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MATHEMATICAL / NUMERICAL MODELING OF A FREE PISTON LINEAR ELECTRIC GENERATOR

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Abstract. *Subject of several academic research and some applications in the industry, linear machines have great potential to replace conventional internal combustion engines. Such machines make it possible to minimize losses and maximize power, therefore, the study of these machines is of paramount importance for efficient and feasible energy utilization. The direct combination of a free piston engine with a linear generator is called free-piston engine generator (FPEG). It is considered an alternative non-conventional electric power generator. This work aims to develop a mathematical and numerical model of a linear free-piston generator in mechanical resonance. The modeling consisted of the development of a thermodynamic model, which gives the pressure inside the combustion chamber, and a dynamic model of a spring mass system, which defines the force due to the pressure inside the combustion chamber as excitation. The coupling of both models allowed the study and simulation of the thermal cycle, as well as the analysis of the following parameters: geometric parameters of the engine, pressure, loads, forces and mass / spring assembly. The results obtained were satisfactory when compared to data obtained in the scientific literature.*

Keywords: *resonance, free piston engine, free-piston engine generator, numerical modeling*

1. INTRODUCTION

Originally proposed by Pescara in 1928, free piston engines developed rapidly between the 1930s and 1960s. During this period, other entrepreneurs, such as the Junkers in Germany, also worked on free piston engines. Subsequently, a large number of patents on these machines or related were published. The original Pescara patent describes a single-piston spark-ignition air compressor, but it only sought to protect a large number of applications using the free-piston principle (Mikalsen and Roskilly, 2007a).

According to Li et al. (2010a), the simple structure and low friction made the engine popular at the time. Over time, engines based on the Otto and Diesel cycle were gaining maturity, at the same time the limitations of the engine were exposed, such as: difficult engine synchronization, low efficiency under various types of loading and difficulty of departure. Thus, the employment of free piston engines was gradually abandoned in the 1960s.

However, in recent years, with the effects of global warming and the shortage of fossil fuels, several groups of researchers have regained interest in free piston engines. The research is dedicated to the exploration of new energy conversion devices and use of ecological fuels (Li et al., 2010b).

As pointed out by Lim et al., (2013), the main advantages of the free piston engine include: simplicity in mechanical structure with few moving parts, low friction losses and high operational flexibility.

According to Mikalsen and Roskilly (2007b), one of the applications of these machines is the free-piston engine generator (FPEG), which integrates a linear combustion engine with a linear electric generator into a single unit. The principle of operation consists of withdrawing energy from the fuel in a manner similar to that of a conventional car engine. However, the linear motion of the pistons directly converted into electricity directly.

According to Xiao et al., (2010a), several researches on the use and construction of the FPEG have been approached in recent years. Most researchers tend to adapt ideal and simplified models of conventional motors to simulate the linear piston-free generator.

Researchers at West Virginia University describe the development of a twin-piston engine with ignition. A motor prototype obtained an output of maximum electrical power of 316 W at 23.1 Hz, with a bore of 36.5 mm and a maximum stroke of 50 mm (Atkinson et al., 1999).

Papers published recently show almost all the global FPEG researches are in the initial phase. Different types of tools are used to study this new emerging engine.

Some researchers aim to model and simulate the dynamic operation of an FPEG in order to obtain an understanding of the relation between the characteristics of the piston movement and the necessary thermal energy. In a similar way, we can find in literature researches such as:

Xiao et al., (2010b) established a numerical model of the FPEG. The natural frequency of the oscillation system was obtained from their model. A simulation program was developed in Matlab / Simulink to solve these mathematical equations, and simulation results showed that the movement of the FPEG was a forced vibration system with variable coefficient of damping and stiffness.

Hansson et al.,(2006) investigated the resonant behavior of an FPEG. They linearized the system after expanding the equation around an equilibrium point. Finally, the approximations of the oscillation characteristics of the free piston were reached. However, only compressive pressure forces were calculated in their model, and the pressure increased by the release of heat from the gaseous fuel mixture was not considered.

