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METHODS FOR PERFORMANCE PREDICTING AND SIZING OF MULTISTAGE CENTRIFUGAL COMPRESSORS

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Abstract. *With the crescent number of modern fertilizer plants and hydrocracking process units in oil refineries, the consumption of energy and consequently the efficiency of centrifugal compressors was made of paramount importance in present day. A consequence of this new reality is the necessity of improvement of the methods that estimate the polytropic efficiency of these compressors. Furthermore, new methods also need to predict the basic geometric parameters of each machine. Therefore, new methods of estimation must be fast and reliable. On the other side, most of the literature, which deals with polytropic efficiencies, is not up to date with the values performed by modern compressors. The main scope of present work is to provide background information and present a comparison between two up to date methods, based on different approach, but capable to predict the performance and sizing of multistage centrifugal compressors. The proposed methods are Sandberg Method (2016) and Approximated Lüdtke Method (2004) with Corrections, proposed by Busaid (2016). Both these methods give good agreement with the values performed by several compressors, as demonstrated when these methods are adopted. In this paper, six real cases will be estimated by both methods and compared against supplier data; the result demonstrates good agreement among them.*

Keywords: *centrifugal compressors, polytropic efficiency, rotating machinery*

1. INTRODUCTION

The main objective of present work is to provide background information and present a comparison between two up to date methods, based on different approach, but capable to predict the performance and sizing of centrifugal compressors, for chemical and petrochemical industries.

Therefore, after an appropriate choice of the method, it will be possible to estimate:

- a) Speed of machine in a specified point of operation;
- b) Mean diameter of impellers;
- c) Number of impellers;
- d) Rough external diameter of casing;
- e) Polytropic efficiency;
- f) Brake-horsepower consumed by driven machine;
- g) Nominal power of driver machine.

2. BASIC CONFIGURATION AND CHARACTERISTIC PARAMETER OF CENTRIFUGAL COMPRESSORS

It will be discussed on the following sections the fundamental type of machine that this paper is related with, as well as, the main characteristic parameters of centrifugal compressors.

2.1 Basic configuration

The great majority of Centrifugal Compressors concerned to this paper, follows the Figure 1(a) configuration, which shows a realist view of the interior of a centrifugal compressor, the path of gas is depicted in Figure 1(b). This type of compressor, with several impellers, represents the family of machines that this paper deals with.

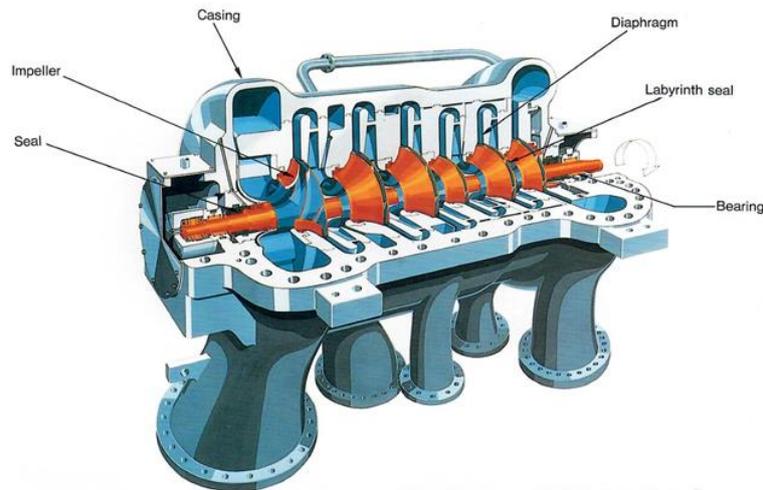


Figure 1(a) – Multistage Centrifugal Compressor (Mitsubishi Heavy Industries Catalog, 2009)

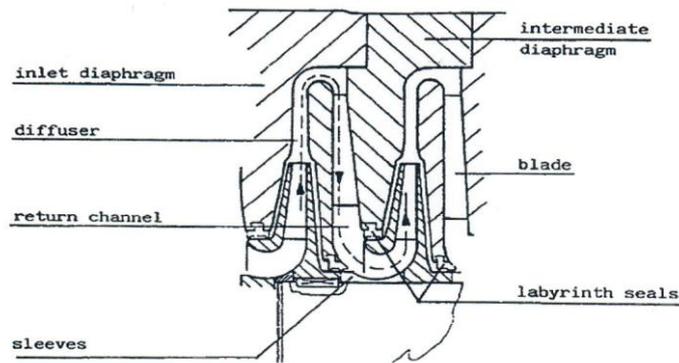


Figure 1(b) – The path of gas through the internal components of compressor (R. Salamat, 2012)

2.2-Characteristic parameters of centrifugal compressors

According to Michael Casey and Chris Robinson (2013), a great number of thermodynamic performance information of a centrifugal compressor may be calculated from the following relations:

$$\eta_p = f(\varphi, M_u) \quad (1)$$

$$\tau = g(\varphi, M_u) \quad (2)$$

Where: η_p is the expected design point polytropic efficiency, defined by Eq. (5), τ is the stage work input coefficient defined by Eq. (11), φ is the global volume flow coefficient defined by Eq. (6) and M_u is the stage tip Mach number defined by Eq. (18). Relations (f) and (g) may be figure out from the formulas of sections 3.1,3.2 and 3.4 , described later.

Nevertheless, the most important dimensionless parameter of a centrifugal compressor is the flow coefficient (φ), as will be demonstrated, since the polytropic efficiency (η_p) is a function of it. Therefore, is essential to know how this coefficient change with the type of impeller, as will be described in the following sections.

2.2.1 The trend of polytropic efficiency (η_p)

According to Figure 2, it will be possible to infer that the expected polytropic efficiency for present decade will oscillate around 87%.

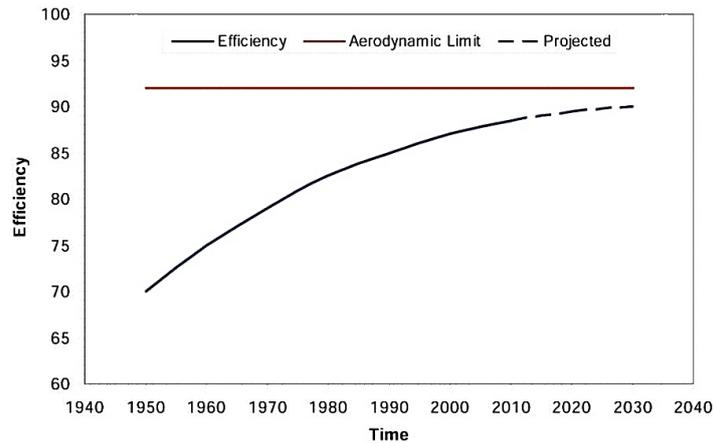


Figure 2 – Polytropic efficiency trend (Sorokes, 2010)

2.2.2 The variation of flow coefficient (ϕ) with the type of impeller

The flow coefficient (ϕ) determines how great an actual volume flow is achieved by an impeller of a given diameter, rotating at a given tip speed. The variation of this parameter and consequently polytropic efficiency (η_p) depends on the following factors:

- Volumetric flow through impeller, implying in low flow coefficients or high flow coefficients;
- Direction of flow, such as radial impellers, mixed flow impellers or axial impellers;
- Conception of impeller geometry, 2D or 3D;
- Type of impellers, such as open or shrouded.

A consequence of the increase of flow coefficient through impellers is evident from the graph of Figure 3, where is showed that if the process demands higher flow coefficient, the characteristic of the impeller must change from radial to mixed flow type, increasing more the coefficient, an axial type is necessary. The present paper focuses mainly the radial type of impellers, with few cases of mixed flow impellers. Since the flow coefficient is a function of volume flow rate, see Eq. (6), this graph indicates indirectly the variation of the intake volume flow rate with the type of impeller, as well as, the variation of the polytropic efficiency (η_p) and polytropic head coefficient Eq. (9), as a function of flow coefficient.

