

# Aircraft braking dynamics and brake system modeling for fault detection and isolation

Lucas Cardoso Navarro<sup>1</sup>, Luiz Carlos Sandoval Goes<sup>1</sup>

<sup>1</sup> ITA, Praça Marechal Eduardo Gomes, 50 - Vila das Acacias, So Jos dos Campos - SP, 12228-900

*Abstract: Due to the increasing complexity of aeronautical systems, it became more and more important to detect possible failures, avoiding costs with maintenance and time out of operation. Modeling techniques and computational softwares made possible to analyze systems behaviour under normal and failure conditions, helping to prevent these problems. In this work, an aircraft anti-skid brake system is considered as a study case. Therefore, the aircraft brake dynamics and the brake system are modeled using Simulink. In the brake model, some common faults are introduced in order to observe its impacts on the braking performance. A model based fault detection and isolation (FDI) method using analytical redundancy relations (ARRs) is proposed. ARR's are equations relating the system constraints. The numerical evaluation of these equations generates residuals indicating the system deviation from its normal operation. The coupling of the Simulink behavior model with the ARR's is presented, permitting the residuals analyses for each failure mode.*

**Keywords:** ARR, Hydraulic brake system, Modeling

## INTRODUCTION

Modeling has become a powerful engineering tool, making possible to study and evaluate complex systems, preview systems projects behaviors without spend money on expensive prototypes and prevent systems failures through model based fault detection and isolation (FDI) methods. FDI methods can be divided into qualitative and quantitative approaches. Qualitative FDI methods are used when the studied system model is too hard to be obtained, or it cannot be analyzed in order to obtain a faithful model. To analyze the systems, these methods are based on artificial intelligence techniques (Medjaher, 2011). Quantitative methods compare the real system behavior with a system ideal model behavior. In this work, the aircraft brake dynamics and its hydraulic brake system will be modeled and simulated in order to preview some failure modes effects on the aircraft brake performance. After that, an FDI method based on analytical redundancy relations (ARRs) will be proposed in order to detect and isolate these faults.

## MODELING METHODOLOGY

Physical modeling is a simplified system representation through idealized elements with the intention of representing the dynamics phenomena of real systems. These elements simulate supply sources, as well as the storage, dissipation and transformation of the system energy. In this work, the concept of effort and flow was used. This concept is common to all energy domains. The Tab. 1 lists for each domain what effort and flow represents.

**Table 1 – Flow and effort concept for each energy domain.**

System	Variable types			
	Flow (f)	Accumulated Flow (q)	Effort (e)	Accumulated Effort (p)
Translational Mechanical	Velocity	Displacement	Force	Momentum
Rotational Mechanical	Angular Velocity	Displacement	Torque	Angular Momentum
Electrical	Current	Electric Charge	Voltage	Magnetic Flux
Hydraulic	Flow rate	Volume	Pressure	Pressure Momentum
Thermal	Heat Flow	Energy/Enthalpie	Temperature	--

Based on this concept it is possible to represent the system in an unified way, taking into account only the elements energy interactions. Thus, in this work, electrical equivalents were used to perform mathematical modeling, representing the entire system. From the electrical equivalents, the state equations that reflects the system behavior can be derived.

## FAULT DETECTION AND ISOLATION METHOD

FDI methods are based the comparison between a system real behavior and its theoretical behavior given by a model (Medjaher, 2011).The method proposed by this work is a model based FDI, also called quantitative FDI method. Model based fault detection and isolation applications can be found in (Medjaher, 2011), (Borutzky, 2016) and Samantaray, et al. (2005).The first step is to create a behavior model where the state equations describing the dynamic behavior of a given system, can be derived. In this model, some sensors are introduced in order to monitor the states. The second step is to create a diagnostic model. Instead of measuring the states, this model will receive from the behavior model the sensor measurement as a source. This way, the equations describing the system dynamics (the system constraints) will be written in terms of the real system sources, parameters and sensor measurements. Thus, these equations will have a redundance, creating a real test from the actual system. This equations are called analytical redundancy relations (ARRs). If the real system works in normal operation, theoretically the ARR equation value must be around zero, also called as residual value. If the residual value of an ARR differs significantly from zero, it can indicate a fault in some system component. The Fig. 1 illustrates the FDI method proposed.

