Substituting Eqs. (5) to (8) in (3) yields

$$\left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{u}_f+\mathbf{A}\mathbf{T}_{\mathbf{pp}}\mathbf{q}_{\mathbf{p}}+\mathbf{A}\mathbf{T}_{\mathbf{pb}}\mathbf{q}_{\mathbf{b}}-f_0\mathbf{i}_{\mathbf{0}}=\mathbf{0}\,, \tag{9}$$

since $\mathbf{T_{bp}=0}$ and $\mathbf{T_{bb}=I}$ (the plate rests on the traction-free surface of the half-space), where **I** is the identity matrix.

By imposing continuity at the pile tip–plate interface, $u_0=\mathbf{i_0}^T\mathbf{u_f}$, Eq. (2) can be written in terms of fictitious loads as

$$-m_b\omega^2\mathbf{i}_{\mathbf{0}}^{\mathbf{T}}\mathbf{u}_{\mathbf{f}}+\mathbf{s}_{\mathbf{b}}^{\mathbf{T}}\mathbf{T}_{\mathbf{pb}}\mathbf{q}_{\mathbf{p}}+\mathbf{s}_{\mathbf{b}}^{\mathbf{T}}\mathbf{T}_{\mathbf{bb}}\mathbf{q}_{\mathbf{b}}+f_0=Q_0\,, \tag{10}$$

Finally, continuity between the displacements $\mathbf{u_p}$ of the half-space (Eq. 5) and the nodal pile displacements $\mathbf{u_f}$, $\mathbf{u_p}=\mathbf{D}\mathbf{u_f}$, means that Eq. (5) can be written as

$$-\mathbf{D}\mathbf{u}_{\mathbf{f}}+\mathbf{U}_{\mathbf{pp}}\mathbf{q}_{\mathbf{p}}+\mathbf{U}_{\mathbf{pb}}\mathbf{q}_{\mathbf{b}}=\mathbf{0}\,, \tag{11}$$

in which the transformation matrix D is

$$\mathbf{D}=\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & & 0 & 0 \\ \vdots & & & \ddots & & \\ 0 & 0 & 0 & & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & & 0 & 1 \end{bmatrix}. \tag{12}$$

For the case in which a Winkler layer of vertical reaction k_w is introduced between the plate and the half-space, the rigid-body displacement u_0 of the plate differs from u_b. In this case,

$$\mathbf{t}_{\mathbf{b}}=k_w\left(\mathbf{u}_{\mathbf{b}}^*-\mathbf{u}_{\mathbf{b}}\right), \tag{13}$$

in which $\mathbf{u}^*_\mathbf{b}$ are the displacements of the discretized plate above the elements of the Winkler layer. Since the plate is rigid, then

$$\mathbf{u}_b^*=\mathbf{1}\cdot\mathbf{i}_{\mathbf{0}}^{\mathbf{T}}\mathbf{u}_{\mathbf{f}}\,, \tag{14}$$

where **1** is a vector of ones. Substituting Eqs. (6) and (8) into (14) yields

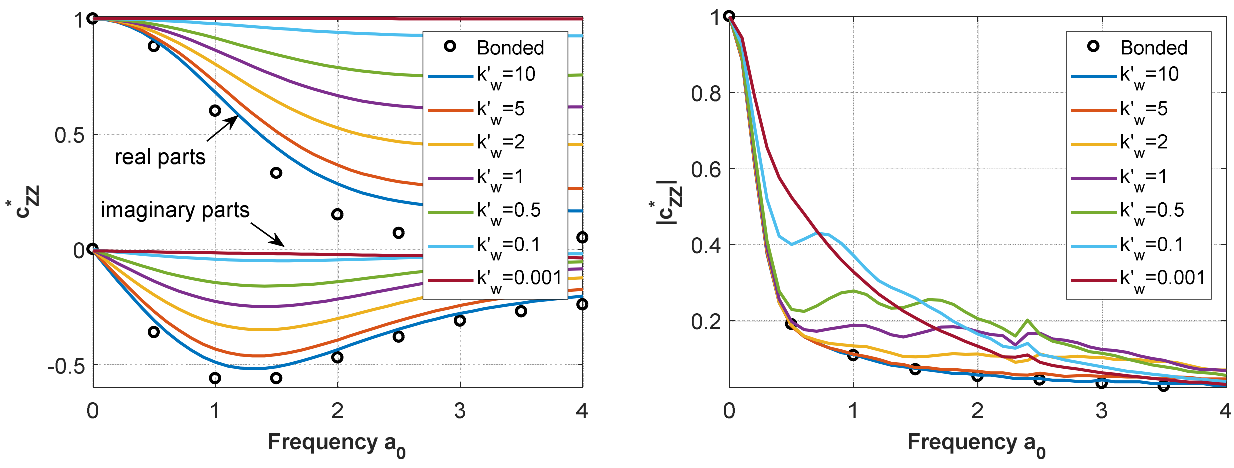
$$-k_w\mathbf{1}\cdot\mathbf{i}_{\mathbf{0}}^{\mathbf{T}}\mathbf{u}_{\mathbf{f}}+\left(k_w\mathbf{U}_{\mathbf{bp}}+\mathbf{T}_{\mathbf{bp}}\right)\mathbf{q}_{\mathbf{p}}+\left(k_w\mathbf{U}_{\mathbf{bb}}+\mathbf{T}_{\mathbf{bb}}\right)\mathbf{q}_{\mathbf{b}}=\mathbf{0}\,, \tag{15}$$