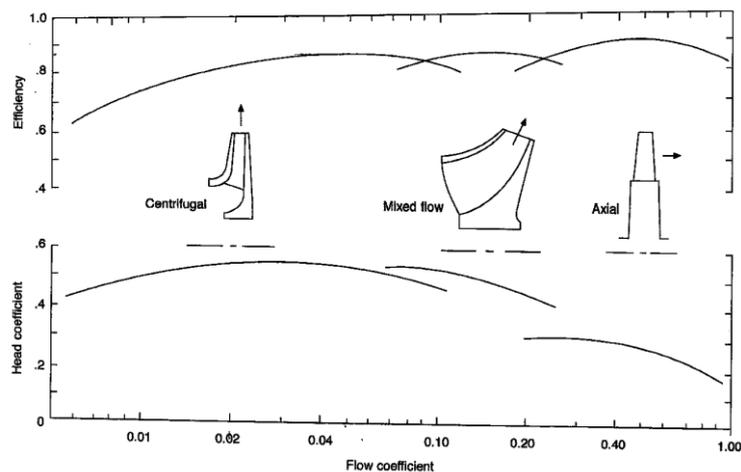


Figure 3 – The flow coefficient, polytropic efficiency and polytropic head coefficient (Gresh, 2001)

With the evolution of milling machines, new blades profiles were possible to be achieved, with impressive increase of polytropic efficiency per stage. Figure 4 and Figure 5, demonstrate this increase in efficiency, for flow coefficients greater than 0,04, mainly for impellers with 3D conception. With reference to Fig. 4, it is necessary to clarify that “Normalized Stage Efficiency” is the ratio of polytropic efficiency, for each flow coefficient, to maximum polytropic efficiency in the range of flow coefficients for these kind of impellers.

Figures 6(a), (b) and (c) show the classical types of impellers that this paper may embrace, therefore, the great majority of the impellers, object of this study, are shrouded type impellers.

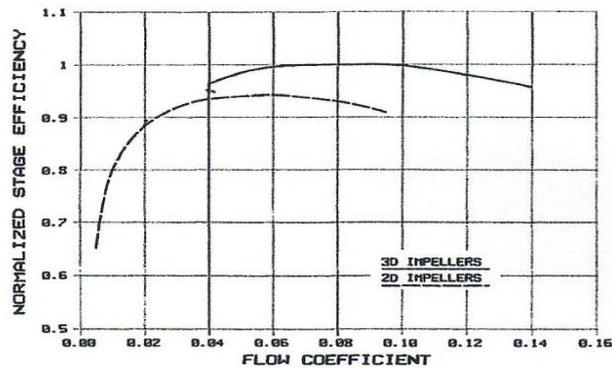


Figure 4 – Comparison of 3D vs 2D Impeller Stage Design Point Efficiency vs Flow Coefficient (Marshall and Cotroneo, 1989)

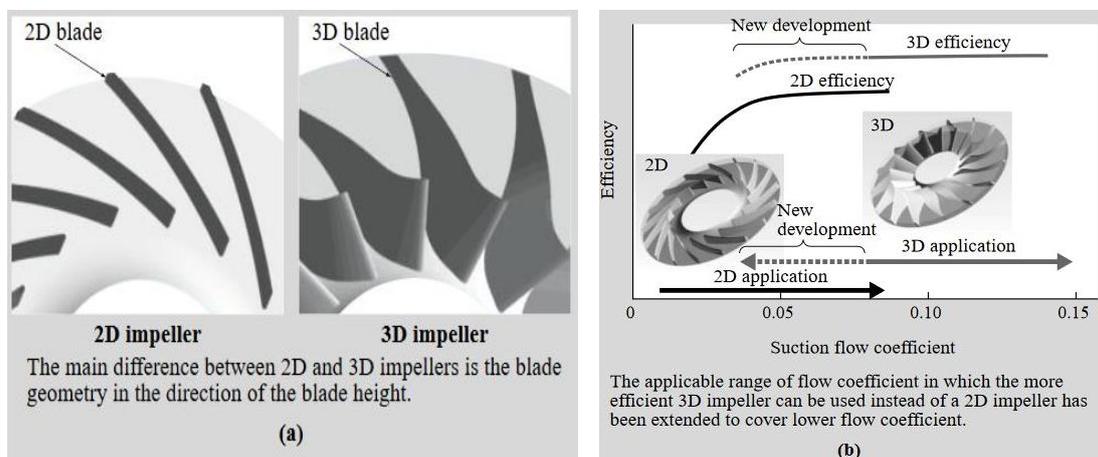


Figure 5 – Blade Geometry and Applicable Range of 2D and 3D Impellers. Comparisons of the blade geometries (a) and applicable ranges (b) of 2D and 3D impellers (Fukushima, Shibata et al, 2009)

Another fact is concerned with the most usual impeller, evolved by the paper, that is the so-called semi-3D impeller, whose geometry stays between 2D and 3D conception see Fig.6(b). Also, may be considered here the radial characteristic of the flow through these impellers. For a realistic view of a semi-3D impeller, see Fig.6(d).

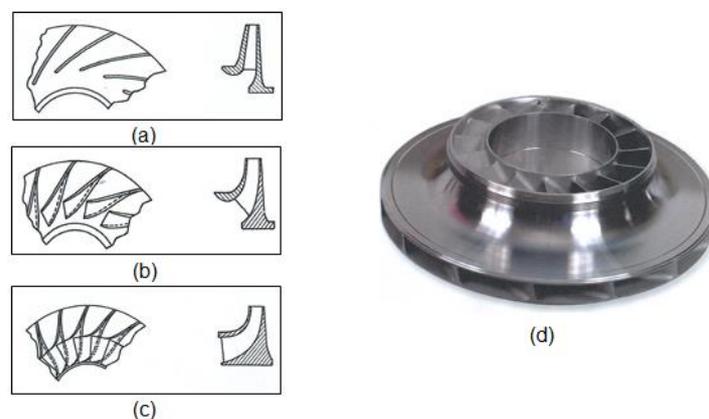


Figure 6 : (a) 2D Impeller, (b) Semi 3D Impeller, (c) Full 3D Impeller (Marshall and Cotroneo, 1989), (d) Realistic view of a semi-3D impeller (Elliott Catalog, 2009)

3. THEORETICAL BACKGROUND

This section presents the main thermodynamics equations dealing with centrifugal compressors, as well as, basic relations necessary to understand both methods and their differences.

3.1 Common Thermodynamics and Aerodynamics Equations

Sandberg Method (2016), Lüdtke Method (2004) and a great majority of methods, such as N-Method, described by Gresh (2001); Delaval Method, described by Bloch (2006); Elliott Catalog Selection Guide Method, from Elliott Company (1996) and all other methods share common thermodynamics equations, as stated bellow.

One of the most fundamental equations used to describe and design a compression section, for an application, is related to the specific work imparted to the gas. This equation was deduced from the second law of thermodynamics and some relations involved in a polytropic process for real gases.

$$H_p = 8,31446 \frac{Z_m T_s}{MW} \frac{n}{(n-1)} \left[\left(\frac{P_d}{P_s} \right)^{\frac{n-1}{n}} - 1 \right] \quad (3)$$

Where: H_p is the specific work imparted to the gas, or Polytropic Head (kJ/kg); Z_m is the mean compressibility factor of the gas; T_s is the suction gas temperature (K); MW is the molecular weight (kg/kmol); n is the gas polytropic exponent; P_d is the discharge pressure (bar a); P_s is the suction pressure (bar a).

To evaluate the polytropic exponent of gas, it will be employed the simplified equation below:

$$\frac{n-1}{n} = \frac{k-1}{k\eta_p} \quad (4)$$

Where: k is the mean isentropic exponent and η_p is the polytropic efficiency.

The polytropic efficiency may be expressed by:

$$\eta_p = \frac{H_p}{(h_d - h_s)} \quad (5)$$

Where: h_d is the discharge gas enthalpy (kJ/kg) and h_s is the suction gas enthalpy (kJ/kg).