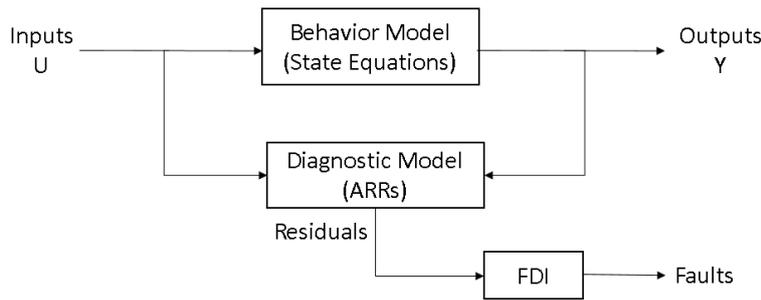


Figure 1 – FDI method based on residual generation. Adapted from Borutzky (2016).

The Fig. 1 shows that the diagnostic model works in parallel with the behavior model, monitoring the outputs results and generating constantly the residuals. Thus, the FDI method will evaluate the residual generation and be able to determine the fault origin. The Fig. 2 a), b) and c) shows a mass, damper and spring system, its electrical equivalent behavior model with a flow sensor (mass velocity sensor) and the diagnostic model with the sensor measurement source, respectively.

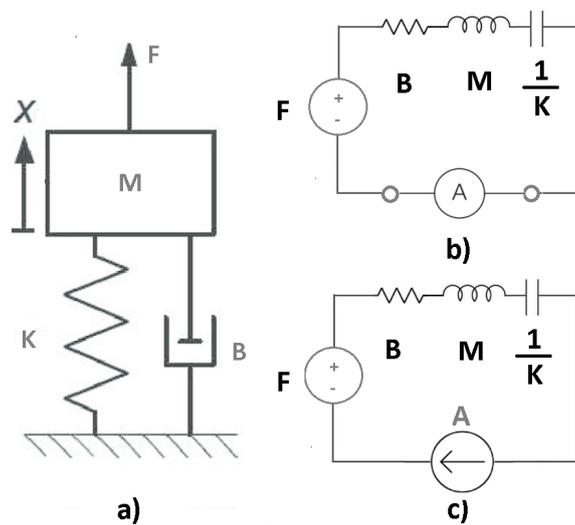


Figure 2 – a) Mass, Spring and Dumper system. b) Electrical equivalent behavior model. c) Electrical equivalent diagnostic model.

From the behavior model, the state equations describing the system dynamics are:

$$\dot{p} = F - \frac{q}{(1/k)} + B \cdot \frac{p}{M} \quad , \quad \dot{q} = \frac{p}{M} \quad (1)$$

From the diagnostic model, the ARR equation can be generated.

$$ARR : F - M.\ddot{A} - K.A - B.\dot{A} = r \quad (2)$$

If the sensor measurement comes from the original system, without faults, the residual value ( $r$ ) must be around zero. however, if the sensor measurement comes from the same system, but with some spring or damper malfunction, the residual value will differs from zero, indicating the fault. The system FDI application in the section shows more details about this method.

## SYSTEM MODELING

### Aircraft and wheel dynamics

The main part of the aircraft kinetics energy is dissipated by the brake system when landing. The brake force is derived from the counter torque to the wheels movement, which is generated from the brake system actuators. The Fig. 3 shows the forces acting on the aircraft and the wheels during braking time and its electrical equivalent. The equations derived from the electrical equivalent circuit are represented by 3.