Thus, this work aims to explore the resonant characteristics of mechanical systems. For this, the dynamic system of the linear free-piston generator will be represented as a damped mass-spring system operating in mechanical resonance.

It is intended to develop a dynamic and thermal model that describes the characteristics of movement and release of heat. The model will then be linearized so that the system is represented by a forced vibration equation with viscous damping. In addition, the approximations of the spring constant and the natural frequency can be obtained. Based on this, the vibration equation and the solution for the displacement can be derived.

2. LINEAR FREE ELECTRIC PISTON GENERATOR

According to Mikalsen and Roskilly (2007c), the FPEG (Free-piston engine Generator) integrates a linear combustion engine with a linear electric generator in a single unit.

The principle of operation consists in withdrawing energy from the fuel in a manner similar to that of a conventional car engine. However, the linear motion of the pistons is converted into electricity directly.

The free piston linear motor consists of two opposing cylinder / piston assemblies connected by a rod. The combustion of the air-fuel mixture happens alternately in each cylinder. The expansion of the gases in one chamber provides compression in the other, thus promoting the linear movement of the piston.

The linear electric generator is composed of permanent magnets that are coupled to the piston rod. Thus, with their linear and alternating motion, the magnets move inside a set of coils, generating electricity. Fig. 1 schematizes a linear generator.
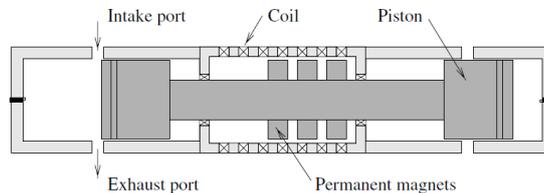

Figure 1 - Prototype illustration of the free piston engine generator developed at the University of West Virginia. Adapted from: Mikalsen and Roskilly (2007).

3. MATHEMATICAL MODEL

The purpose of this section will be to develop a mathematical model that describes the operation of the free-piston electric generator. The model will consist of two parts: a thermodynamic model and a dynamic model.

3.1 Dynamic model

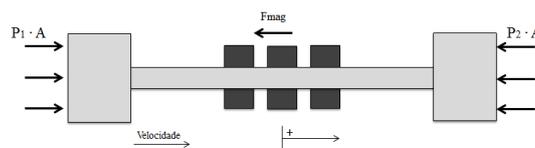The piston movement obeys Newton's law and the body diagram is shown in Fig. 2.

Figure 2 - FPEG free body diagram

The piston acceleration is determined as:

$$\sum F = m \cdot \ddot{x} \tag{1}$$

where $\sum F$ represents all the forces applied to the piston, m the moving mass that moves along with the piston and x the displacement of the engine.

The forces acting on the system, as shown in Fig. 1, are: the force of the gases in the right cylinder (F_r,) the force of the gases in the left cylinder (F_l), the magnetic force of the linear generator (F_{mag}), and the force of friction (F_f.) As the motor movement is linear, the lateral loads applied on the piston are minimal, in addition to being insignificant when compared to the other forces. Thus, neglecting the force F_f. and rewriting Eq. (1), we have:

$$m\ddot{x} = \vec{F}_l + \vec{F}_r + \vec{F}_{mag} \tag{2}$$

The force resulting from the action of the gas and the pressure in the cylinder are a function of the displacement of the engine (x):

$$F_l = P_l \cdot A \tag{3}$$
$$F_r = P_r \cdot A \tag{4}$$

where P_l is the pressure of the left cylinder, P_r is the pressure of the right cylinder and A is the surface area of the cylinder which is calculated by:

$$A = \frac{\pi D^2}{4} \tag{5}$$

The variable D is the diameter of the piston.

According to Li, et al., (2008a), the resulting magnetic force is a function of the generator design parameters such as resistance, mass and speed of operation.