As defined in section 2.2.2, it is necessary to consider here the formula for flow coefficient (φ), since most of variables are related to it and it will be expressed by:

$$\varphi = \frac{4\dot{V}_s}{\pi U_{tip} D^2} \quad (6)$$

Where: φ is the flow coefficient, \dot{V}_s is the volume flow rate (m³/s) at suction of impeller, U_{tip} is the tangential velocity at impeller tip (m/s) and D is the diameter of impeller (m).

After the values of polytropic head and efficiency are obtained, the gas power absorbed by the compression process can be calculated by the following expression:

$$P_i = \frac{\dot{m} H_p}{\eta_p} \quad (7)$$

Where: P_i is the gas power (kW) and \dot{m} is the mass flow (kg/s) plus leakage trough labyrinths and seals.

Another parameter necessary for properly sizing the compressor is the polytropic head per stage, however there are a technical limitation in its value, approximately 40 kJ/kg.

$$h_p = \frac{H_p}{n_{stg}} \quad (8)$$

Where: h_p is the polytropic head per stage (kJ/kg) and n_{stg} is the number of stages.

3.2 Relations, based on the theory of aerothermodynamics for rotary machines.

The parameter that links thermodynamics with the rotary machinery, deduced from the velocity triangles, is the so-called polytropic head coefficient (μ_p), Sandberg (2016).

$$\mu_p = \frac{1000h_p}{U_{tip}^2} \quad (9)$$

For only one stage, according to Eq. (8), (H_p) becomes the polytropic head per stage (h_p) and shall be expressed by:

$$h_p = \frac{\mu_p U_{tip}^2}{1000} \quad (10)$$

To conclude all the fundamental equations, that relates energy imparted to the compressor with the mechanical of rotation, it is necessary to introduce the work input coefficient per stage, (Sandberg, 2016):

$$\tau = \frac{1000(h_d - h_s)}{U_{tip}^2} \quad (11)$$

Finally, considering Eq. (5), Eq.(8) and only one stage, an additional relation may be deduced from Eq.(9) and Eq.(11),resulting in Eq.(12),that express the polytropic head coefficient as a function of polytropic efficiency and work input coefficient.

$$\mu_p = \eta_p \tau \quad (12)$$

3.3 Sandberg Method Equations

Sandberg Method (2016) demands some specific equations, which are listed below. From similitude theory, the impeller diameter may be expressed as:

$$D = \frac{0,17783d_s \sqrt{\dot{V}_s}}{\sqrt[4]{\frac{H_p}{n_{stg}}}} \quad (13)$$

Where: d_s is the specific diameter.

The impeller speed, based in the same theory, is:

$$N = \frac{1698,13102n_s \left(\frac{H_p}{n_{stg}}\right)^{0,75}}{\sqrt{\dot{V}_s}} \quad (14)$$

Where: N is the impeller speed, expressed in (rpm) and n_s is the specific speed.

For relation of polytropic head coefficient with specific diameter and specific speed, Sandberg Method (2016) uses Eq. (15), based again in similitude theory:

$$\sqrt{\mu_p} = \frac{2}{n_s d_s} \quad (15)$$

According to Sandberg Method (2016), the flow coefficient (φ) may be expressed as a function of volume flow ratio, impeller diameter and impeller speed:

$$\varphi = \frac{24,31728 \dot{V}_s}{ND^3} \quad (16)$$

3.4 Approximate Lüdtke Method Equations

Approximate Lüdtke Method (2004) demands some specific equations that are listed below. From basic rotary machine theory:

$$N = \frac{60U_{tip}}{\pi D} \quad (17)$$

From the definition of tip speed Mach number for compressors:

$$M_u = \frac{U_{tip}}{\sqrt{1000k_s Z_s \frac{8,31446}{MW} T_s}} \quad (18)$$

Where: M_u is the compressor Mach number, k_s is the isentropic exponent of gas at compressor suction and Z_s is the compressibility factor of gas at compressor suction.

For the evaluation of the head achieved by stage (h_p), Approximate Lüdtke Method (2004) uses the basic Eq. (10) and to evaluate the polytropic head coefficient (μ_p), Eq. (12) is modified by an average value, adopted for work input coefficient:

$$\tau_{avg} \approx 0,63 \quad (19)$$

Therefore, Eq. (12) is modified to:

$$\mu_{avg} = 0,63\eta_p \quad (20)$$

Furthermore, Approximate Lüdtke Method (2004) is based in the evaluation of three flow coefficients, for suction, discharge and medium section of compression:

From relations among pressure, volume and polytropic exponents applied in flow coefficient formula, it is possible to deduct the formula for discharge flow coefficient φ_d :

$$\varphi_d = \varphi_s \left(\frac{P_d}{P_s} \right)^{\frac{n-1}{n}-1} \quad (21)$$

Where: φ_s is the suction flow coefficient.

The medium flow coefficient (φ_m) is defined according to Eq. (22).

$$\varphi_m = \frac{\varphi_d + \varphi_s}{2} \quad (22)$$

Approximate Lüdtke Method (2004) also demands three polytropic efficiencies, for suction, discharge and medium section of compression, these efficiencies correspond to the three flow coefficients defined above. According to this method, the basic polytropic efficiency is defined, as stated below:

$$\eta_{pb} = \frac{\eta_{ps} + \eta_{pm} + \eta_{pd}}{3} \quad (23)$$

Where: η_{pb} is the basic polytropic efficiency, η_{ps} is the suction polytropic efficiency, η_{pm} is the medium polytropic efficiency and η_{pd} is the discharge polytropic efficiency.

Recently, W. Al-Busaid and Pericles Pilids (2016) suggest the correction of the basic polytropic efficiency (η_{pb}), according to the following equation:

$$\eta_p = \eta_{pb} + \Delta\eta_{sc} + \Delta\eta_{DD} + \Delta\eta_{ic} + \Delta\eta_{Mu} + \Delta\eta_{Re} \quad (24)$$

Where; η_p is the final polytropic efficiency, η_{pb} is the basic polytropic efficiency, $\Delta\eta_{sc}$ is the size effect correction, $\Delta\eta_{DD}$ is the diffuser diameter ratio effect correction, $\Delta\eta_{ic}$ is the inlet loss correction (adopted -0,01), $\Delta\eta_{Mu}$ is the Mach number correction, $\Delta\eta_{Re}$ is the Reynolds number correction (adopted 0,015 as an approximated value).

4. METHODOLOGY

In this section will be described the main differences between the two methods.

4.1 Method of Sandberg

Sandberg Method (2016) is based in Specific Speed and Specific Diameter parameters, these concepts was developed by Balje (1981), that demonstrates through a graph, Fig.7, that exists relations among polytropic efficiencies, specific speed, specific diameters and head coefficient, for rotating machines. Therefore, starting with an assumed value of flow coefficient, it is possible to determine a value for a dimensionless specific diameter and a value for dimensionless specific speed, based in curves developed by Cordier, Aungier and Casey.

Specific speed and specific diameters may also be expressed by formulas, stated in section 3.3, which relates speed, diameter, and volume flow rate and polytropic head per stage.

Based in these relations, curves and additional equations, it is possible to evaluate the main parameters of a centrifugal compressor.

To evaluate the gas power and other fundamental parameters, related to compressor, Sandberg Method (2016) provides the graph of flow coefficient versus polytropic efficiency, depicted in Fig.8, with curves developed by Aungier and actual case data of several compressors.

The flowchart, depicted in Fig.12, demonstrates how these calculations are made to achieve the final parameters of a centrifugal compressor.

As will be preliminary verified during the calculation of cases, Sandberg Method (2016) is more applicable during the conception phase, when the compressor need to be calculated for the basic engineering, in this scenario all the geometric characteristics of compressor are unknown. Therefore, this method must embrace a broad range and type of centrifugal compressors, including those with open type impellers and high flow coefficients. Because of this philosophy, deviations in some parameters may occur.