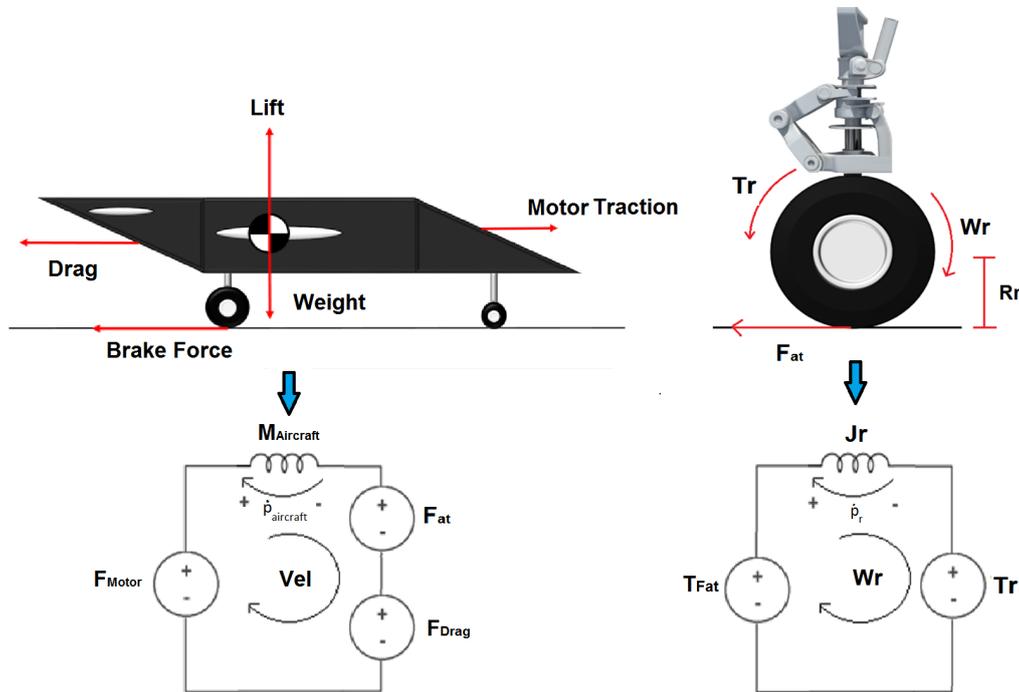


Figure 3 – Forces acting on the aircraft and wheels during braking and the corresponding electrical equivalent.

$$p_{aircraft} = F_{Motor} - F_{at} - F_{Drag} \quad , \quad \dot{p}_r = T_{fat} - T_r \quad (3)$$

where:

$$F_{at} = N \cdot \mu_{at}(S) \quad , \quad T_{at} = N \cdot u_{at}(S) \cdot R_r \quad , \quad F_{drag} = \frac{1}{2} \cdot \rho \cdot V^2 \cdot C_d \cdot S \quad (4)$$

The normal force on the aircraft depends on its weight and lift acting on the braking moment. The normal force corresponding to each wheel, considering that each wheel receives one third of the resulting force is described as follows:

$$N = \frac{W - L}{3} \quad (5)$$

where:

$$L = \frac{1}{2} \cdot \rho \cdot V^2 \cdot C_l \cdot S \quad (6)$$

$T_r$  is the counter torque generated by the brake actuator and  $F_{Motor}$  is the motor traction. The wheel and aircraft equations are related by the wheel slip. The slips relates the aircraft speed and the wheel speed, as described by the Eq. (7).

$$S = 1 - \frac{W_r}{W_t} \quad (7)$$

where:

$$W_r = \frac{1}{J_{roda}} \cdot \int (\dot{p}_r) \quad , \quad W_t = \frac{Vel}{R_r} \quad , \quad Vel = \frac{1}{M_{aircraft}} \cdot \int (p_{aircraft}) \quad (8)$$

### Brake system

The Fig. 4 a) shows the servo valve schematic and 4 b), its corresponding electrical equivalent circuit. This valve controls the brake pressure. The torque motor has the function of moving the flapper coupled to its axis. The flapper moves in the longitudinal direction. The flapper displacement is proportional to the voltage applied to the torque motor terminal.