The magnetic force acts on the system as a force of resistance to movement. Thus, it is assumed that F_{mag} is proportional to the speed of the motor:

$$F_{amort} = F_{mag} = C_{mag} \cdot \dot{x} \tag{6}$$

where C_{mag} is a coefficient of proportionality. Therefore, Eq. (1) can be rewritten by:

$$m\ddot{x} = \vec{F}_l(x) + \vec{F}_r(x) + \vec{F}_{amort}(\dot{x}) \tag{7}$$

Eq. 7 describes a non-linear system of one degree of freedom. Based on such information, the dynamic equation of the free piston motor generator (FPEG) can be linearized to a forced vibration system with viscous damping as shown in Fig.3.
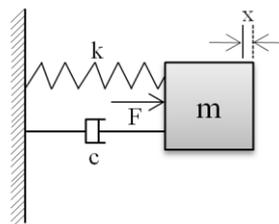

Figure 3 - Forced vibration system

3.2 Thermodynamic Model

As in the Otto cycle, the FPEG thermal cycle can be represented in a simplified way by a process of compression, combustion and expansion. A control volume is used to describe the thermodynamic process and it is assumed that there is no mass loss after the piston rings. It is considered that the pressure variation in the cylinders is influenced by two facts:

1. Heat release relative to the combustion of the air-fuel mixture, denoted by p_{comb};

2. Change in cylinder volume caused by the movement of the piston, p_{vol}.

Thus, the pressure in each cylinder can be written as:

$$P_l = p_{lvol} + p_{lcomb} \cdot \sigma_l \tag{8}$$
$$P_r = p_{rvol} + p_{rcomb} \cdot \sigma_r \tag{9}$$

where p is the pressure in each cylinder and the subscript l and r indicate the values for the left and right cylinders.

At the beginning of the combustion the pressure rise is very small, so the compression will continue until the gas pressure in the piston is high enough to reverse the piston direction and start the expansion process.

As presented by Jia, et al., 2016a, a unit function σ is introduced to activate / deactivate the influence of the heat release for both cylinders, as shown in Eq. (10) and Eq.(11):

$$\sigma_l = \begin{cases} 1, \dot{x} \geq 0 \\ 0, \dot{x} < 0 \end{cases} \tag{10}$$

$$\sigma_r = \begin{cases} 0, \dot{x} \geq 0 \\ 1, \dot{x} < 0 \end{cases} \tag{11}$$

According to Jia et al., 2016b, if no heat transfer to the cylinder walls is considered in the thermodynamic model and there is no gas leakage through the piston rings, the ideal FPEG operating cycle can be described by two adiabatic processes connected by a process of heat release at constant volume.. Thus, the force resulting in each cylinder can be expressed by:

$$F_l = A \cdot \left(p_0 \left(\frac{V_0}{V_l} \right)^{\gamma} + \Delta p_{cm} \left(\frac{V_c}{V_l} \right) \cdot \sigma_l \right) \tag{12}$$

$$F_r = A \cdot \left(p_0 \left(\frac{V_0}{V_r} \right)^{\gamma} + \Delta p_{cm} \left(\frac{V_c}{V_r} \right) \cdot \sigma_r \right) \tag{13}$$

where p_0 is the pressure in the cylinder in the equilibrium position, is assumed to be equal to the ambient pressure V_0, is the cylinder volume in the equilibrium position, V is the cylinder volume, γ is the polytropic exponent, Δp_{cm} is the pressure increase during the process of heat release at constant volume, the value is the same for both sides, and V_c, is the volume of clearance.

The volume of each cylinder and the clearance volume is calculated by:

$$V_0 = L_s \cdot A \tag{14}$$
$$V_l = (L_s + x) \cdot A \tag{15}$$
$$V_r = (L_s - x) \cdot A \tag{16}$$
$$V_c = L_c \cdot A \tag{17}$$

Where L_s is the length of half stroke and L_c is the length of clearance.