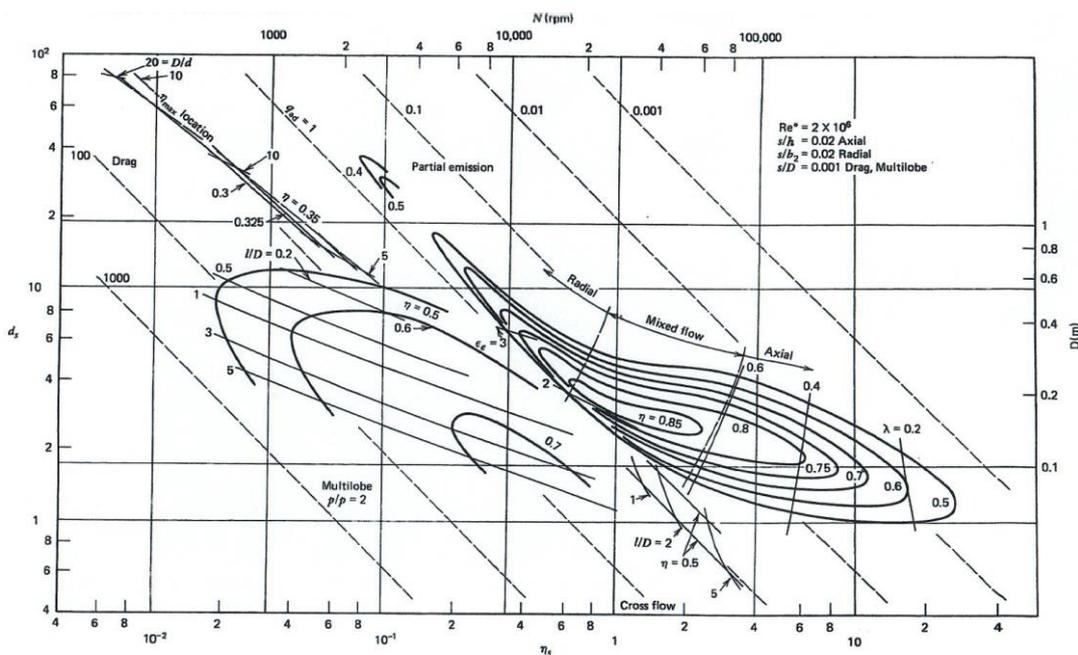


Figure 7 – Adoption of $n_s d_s$ diagram to problem statements (Balje 1981)

4.1.1 Flow Coefficient versus Polytropic Efficiency

After to establish the values of d_s , D , n_s and N , whose graphs will be described in the next sections, as well as, to assume an initial value of flow coefficient to start the method, it is necessary to determine or confirm the polytropic efficiency for close the main calculations, related to the sizing of compressors.

From the relations proposed by Aungier (1995, 2000), showed in Fig.8, it is possible to determine the polytropic efficiency as a function of flow coefficient. Analyzing this graph, where is also plotted several curves and real case data from equipment supplier selections, it is evident that more scatter points appear as compared to the other parameters. However, this fact does not affect significantly the impeller sizing or speed, since the influence in the value of polytropic head is limited. Therefore, the main parameter that is directly impacted by this variation is the gas power. To cover the great majority of the case data, a curve has been plotted, which corresponds to 95% Aungier average efficiency.

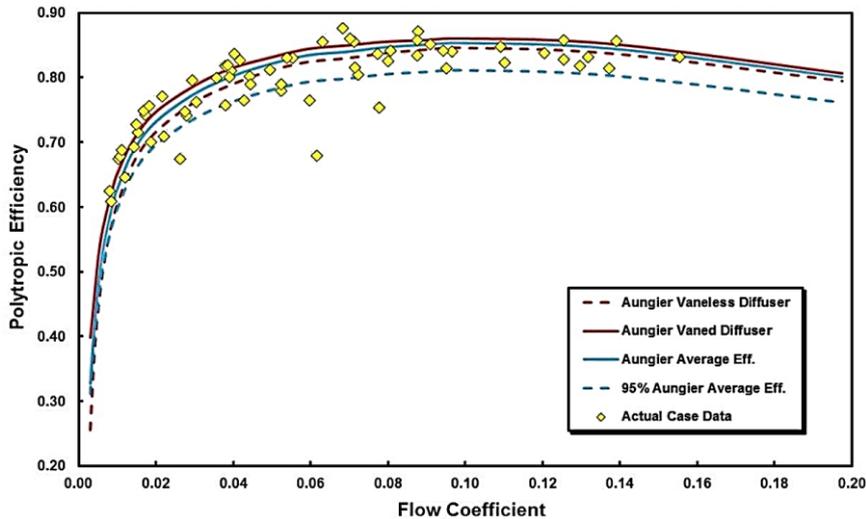


Figure 8 – Flow Coefficient versus Polytropic Efficiency (Sandberg, 2016)

4.1.2 Flow Coefficient versus Dimensionless Specific Diameter

The first of correlations involving similarity parameters is depicted in Fig.9, which shows specific diameter as a function of the inlet flow coefficient. The figure includes plots derived from the relations presented by Cordier (1955), Casey et. al (2010), as well as, developed from Aungier (1995, 2000) equations, for both vaneless and vaned diffusers. Real equipment supplier selection data is also depicted in the graph, with yellow diamond marks. The close agreement among the curves is remarkable.

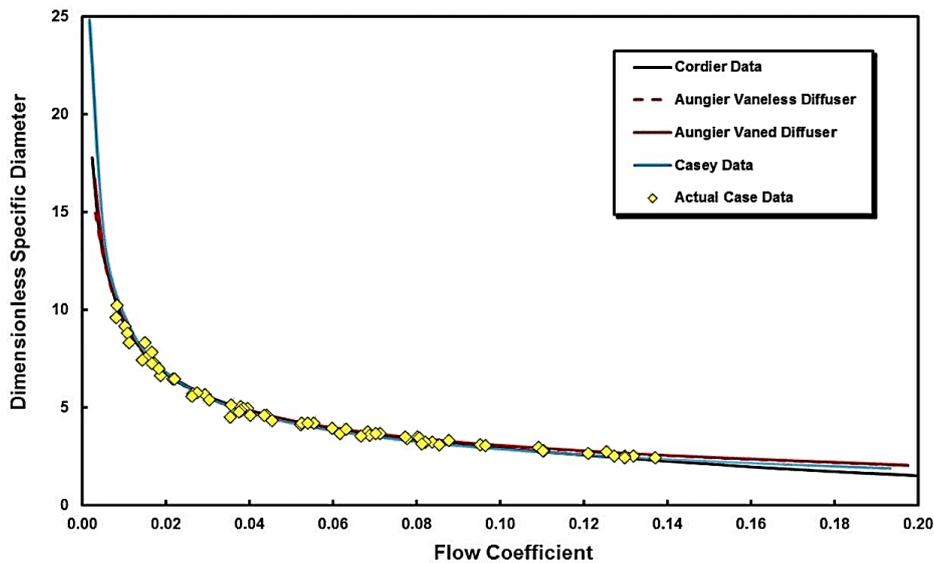


Figure 9 – Dimensionless Specific Diameter versus Flow Coefficient (Sandberg, 2016)

4.1.3 Specific Diameter versus Specific Speed

Another important relation between specific speed and specific diameter was developed by Cordier (1955) with equations proposed by Aungier (1995, 2000) and Casey et. al (2010). There is also a good accuracy of these curves, when compared to real case data from equipment supplier selections that are included in Fig.10.