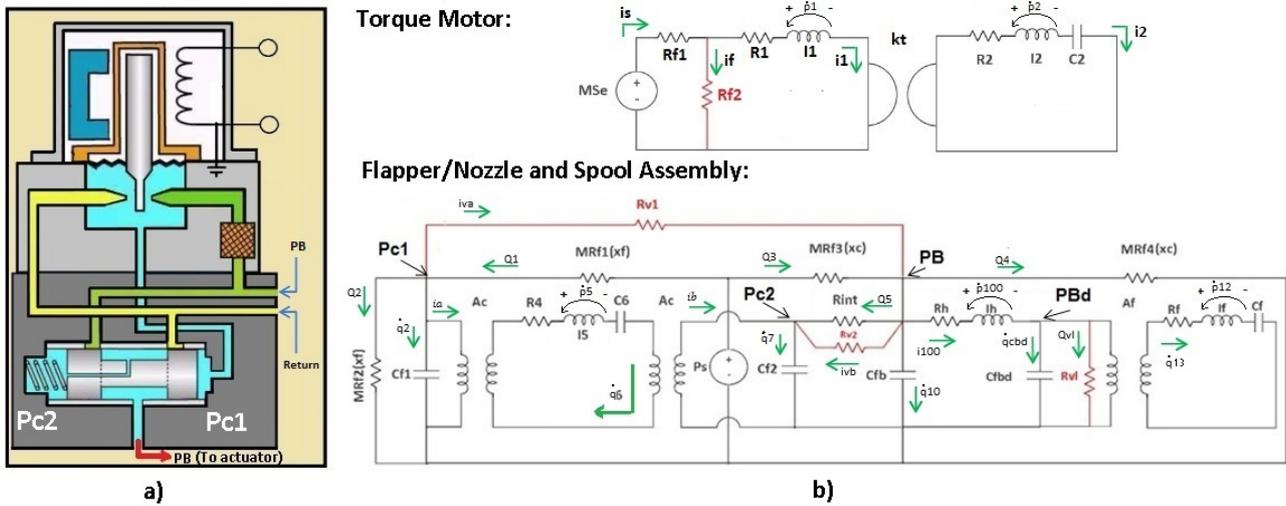


Figure 4 – a) Servo-valve schematic. b) Servo-valve electrical equivalent

The torque motor modeling can be simplified considering that it is a DC motor. In the Fig. 4 b) the electrical equivalent of a DC motor is presented, where  $R_{f1}$  and  $R_1$  represents the motor wiring resistances before and after a possible current leak ( $i_f$ ) represented by  $R_{f2}$ .  $I_1$  represents the motor windings resistance, inductance and the voltage applied to the motor respectively. The gyrator is the representation of the energy transformation, where the motor current (flow) turns into torque (effort), in a  $K_t$  ratio proportion.  $R_2$ ,  $I_2$  and  $C_2$  represents respectively the viscous friction, flapper mass and flapper return spring. When the flapper moves, the pressure nozzle is released, generating a fluid flow toward the spool side called  $P_{c1}$ . This will generate a pressure proportional to the flapper movement. At this point, the spool will move, releasing another orifice that leads to a pressure port linked to the spool camera. When this happens, a flow is generated toward the volume that goes until the pressure actuator and the volume at the other spool side ( $P_{c2}$ ), via a feedback orifice. When the  $P_{c2}$  side reaches the same pressure rate as the  $P_{c1}$  side, the spool will stop moving. at this time, the return spring at the  $P_{c2}$  side will make the spool returns to the original position, closing the pressure port. This will make the brake actuator pressure be equal to the control pressure ( $P_{c1}$  pressure). When the flapper comeback to the idle position, the pressure at the  $P_{c1}$  side returns to zero, as the flow goes trough the return nozzle. This will make the spool moves to the other direction, releasing the return port of the spool camera, making the brake pressure decrease until zero. This way, the brake pressure is controlled.

In the electrical equivalent,  $P_s$  is the pressure supply,  $MR_{f1}(xf)$  and  $MR_{f2}(xf)$  are the fluid resistances modulated by the flapper movement, where the flow trough them are represented by the flows  $Q_1$  and  $Q_2$ .

$$Q_1 = C_d \cdot \pi \cdot dn \cdot xf \cdot \sqrt{\frac{2}{\rho} \cdot (P_s - P_{c1})}; \quad Q_2 = C_d \cdot \pi \cdot dn \cdot (L - xf) \cdot \sqrt{\frac{2}{\rho} \cdot (P_{c1})} \quad (9)$$

where L is the distance between the nozzles, dn is the nozzle diameter and xf is the flapper position. The capacitances Cf1 and Cf2 represent the volumes in both side of the spool.

$$C_{f1} = \frac{V_{flapper}}{\beta}; \quad C_{f2} = \frac{V_{feedback}}{\beta} \quad (10)$$

The spool dynamics is represented in the circuit by the RLC circuit, where R4 is the viscous friction, I5 is the spool mass and C6 represents the spring rigidity. The transformers represent the spool head cross section area, where pressure is transformed into force, moving the spool.

$$R_4 = b_{spool}; \quad I_5 = M_{spool}; \quad C_6 = \frac{1}{K_{spool}} \quad (11)$$

MRf3(xc) and MRf4(xc) are the fluid resistances modulated by the spool movement. The flow trough MRf3(xc) happens when the spool movement is positive, whereas for MRf4(xc), when it is negative.

$$Q_3 = C_d \cdot w \cdot xc \cdot \sqrt{\frac{2}{\rho} \cdot (P_s - PB_{bl})}; \quad Q_4 = C_d \cdot w \cdot xc \cdot \sqrt{\frac{2}{\rho} \cdot (PB_{bl})} \quad (12)$$

Cfb is the hydraulic line volume and Cfbd is the volume referring to the brake actuator.

$$C_{fb} = \frac{V_{bl}}{\rho} \quad C_{fbd} = \frac{V_{brake}}{\rho} \quad (13)$$

Rint is the fluid resistance between the control volumes represented by Cf2 and Cfb.

$$R_{int} = \frac{128 \cdot \mu \cdot L_{int}}{\pi \cdot D_{int}^4} \quad (14)$$

Rh represents the fluid resistance offered by the hydraulic line until the brake actuator and lh represents the fluid inertance to move inside the pipe.

$$I_h = \frac{L_h \cdot \rho}{A_h} \quad R_h = \frac{128 \cdot \mu \cdot L_h}{\pi \cdot D_h^4} \quad (15)$$

where Ah, lh and Dh are the cross section area, length and the pipe cross section diameter respectively.