3.3 Forced vibration equation

For the equation the motion of the linear free-piston generator can be represented by a mass-spring-damper system it is necessary to linearize the nonlinear expressions present in the equation of the forces imposed on each cylinder.

According to Jia, et al., 2016c, a Taylor series can be used as a form of linearization and it is expanded around the equilibrium point of the system. After linearization Eq. (12) and eq. (13) can be written as:

$$F_l(x) = \left(p_0 + \Delta p_{cm} \left(\frac{L_c}{L_s} \right)^{\gamma} \cdot \sigma_l \right) A - \left(\frac{\gamma p_0}{L_s} + \Delta p_{cm} \frac{\gamma L_c^{\gamma}}{L_s^{\gamma+1}} \cdot \sigma_l \right) A \cdot x \tag{18}$$

$$F_r(x) = \left(p_0 + \Delta p_{cm} \left(\frac{L_c}{L_s} \right)^{\gamma} \cdot \sigma_r \right) A - \left(\frac{\gamma p_0}{L_s} + \Delta p_{cm} \frac{\gamma L_c^{\gamma}}{L_s^{\gamma+1}} \cdot \sigma_l \right) A \cdot x \tag{19}$$

Substituting Eq. (18) and Eq. (19) in Eq. (7) is obtained:

$$m\ddot{x} + F_{amor} + \left(\frac{2\gamma p_0 A}{L_s} + \frac{\Delta p_{cm} A \gamma L_c^{\gamma}}{L_s^{\gamma+1}} \left(\sigma_l + \sigma_r \right) \right) x = \cdot \Delta p_{cm} A \left(\frac{L_c}{L_s} \right)^{\gamma} \left(\sigma_l - \sigma_r \right) \tag{20}$$

The damping force of the system is expressed by the magnetic force (F_{mag}), which converts the kinetic energy into electric and the viscous damping force (F_{vis}) which by the resistance of the air to the motion dissipates energy. Therefore, the damping force can be expressed as:

$$F_{amor} = \left(C_{visc} + C_{mag} \right) \dot{x} \tag{21}$$

where C_{mag} and C_{vis} are constants of proportionality. As the combustion occurs alternately in each cylinder, as already presented, we have:

$$\sigma_l + \sigma_r = 1 \tag{22}$$

and

$$\sigma_l - \sigma_r = \begin{cases} 1, \dot{x} \geq 0 \\ -1, \dot{x} < 0 \end{cases} \tag{23}$$

According to Jia et al., (2016d), the rectangular wave of excitation can be described by a series of Fourier. Thus, it is considered that the first mode of is defined:

$$\sigma_l - \sigma_r \approx \frac{4}{\pi} sen\,\omega t \tag{24}$$

where is the angular frequency of combustion that occurs in each cylinder. Finally, replacing Eq (21), Eq (22) and Eq. (24) in Eq. (20), we obtain:

$$m\ddot{x} + \left(C_{vis} + C_{mag} \right) \cdot \dot{x} + \left(\frac{2\gamma p_0 A}{L_S} + \frac{\Delta p_{cm} A \gamma L_C^{\gamma}}{L_S^{\gamma+1}} \right) x = \Delta p_{cm} A \left(\frac{L_C}{L_S} \right)^{\gamma} \left(\frac{4}{\pi} sen\,\omega t \right) \tag{25}$$

Like this:

$$m\ddot{x} + c \cdot \dot{x} + kx = F_0 sen\,\omega t \tag{26}$$

where:

$$c = C_{vis} + C_{mag} \tag{27}$$

$$k = \frac{2\gamma p_0 A}{L_S} + \frac{\Delta p_{cm} A \gamma L_C^{\gamma}}{L_S^{\gamma+1}} \tag{28}$$

$$F_0 = \frac{4}{\pi} \Delta p_{cm} A \left(\frac{L_C}{L_S} \right)^{\gamma}$$

(29)

Then the FPEG dynamic model is linearized to the same form with the single degree-of-freedom forced vibration system with viscous damping. Where k is the stiffness of the air-spring system; c is the damping coefficient; the excitation $F_0 sen\omega t$ is a continuing force whose magnitude F_0 varies sinusoidally with time.