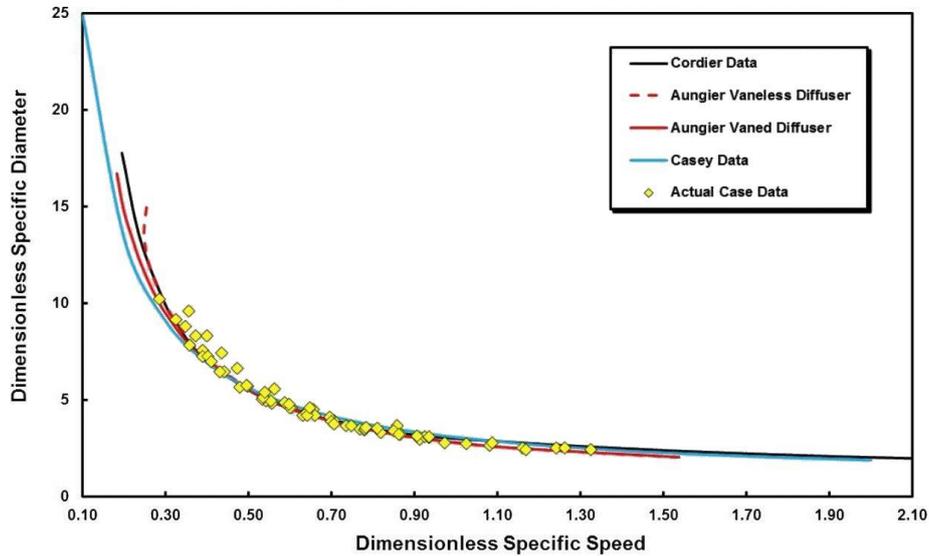


Figure 10 – Dimensionless Specific Diameter versus Specific Speed (Sandberg,2016)

4.1.4 Flow Coefficient versus Polytropic Head Coefficient

The graph of Fig.11 was developed by Aungier (1995, 2000), even though it appears a little inaccurate for low flow coefficient, their ability to predict design performance characteristics allows their use in preliminary sizing and design of a compressor section. In this figure are also plotted curves for vaned and vaneless diffusers.

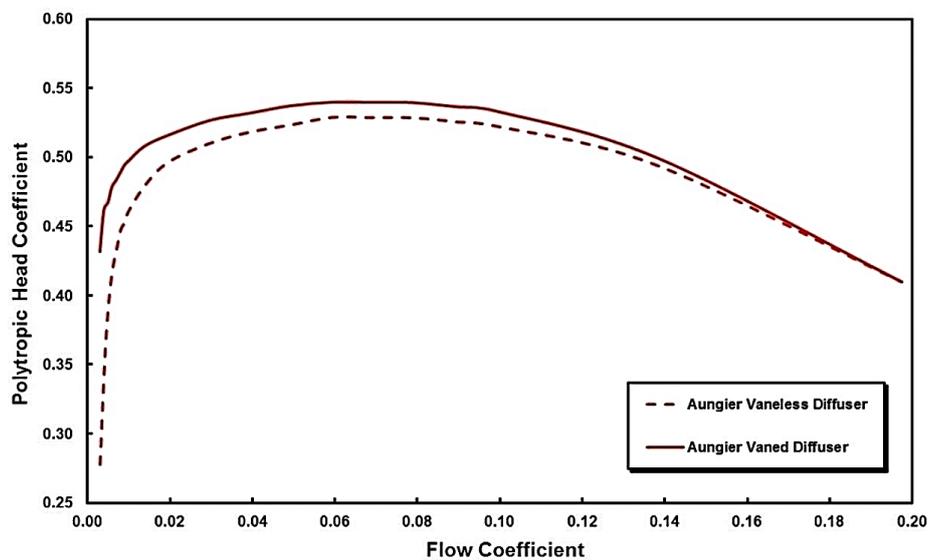


Figure 11 – Flow Coefficient versus Polytropic Head Coefficient (Sandberg, 2016)

4.1.5 Flow Chart of Sandberg Method

Below, in Fig.12, is summarized the procedure for compressor sizing, based in Sandberg Method (2016).

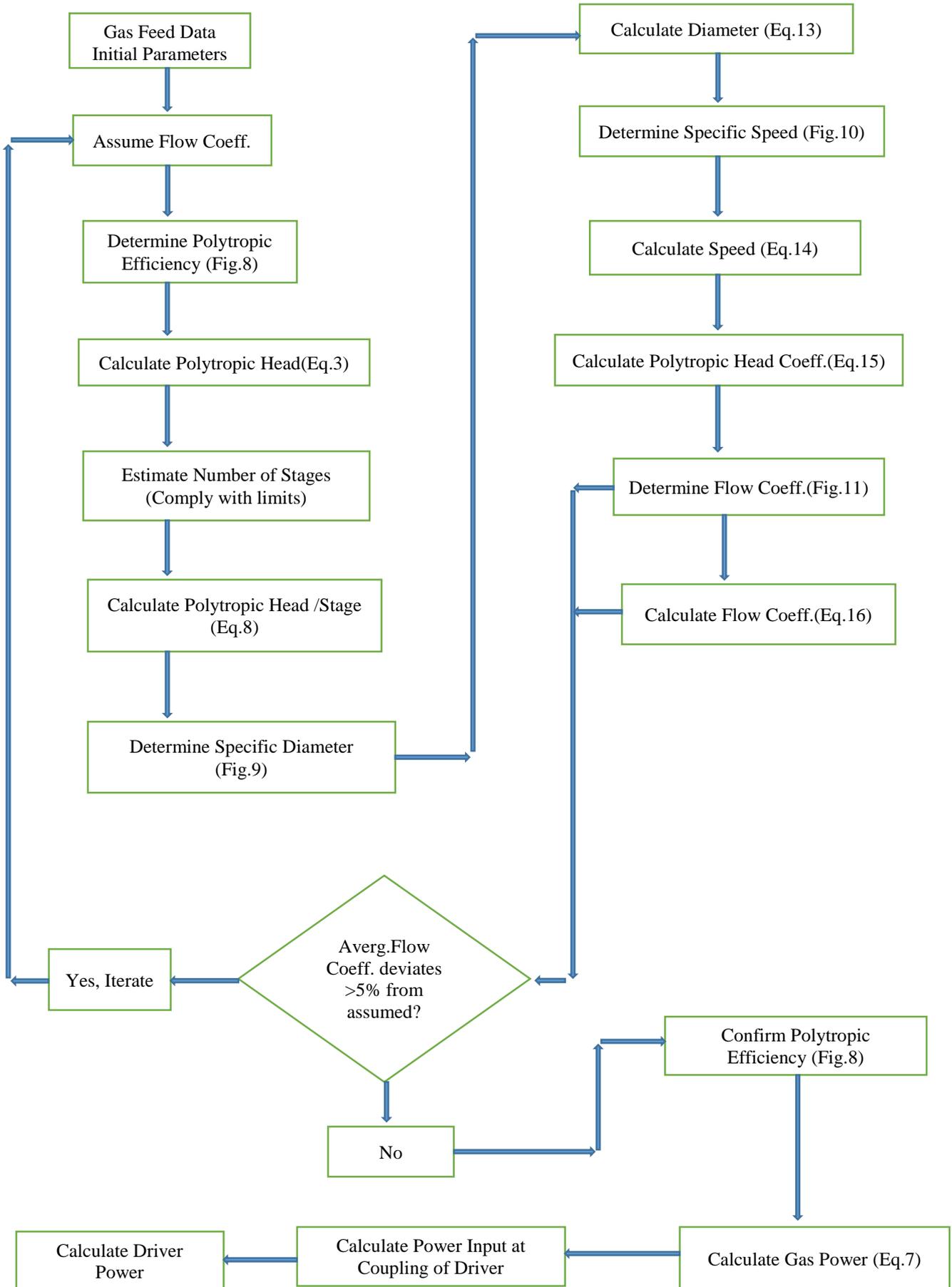


Figure 12 - Flow Chart of Sandberg Method

For the first step, that is the gas feed data initial parameters, it is necessary to inform the following input data:

- a) Type of gas and molecular weight;
- b) Suction gas temperature;
- c) Mean isentropic exponent;
- d) Mean compressibility factor of the gas;
- e) The suction gas pressure;
- f) The discharge gas pressure;
- g) Volume flow rate or mass flow rate

Considering item (g) above, the conversion of mass flow rate to volume flow rate must be done by the choice of the most adequate equation of state for the gas in compression process.

4.2 Correct Method of Lüdtke

One of the most reliable methods, provide by literature, for performance prediction of a multistage centrifugal compressor, with shrouded impellers, is the approximate method of Lüdtke (K. H. Lüdtke, 2004) with corrections (Busaid, 2016).