The transformer followed by a RLC circuit represent the actuator assembly where Af, Rf, Cf and If represents respectively the brake actuator effective area, brake viscous friction, brake rigidity and brake piston mass.

The resistances Rv1 and Rv2 represented in the Fig. 4 b) in red are the leakage in the spool walls. The resistance Rv1, also in red, in parallel with the capacitance Cbd, represents a possible leakage in the hydraulic line.

$$R_{v1} = R_{v2} = \frac{128 \cdot \mu \cdot L_s h}{\pi \cdot D_{vs}^4}; \quad Q_{vl} = C_d \cdot D_{vl} \cdot \sqrt{\frac{2}{\rho} \cdot (PB)} \quad (16)$$

where Dvs and Dvl are respectively the leakages diameters of the spool and the hydraulic line.

The state equations describing the servo valve dynamics are as follows, where the states are illustrated in the Fig. 4 b) :

$$\dot{p}_1 = \frac{M_{SE} - R_{f1} \cdot \frac{p_1}{I_1} - \left(\frac{R_{f1}}{R_{f2}} + 1\right) \cdot (R_1 \cdot \frac{p_1}{I_1} + \frac{p_2}{I_2} \cdot K_t)}{\left(\frac{R_{f1}}{R_{f2}} + 1\right)}, \quad \dot{p}_2 = V_2 - R_2 \cdot \frac{p_2}{I_2} - \frac{q_{C2}}{C_2}, \quad \dot{q}_{C2} = \frac{p_2}{I_2} \quad (17)$$

$$\dot{q}_2 = Q_1 - Q_2 - i_{va} - i_b \quad \begin{cases} Q_1 = C_d \cdot \pi \cdot dn \cdot xf \cdot \sqrt{\frac{2}{\rho} \cdot (P_s - P_{c1})} \\ Q_2 = C_d \cdot \pi \cdot dn \cdot (L - xf) \cdot \sqrt{\frac{2}{\rho} \cdot (P_{c1})} \\ i_{va} = \frac{P_{c1} - P_B}{R_{v1}} \\ i_a = Ac \cdot \frac{p_5}{I_5} \end{cases}, \quad \dot{p}_5 = V_1 - V_{R4} - V_{C6} - V_2 \quad \begin{cases} V_1 = Ac \cdot P_{c1} \\ V_{R4} = \frac{p_5}{I_5} \cdot R_4 \\ V_{C6} = \frac{q_6}{C_6} \\ V_2 = Ac \cdot \frac{q_7}{C_7} \end{cases} \quad (18)$$





placement and the Fig. 8 b) shows the diagnostic model, with measured sources from behavior model.