Equations (9) to (11) and (15) can be combined into

$$\begin{bmatrix} \mathbf{K}-\omega^2\mathbf{M} & \mathbf{A}\mathbf{T}_{\mathbf{pp}} & \mathbf{A}\mathbf{T}_{\mathbf{pb}} & -\mathbf{i}_{\mathbf{0}} \\ -\mathbf{D} & \mathbf{U}_{\mathbf{pp}} & \mathbf{U}_{\mathbf{pb}} & \mathbf{0} \\ -k_w\mathbf{1}\cdot\mathbf{i}_{\mathbf{0}}^T & k_w\mathbf{U}_{\mathbf{bp}}+\mathbf{T}_{\mathbf{bp}} & k_w\mathbf{U}_{\mathbf{bb}}+\mathbf{T}_{\mathbf{bb}} & \mathbf{0} \\ -m_b\omega^2\mathbf{i}_{\mathbf{0}}^T & \mathbf{s}_{\mathbf{b}}^T\mathbf{T}_{\mathbf{bp}} & \mathbf{s}_{\mathbf{b}}^T\mathbf{T}_{\mathbf{bb}} & 1 \end{bmatrix}\begin{Bmatrix} \mathbf{u}_{\mathbf{f}} \\ \mathbf{q}_{\mathbf{p}} \\ \mathbf{q}_{\mathbf{b}} \\ f_0 \end{Bmatrix}=\begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ Q_0 \end{Bmatrix}. \tag{16}$$

NUMERICAL RESULTS

Figure 2a considers the case of external excitation of a rigid plate at the surface of the half-space. The results are presented in terms of the normalized vertical compliance of the plate, $c^*_{ZZ}=c_{ZZ}/c_{ZZ}(a_0=0)$, in which $c_{ZZ}=u_0a_p\mu_s/Q_0$, with respect to the normalized frequency $a_0=\omega a_p\sqrt{\rho/\mu_s}$. An analysis of this problem can be obtained with the present implementation by considering that the dimensions of the pile are much smaller than those of the plate, and the material properties of the pile are equal to those of the soil. On the other hand, Fig. 2b shows the corresponding results for a piled raft, in which a pile of length $l_p=10a_p$ is considered. The influence of the stiffness of the Winkler layer k_w is shown in comparison with the perfectly bonded case, in terms of its normalized value $k'_w=k_w a_b/\mu_s$. Figure 2b shows that the frequency response of the system for any elastic bond between the plate and the soil differs significantly from the perfectly bonded case.



(a) (b)

Figure 2 – Influence of Winkler layer in the response of (a) the surface raft and (b) the pile raft

CONCLUSIONS

This article presented a model of the time-harmonic response of piled raft foundations with elastic bond between the raft and the soil. An IBEM model of the soil was coupled with the plate and the pile at discrete points. Elastic continuity conditions were established at the plate-soil interface to represent imperfect bonding conditions. The results showed that different bonding conditions affect the dynamic response of the system significantly.

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REFERENCES

Barros, P.L.A., Labaki, J. and Mesquita, E. "IBEM-FEM Model of a Piled Plate Within a Transversely Isotropic Half-Space". Engineering Analysis with Boundary Elements (submitted), 2018.

Barros, P.L.A., Labaki, J., L.F.V. Lima, and Mesquita, E. "A Study on Spring-Dashpot Equivalency for the Dynamic Response of Piled Rafts: the Extent to which Energy Transmission through the Soil can be Disregarded." International Conference on Boundary Element and Meshless Tecniques, BETeq 2018, Malaga.

Christensen, R. M. Theory of Viscoelasticity. New York, Dover Publications, 2010.

Labaki, J., Mesquita, E. and Rajapakse, R.K.N.D. "Vertical Vibrations of an Elastic Foundation with Arbitrary Embedment within a Transversely Isotropic, Layered Soil". Computer Modeling in Engineering & Sciences (Print), v. 103, p. 281-313, 2014.

Lysmer, J. "Vertical Motions of Rigid Footings", Ph.D. Thesis, University of Michigan, Ann Harbor, USA, 1965.

Pak, R. Y. S and Gobert, A. T. "Forced Vertical Vibration of Rigid Discs with Arbitrary Embedment", J. Eng. Mech, 117, issue 11, 2527, 1991.

Rajapakse, R. K. N. D; Wang. Y. "Green's Functions for Transversely Isotropic Elastic Half Space". Journal of Engineering Mechanics, 119, No. 9, pp. 1724-1746, 1993.

Vable M, Ammons BA. "A study of the direct and indirect BEM". WIT Transactions on Modelling and Simulation. 1970 Jan 1;10.

Wynn P. "On a device for computing the $e_{\{m\}}(S_{\{n\}})$ transformation". Mathematics of Computation. 1956;10(54):91-6.

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