Assuming an impeller tip speed and a flow coefficient as start point, it is possible to calculate all other parameters of compressor. However, there must exist a relation among compressor suction volume, speed and frame size (diameter), as depicted in diagram of Fig.13. The flowchart, depicted in Fig.18, demonstrates how this calculation is made to achieve the final parameters of a centrifugal compressor. It need to be estimated three flow coefficients, corresponding to suction section, medium section and discharge section. These three coefficients will determine three polytropic efficiencies, provide by the graph of Fig.14. The basic polytropic efficiency is the arithmetic mean of the three values.

However, the basic efficiency must be corrected to achieve a more realistic efficiency, in a recent work W. Al-Busaid and P. Pilidis (2016) presented five corrections factors to polytropic efficiency: size effect correction, inlet loss correction, diffuser diameter ratio correction, Mach number correction and Reynolds Number correction. The fundamental reason for all these corrections is related to the Lüdtke (2004) curve of Fig.14, since it was determined, according to recorded values of a certain compressor, with fixed geometry. However even considering these limitations, the Approximate Method of Lüdtke (2004), with corrections of Busaid (2016), provides good results, mainly in the range 0,03 to 0,12 of flow coefficients. This will be confirmed by tabulation of results, related to the data of six actual cases, as described in section 5.

4.2.1 Equivalence of Approximate Lüdtke Method Nomenclature

Comparing some parameters of Fig.14 and the formulas of sections 3.2 and 3.4, it may be realized that is necessary to establish some relations of equivalence, due to the nomenclature adopted by Ludtke (2004), since the method is based on the curves of this graph. Nevertheless, these changes do not affect the results nor the physical meaning of the main parameters.

As follow, the approximate Method of Lüdtke (2004) consider:

$$\psi_p = 2\mu_p \quad (25)$$

Where: ψ_p is the polytropic head coefficient. Also, for the nomenclature of work input coefficient (τ), Lüdtke (2004) adopts (s), therefore:

$$s = \tau \quad (26)$$

Considering Eq. (12), Eq. (25) and Eq. (26), results in Eq. (27), which relates polytropic efficiency, work input coefficient and polytropic head coefficient, therefore:

$$\psi_p = 2\eta_p s \quad (27)$$

The previous Eq. (10) will be modified by Eq. (25), resulting in the following formula for polytropic head per stage:

$$h_p = \frac{\psi_p U_{ip}^2}{2000} \quad (28)$$

Considering Eq. (19) and Eq. (26), the average value for work input coefficient results in:

$$s_{avg} = 0,63 \quad (29)$$

From Eq. (27) and Eq. (29), the average value of polytropic head coefficient shall be:

$$\psi_{avg} = 2\eta_p 0,63 \quad (30)$$

Finally, from Eq. (28) and Eq. (30), it is possible to evaluate the average value for polytropic head per stage:

$$h_{avg} = \frac{\psi_{avg} U_{tip}^2}{2000} \quad (31)$$

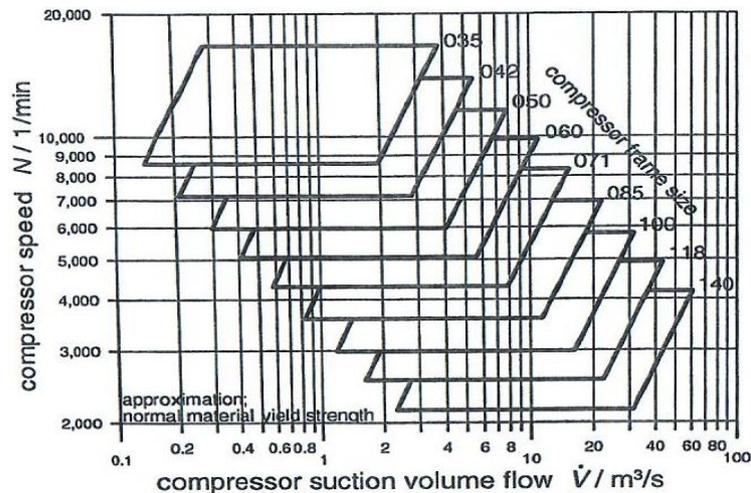


Figure 13 – Compressor range chart for single-flow horizontally split casings (Lüdtke, 2004)

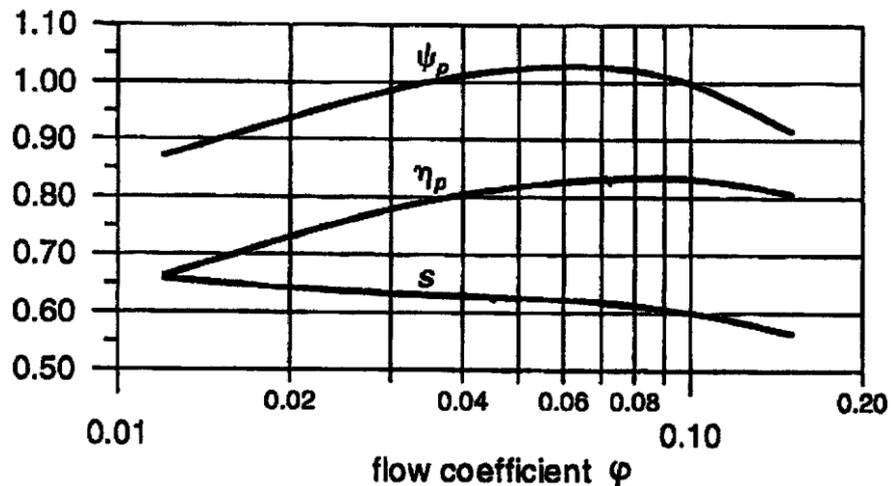


Figure 14 – Flow coefficient versus polytropic efficiency (Lüdtke, 2004)

4.2.2 Polytropic Efficiency Corrections

As mentioned in section 4.2, the curves of Figure 14 were plotted by Lüdtke (2004), based in a real prototype of a centrifugal compressor with fixed geometry. As result of tests performed in this machine, some deviations due to different geometries of compressors were to be expected, as compared to the original prototype machine.

To correct these minor distortions Al. Busaid and Pilids (2016) proposed some corrections in the basic efficiency of Lüdtke (2004), as demonstrated in Eq. (24), whose coefficients $\Delta\eta_{sc}$, $\Delta\eta_{DD}$ and $\Delta\eta_{Mu}$ are shown in curves of Fig.15, Fig.16 and Fig.17 below. For $\Delta\eta_{ic}$, the inlet loss correction, was adopted -0,01 and for $\Delta\eta_{Re}$, the Reynolds number correction, was adopted 0,015, as approximate values.

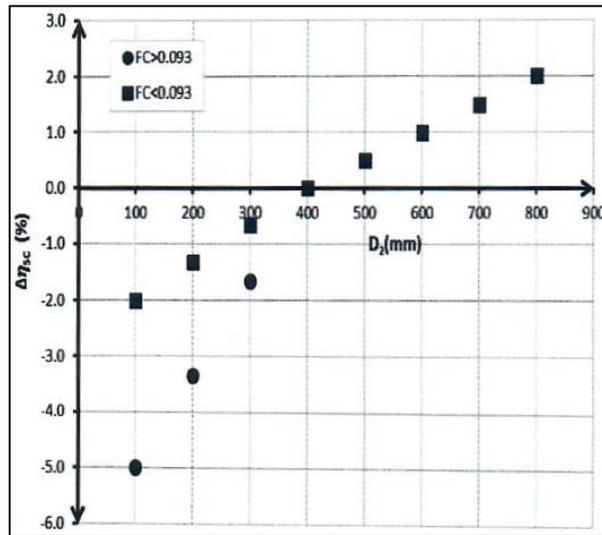


Figure 15 – Size effect correction, (W. A. Busaid and P. Pilids, 2016)