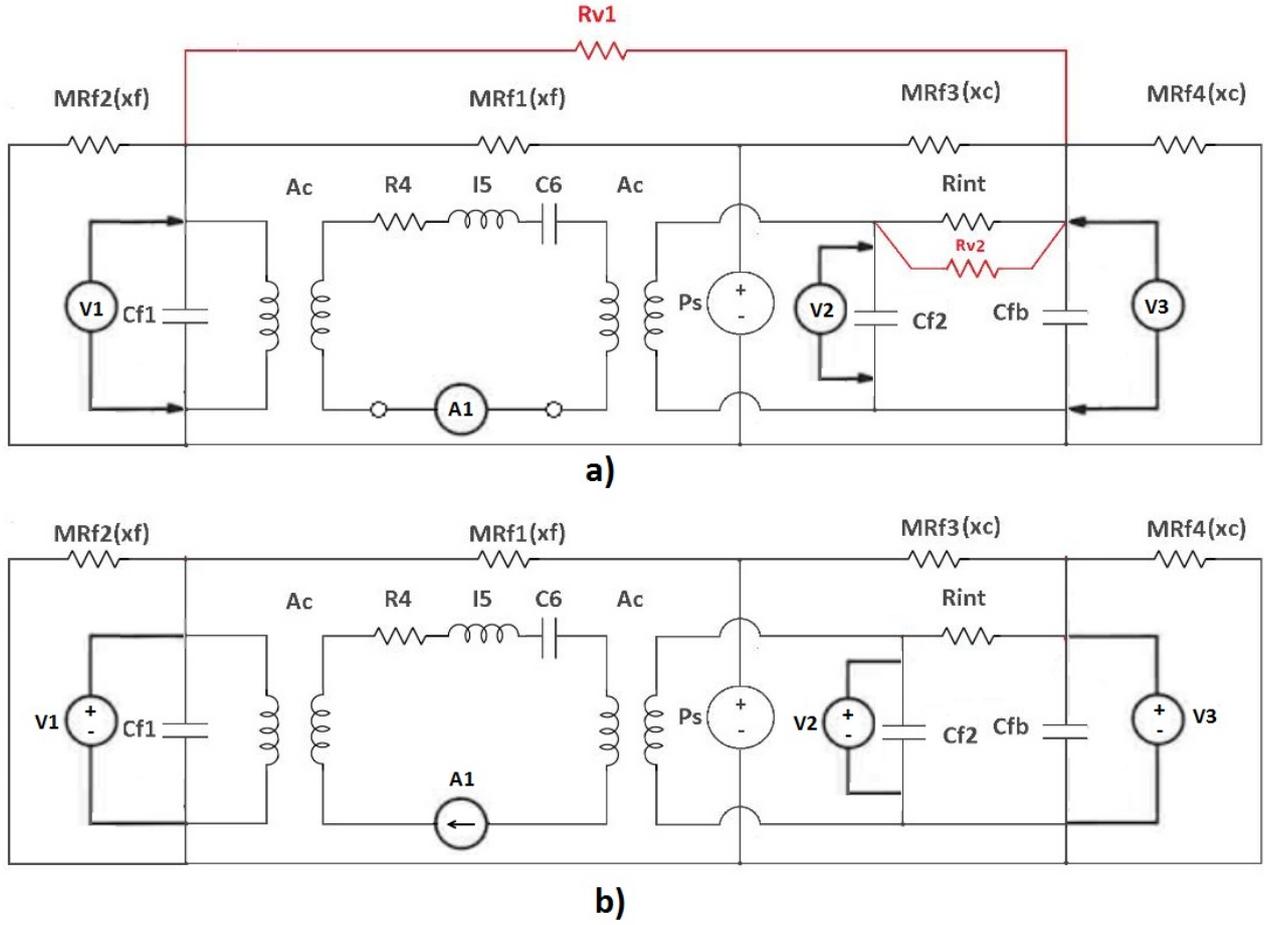


Figure 8 – a) Sensor placement in the behavior model. b) Diagnostic model and measured sources from behavior model.

With this sensor placement configuration, it is possible to extract the ARRs. Thus, from each measured source, there will be an ARR. The current measurement with the sensor A1 in the behaviour model shown in Fig. 8 is the spool velocity. However, its usual to have the spool position instead of the spool velocity. Thus, the ARRs will be derived for this constraint, considering the spool position ( $x_c$ ), that is the A1 sensor measurement integration.

$$ARR 1: \begin{cases} Q_1 - Q_2 - i_1 - i_{V1} - i_{Cf1} = r_1 \\ Q_1 = C_d \cdot \pi \cdot d_n \cdot x_f \cdot \sqrt{\frac{2}{\rho} \cdot (P_s - V_1)} \\ Q_2 = C_d \cdot \pi \cdot d_n \cdot (L - x_f) \cdot \sqrt{\frac{2}{\rho} \cdot (V_1)} \\ i_1 = A_c \cdot \dot{x}_c \\ i_{V1} = 0 \\ i_{Cf1} = \dot{V}_1 \cdot C_{f1} \end{cases} \quad (24)$$

$$ARR 2: A_c \cdot V_1 - R_4 \cdot \dot{x}_c - I_5 \cdot \ddot{x}_c - \frac{1}{C_{f1}} \cdot x_c - A_c \cdot V_2 = r_2 \quad (25)$$

$$ARR 3: \begin{cases} i_2 - i_{Cf2} + i_{R_{int}} + i_{V2} = r_3 \\ i_2 = A_c \cdot V_1 \\ i_{Cf2} = C_{f2} \cdot \dot{V}_2 \\ i_{R_{int}} = \frac{V_3 - V_2}{R_{int}} \\ i_{V2} = 0 \end{cases} \quad (26)$$

$$ARR\ 4: \begin{cases} Q_3 - Q_4 - i_{Rint} - i_{cfb} - i_{V3} = r_4 \\ Q_3 = C_d \cdot w \cdot xc \cdot \sqrt{\frac{2}{\rho} \cdot (P_s - V_3)} \\ Q_4 = C_d \cdot w \cdot xc \cdot \sqrt{\frac{2}{\rho} \cdot V_3} \\ i_{Rint} = \frac{V_3 - V_2}{R_{int}} \\ i_{cfb} = Cfb \cdot \dot{V}_3 \\ i_{V3} = 0 \end{cases} \quad (27)$$