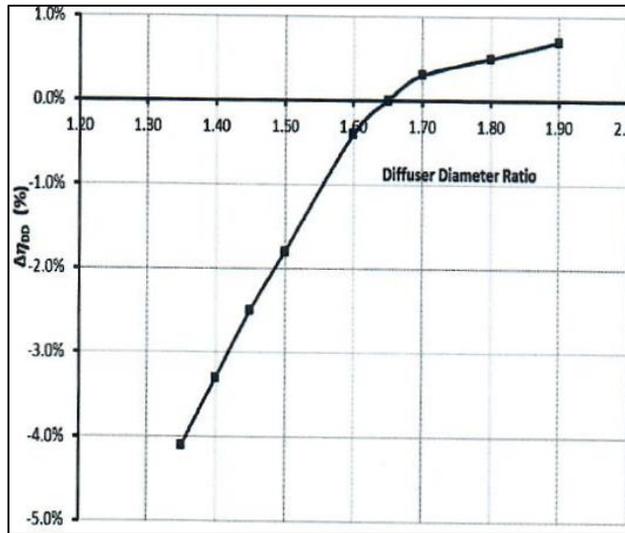


Figure 16 – Diffuser diameter ratio effect correction, (W. A. Busaid and P. Pilids, 2016)

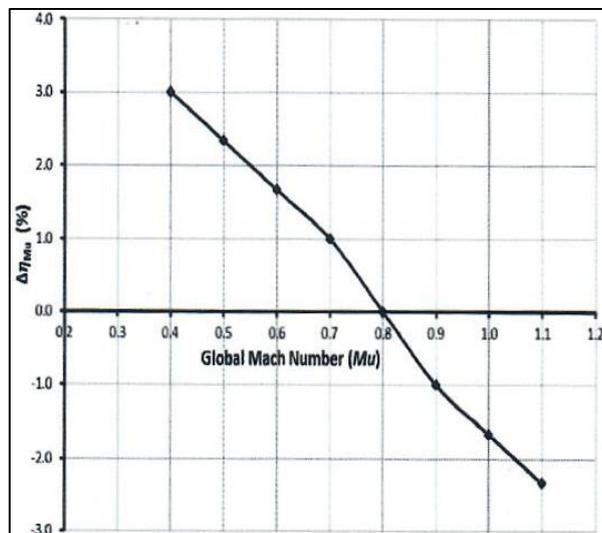


Figure 17 – Mach number correction (W. A. Busaid and P. Pilids, 2016)

4.2.3 Flow Chart of Approximate Lüdtke Method with Corrections

Below is summarized the procedure for compressor sizing based in Approximate Lüdtke Method with Corrections. For the first step, that is the gas feed data initial parameters, it is necessary to inform the same parameters of Sandberg Method, mentioned above.

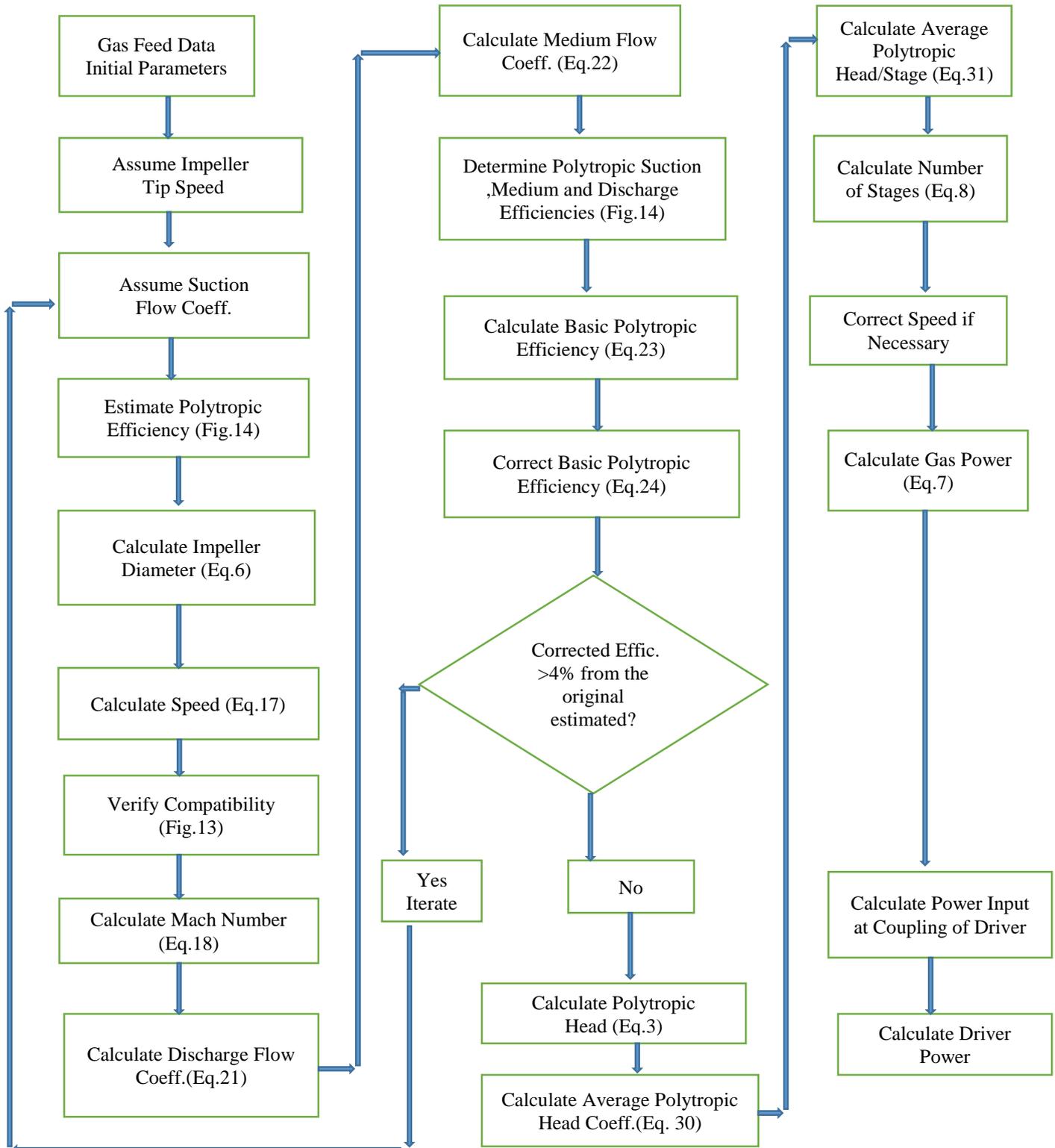


Figure 18 – Flow Chart of Approximate Lüdtke Method with Corrections

5. RESULTS OF SANDBERG AND APPROXIMATE LÜDTKE METHOD WITH CORRECTIONS APPLIED TO REAL CASES

It will be discussed here six cases of centrifugal compressors. These machines were tested according to ASME PTC10, except case 3. The data related to them is depicted on Table 1.

Table 1. Gas Feed Data Initial Parameters for Six Cases

Variable	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Mass flow (\dot{m}) [kg/h]	115881	164660	66383	98754	33660	72846
Molecular Weight (MW) [kg/kmol]	42,13	28,7	2,53	8,62	4,34	17,03
Isentropic mean coefficient (k_m) [-]	1,279	1,398	1,534	1,42	1,386	1,290
Mean compressibility factor (Z_m) [-]	0,9936	1,0	1,0591	1,0215	1,0135	0,9587
Suction pressure (P_s) [bar (a)]	1,471	2,667	110,06	31,96	28,333	3,833
Discharge pressure (P_d) [bar (a)]	4,677	8,069	130,45	71,03	47,255	8,029
Suction temperature (T_s) [K]	313	313	331	274,8	321	286,3

The objective of case 1 is to predict the performance of a carbon dioxide (CO₂) gas compressor; the calculation is related to section 1 of compression, operating in normal condition. This compressor was supplied to Petrobras UFN3 Fertilizer Plant.

For case 1, the following data were evaluated for both methods and depicted in Table 2.

Table 2. Predicted Data of Case 1

Compressor Parameters	Sandberg Method	Approx. Lüdtke Method w/Correct.	Supplier Actual Data
Suction Volume Flow (m ³ /s)	13,434	13,434	13,438
Polytropic Head (kJ/kg)	82,978	82,996	82,870
Flow Coefficient (First Impeller)	0,11	0,115	0,113
Polytropic efficiency (%)	83	82,9	82,7
Number of Stages	2	2	2
Mean Diameter (mm)	719	725	725
Speed (rpm)	7574	7462	7462
Power Input at Coupling of Driver (BkW)	3353	3386	3251

After an analysis of results, it is possible to conclude that the two methods are in good agreement with supplier actual data. The deviation in Sandberg Method (2016) data, related to diameter and speed, is justified by a balance between them. The deviation between Sandberg Method (2016) and Approximate Lüdtke Method with Corrections (2016), related to power input at coupling, is justified by the margin adopted by supplier.

The objective of case 2 is to predict the performance of a process air compressor; the calculation is related to section 2 of compression, operating in normal condition. This compressor was supplied to Petrobras UFN3 Fertilizer Plant.

For case 2, the following data were estimated for both methods and depicted in Table 3.

Table 3. Predicted Data of Case 2

Compressor Parameters	Sandberg Method	Approx. Lüdtke Method w/Correct.	Supplier Actual Data
Suction Volume Flow (m ³ /s)	15,536	15,536	15,56
Polytropic Head (kJ/kg)	121,522	121,412	121,41
Flow Coefficient (First Impeller)	0,081	0,082	0,082
Polytropic efficiency (%)	85	85,4	85,4
Number of Stages	4	4	4
Mean Diameter (mm)	986	999	1000
Speed (rpm)	4572	4595	4595
Power Input at Coupling of Driver (BkW)	6698	6663	6500

After an analysis of results, it is possible to conclude that the two methods are in good agreement with supplier actual data. As observed in the case 1, again the margin adopted by supplier for the power input at coupling, is smaller than the adopted by both methods.

The objective of case 3 is to predict the performance of a hydrogen (H₂) recycle gas compressor, propose by a supplier, operating in a condition that was chosen to design the compressor. This compressor was originally calculated by supplier to Petrobras HDT Diesel REPLAN refinery, however due to other technical reasons this compressor was not tested according to ASME PTC10.

For Case 3, the following data were estimated for both methods and depicted in Table 4.

Table 4. Predicted Data of Case 3

Compressor Parameters	Sandberg Method	Approx.Lüdtke Method w/Correct.	Supplier Actual Data
Suction Volume Flow (m ³ /s)	1,921	1,921	1,972
Polytropic Head (kJ/kg)	202,994	202,699	203,36
Flow Coefficient (First Impeller)	0,05	0,0575	0,059
Polytropic efficiency (%)	82,5	85,5	85,7
Number of Stages	6	6	6
Mean Diameter (mm)	419	414	414
Speed (rpm)	11344	11453	11453
Power Input at Coupling of Driver (BkW)	4701	4507	4442

After an analysis of results, it is possible to conclude that the results of the Approximate Lüdtke Method with corrections (2016) are in good agreement with supplier actual data, whereas considering Sandberg Method (2016), the agreement is only acceptable. The reason is that Sandberg Method (2016) is still sensitive to minor changes in flow coefficient, in this range, resulting in a lower polytropic efficiency and consequently higher power absorbed. Furthermore, since all the curves are based on this parameter, minor deviations on it propagate, generating major deviations.

The objective of case 4 is to predict the performance of an ammonia (NH₃) synthesis gas compressor; the calculation is related to section 1 of compression, operating in normal condition. This compressor was supplied to Petrobras UFN3 Fertilizer Plant. For Case 4, the following data were estimated for both methods and depicted in Table 5.

Table 5. Predicted Data of Case 4

Compressor Parameters	Sandberg Method	Approx.Lüdtke Method w/Correct.	Supplier Actual Data
Suction Volume Flow (m ³ /s)	2,302	2,302	2,305
Polytropic Head (kJ/kg)	250,611	249,804	249,2
Flow Coefficient (First Impeller)	0,053	0,052	0,051
Polytropic efficiency (%)	82	83,2	83,8
Number of Stages	7	7	7
Mean Diameter (mm)	463	472	471,5
Speed (rpm)	10762	10371	10373
Power Input at Coupling of Driver (BkW)	8594	8474	8261

After an analysis of results, it is possible to conclude that the results of the Approximate Lüdtke Method with Corrections (2016) are better than Sandberg Method (2016), as compared to actual supplier data. The reason is the same of case 3.

The objective of case 5 is to predict the performance of a hydrogen (H₂) recycle gas compressor, operating in the design condition. This compressor was supplied to Petrobras RNEST refinery. For Case 5, the following data were estimated for both methods and depicted in Table 6.

Table 6. Predicted Data of Case 5

Compressor Parameters	Sandberg Method	Approx.Lüdtke Method w/Correct.	Supplier Actual Data
Suction Volume Flow (m ³ /s)	2,0497	2,0497	2,0492
Polytropic Head (kJ/kg)	349,767	348,41	347,5
Flow Coefficient (First Impeller)	0,031	0,034	0,034
Polytropic efficiency (%)	78	81,4	80,9
Number of Stages	9	9	9
Mean Diameter (mm)	551	540	540
Speed (rpm)	9416	9361	9360
Power Input at Coupling of Driver (BkW)	4277	4142	4102

After an analysis of results, it is possible to conclude that Approximate Lüdtke Method with Corrections (2016) are in good agreement with supplier actual data, whereas considering Sandberg Method (2016), the deviation is almost unacceptable, even though inside the margin for polytropic efficiencies. The reason is again the response of the method to small values of flow coefficient.

The objective of case 6 is to predict the performance of an ammonia (NH₃) refrigerating gas compressor, the calculation is related to section 3 of compression, operating in normal condition. This compressor was supplied to Petrobras UFN3 Fertilizer Plant.

For Case 6, the following data were estimated for both methods and depicted in Table 7.

Table 7. Predicted Data of Case 6

Compressor Parameters	Sandberg Method	Approx.Lüdtke Method w/Correct.	Supplier Actual Data
Suction Volume Flow (m ³ /s)	7,083	7,083	7,073
Polytropic Head (kJ/kg)	109,72	109,398	109,87
Flow Coefficient (First Impeller)	0,078	0,0784	0,0783
Polytropic efficiency (%)	83	85,4	85,4
Number of Stages	3	3	3
Mean Diameter (mm)	635	630	630
Speed (rpm)	8208	8792	8792
Power Input at Coupling of Driver (BkW)	2811	2740	2624

After an analysis of results, it is possible to conclude that the results of the two methods are in reasonable agreement with supplier actual data, in terms of efficiency, with a deviation in Sandberg Method (2016) related to speed. This deviation is due mainly to the curve of polytropic head coefficient of the Sandberg Method (2016), as compared with of Approximated Lüdtke Method with Corrections (2016) and the real value adopted by supplier.

6. CONCLUSION

Considering the analysis of the six cases that were presented, several conclusions may be stated, when both methods are compared. One of the most important is that Sandberg Method (2016) does not need a prior knowledge of compressor geometry, only the initial input parameters are necessary. However, a great number of iterations is demanded if an initial clue of geometry is not provided. An important aspect is that Sandberg Method (2016) presents more favorable results with higher values of flow coefficients ($\varphi > 0,08$). Therefore, for low flow coefficients ($\varphi < 0,04$), the convergence of results is hard to achieve. This statement may be verified from the curves of polytropic efficiency (Fig.8) and polytropic head coefficient (Fig.11), that is not accurate for low flow coefficients, however by other side, it extends the range of flow coefficient values up to 0,2.

Another remarkable point, related to Sandberg Method (2016) is the scattering data of polytropic efficiency curves (Fig.8); therefore, each selected curve has a margin of uncertainty, which affects directly the power absorbed by the gas being compressed. After an analysis of the six cases presented, is evident the good results achieved by approximated Lüdtke Method (2004), with corrections proposed by Busaid (2016), indicating that this Method is more adequate for multistage compressors calculation, provided with shrouded impellers and with flow coefficients in the range of 0,03 to 0,12. The apparent reason for this lies in the prototype compressor selected by Lüdtke (2004), resulting in plotted curves, depicted in Fig.14, that form the basis for the main performance prediction of this method.

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8. RESPONSIBILITY NOTICE

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