From the ARR's it is possible to obtain the fault signature matrix, shown in Tab. 2. The fault signature matrix makes the relationship of the residuals sensibility for each parameter in the model. Taking as an example the residual 1 (r1), the parameters that most affect its value are Q1, Q2, Ac, and the Pc1 measurement V1. So, thinking about failure modes, if there is a pressure nozzle obstruction (the nozzle that controls the flow Q1), the flow Q1 will be smaller and, depending on the obstruction level, the pressure Pc1 will be significantly smaller than it would be without the nozzle obstruction, thus, the residual r1 will not be zero anymore, if there is a pressure nozzle obstruction. The two last matrix rows relate the detectability and the isolation of the parameter fault. So, if at least in one residual row the true value appears, then it is detectable. For example, the parameter Q1 is detectable because, if it has a disturbance from the original value, the residue 1 would be affected. For one parameter fault isolation, the pattern of the residual appearance must be unique. For example, the parameter xc (spool position) is isolable because, if there is an abnormal condition and the spool position differs from its normal movement (in a hydraulic line leakage or increased spool friction), the residuals r2 and r3 would be modified and this residual appearance pattern is unique for the xc failure mode.

Table 2 – Servo valve fault signature matrix.

	Q1	Q2	xc	Ac	Ps	V1	R4	I5	C6	V2	V3	Rint	Q3	Q4
r1	1	1	0	1	1	1	0	0	0	0	0	0	0	0
r2	0	0	1	1	0	1	1	1	1	1	0	0	0	0
r3	0	0	0	1	0	1	0	0	0	1	1	1	0	0
r4	0	0	1	0	1	0	0	0	0	1	1	1	1	1
D	1	1	1	1	1	1	1	1	1	1	1	1	1	1
I	0	0	1	0	1	0	0	0	0	1	0	0	0	0

The Simulink model linking the behavior model and the diagnostic model, generating the states and the residuals is shown in Fig. 9. The simulations were made introducing some failure modes in the behavior system, making it possible to observe the residual response. The Fig. 10 shows the simulation results for 4 different faults. Pressure source fault, spool internal leakage, hydraulic line leakage, and position sensor measurement fault.

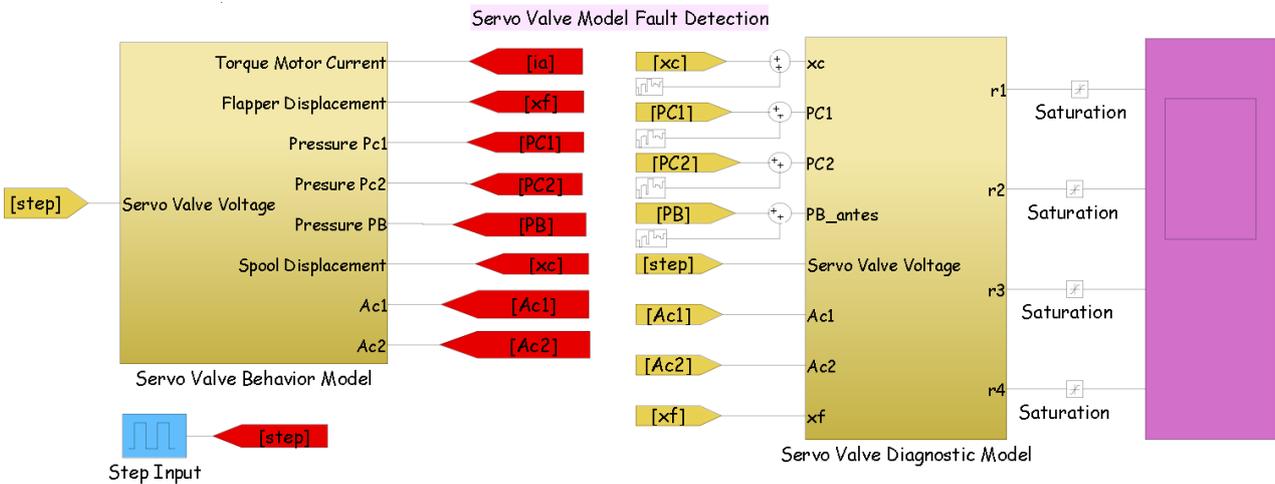


Figure 9 – Simulink for residual analyses.

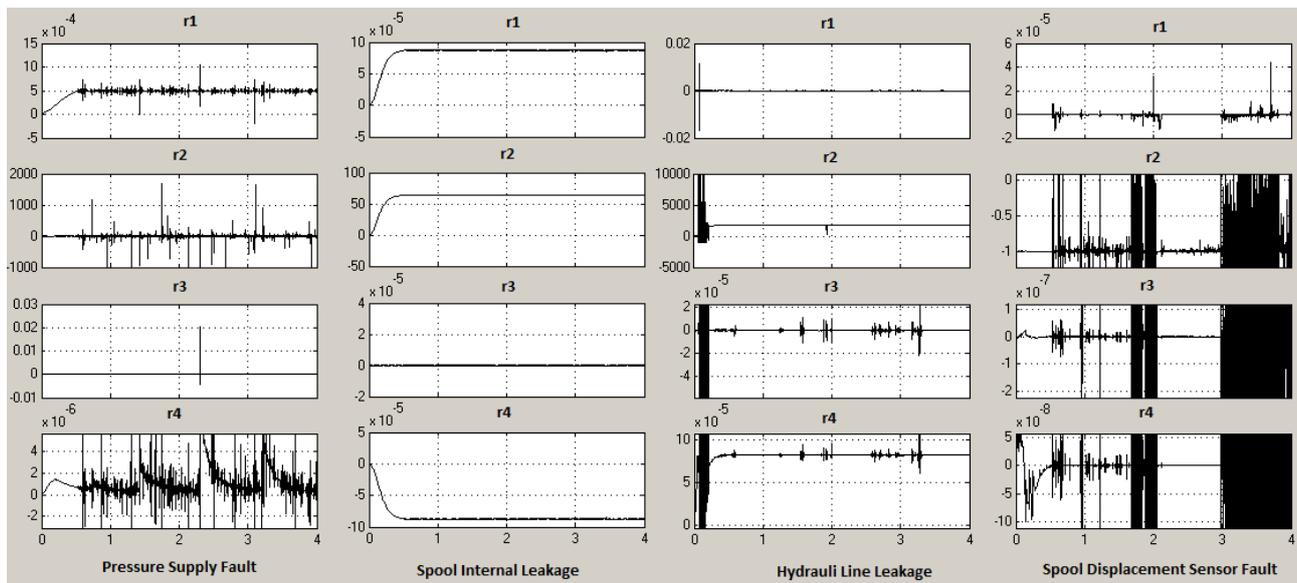


Figure 10 – Residual response for different failure modes.

With the simulations results, it is possible to see that the residual response reflects the fault signature matrix. For the pressure supply fault, the matrix indicates that the most sensitive residuals are r1 and r4. The Fig. 10 indicates variation on these two residuals, as expected. Analysing the spool internal leakage fault, the deviation occurs in more than one parameter. In this case, the most affected parameters are the spool displacement and the flows Q1, Q2 and Q3. Thus, based on the matrix, the residuals sensible to these faults are r1, r2 and r4 as confirmed by the simulation results in Fig. 10. The hydraulic line leakage directly affects the spool displacement and the flows Q3 and Q4. This parameters affects the residuals r2 and r4 as confirmed by the simulation. Introducing a measure fault in the displacement sensor, caused a deviation in the residue r2, as expected according to the matrix.

## CONCLUSION

The objective of this work was to develop a model for the aircraft braking dynamics and its hydraulic brake system, demonstrating, besides its normal operation behavior, what some common failure modes could cause on the system performance. The FDI method proposed presented good results, being able to detect all the faults introduced on the system, showing coherence with the fault signature matrix. Thus, this methodology is a viable solution, helping to support early failure identification.

## REFERENCES

- Borutzky W. Borutzky, *Bond Graph Model-based Fault Diagnosis of Hybrid Systems*, (2016). Springer Verlag
- Medjaher K. Medjaher, *A bond graph model-based fault detection and isolation*, Brenguer and L. Jackson. Maintenance Modelling and Applications. Chapter 6 : Fault Diagnostics., Det Norske Veritas (DNV), pp.503-512, 2011. hal-00635549
- Merritt Merritt, H. E., *Hydraulic Control Systems*, (1967). New York - London - Sydney: John Wiley and Sons
- Samantaray A.K. Samantaray, K. Medjaher, B. Ould Bouamama, M. Staroswiecki, Dauphin-Tanguy *Diagnostic bond graphs for online fault detection and isolation*, (2005). Department of Mechanical Engineering, Indian Institute of Technology, 721 302 Kharagpur, India