Then the angular natural frequency, ω_n of FPEG can be defined as:

$$\omega_n = \sqrt{\frac{k}{m}}$$

(30)

Since combustion is assumed to take place when the piston reaches its top dead centre (TDC), the frequency of the ignition is required to be the same as an air-spring system of FPEG in order to maintain stable operation (Li et al. 2008b.). Thus yielding:

$$\omega_n = \omega$$

(31)

4. NUMERICAL MODEL

The Matlab software was used to solve the coupling between the thermal and the dynamic models. The Equation 29 was rewritten as a matrix, as presented in Eq. 31, and the displacement and velocity of the system can be obtained using the Runge-Kutta integration technique.

$$\begin{bmatrix} m & c \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} 0 & k \\ -m & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix} = \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix}$$

(32)

The input parameters used are shows in Table 1 and have been taken from the literature as presented by Li et al. (2008c). Moreover, it assumes a constant value for the damping coefficient, represented by the load force coefficient. The coefficient of the load force is in function of the operating parameters of the linear generator.

Table 1. Parameters of the prototype presented by Jia, et al., (2016).

Parameters	Value
Bore [mm]	52,5
Maximum total stroke [mm]	70,0
Stroke [mm]	35
Moving mass [kg]	5
Coefficient of the load force $\left[N / (ms^{-1}) \right]$	395
Ambient temperature [°C]	25
Inlet pressure [kPa]	101,325

Data such as fuel, slack length, compression ratio, air-fuel ratio (AFR), heat capacity ratio and combustion efficiency are set as input variables and the values assumed were:

Table 2- Parameters assumed.

Parameters	Value
Clearance length [mm]	2,7
Compression ratio	13
AFR	14,7
Ratio of heat capacities	1,3
Combustion efficiency	1

Thus, with this analysis, it is desired to validate the model presented here with the model developed by Jia et al. (2016d).

5. RESULT

The FPEG natural frequency is directly related to the inlet pressure and piston stroke. They are variables that change with excitement. The natural frequency map of the FPEG is drawn in Fig. 1 for a better understanding All variables are adjusted to healthy ranges, where the pressure is between 1.2 and 2KPa and the stroke of the piston is between 0.03 and 0.036 m.
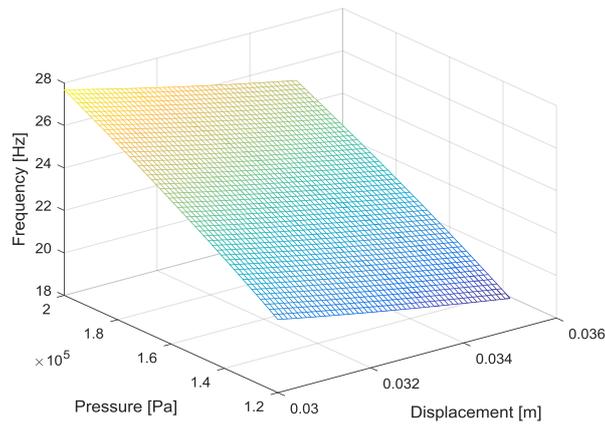

Figure 1 - Natural frequency characteristic of FPEG.

For each initial pressure and set stroke, the system will oscillate at a natural frequency. Thus, it can be ensured that the FPEG works in resonance or not, therefore, by changing some input parameter and keeping constant pressure and stroke, the frequency of the system will be changed to another that does not match the natural one.

Fig. 2 shows the simulation of the displacement of the piston as a function of time during the combustion process compared to the prototype data presented by Jia et al. (2016d), in the same operating condition.
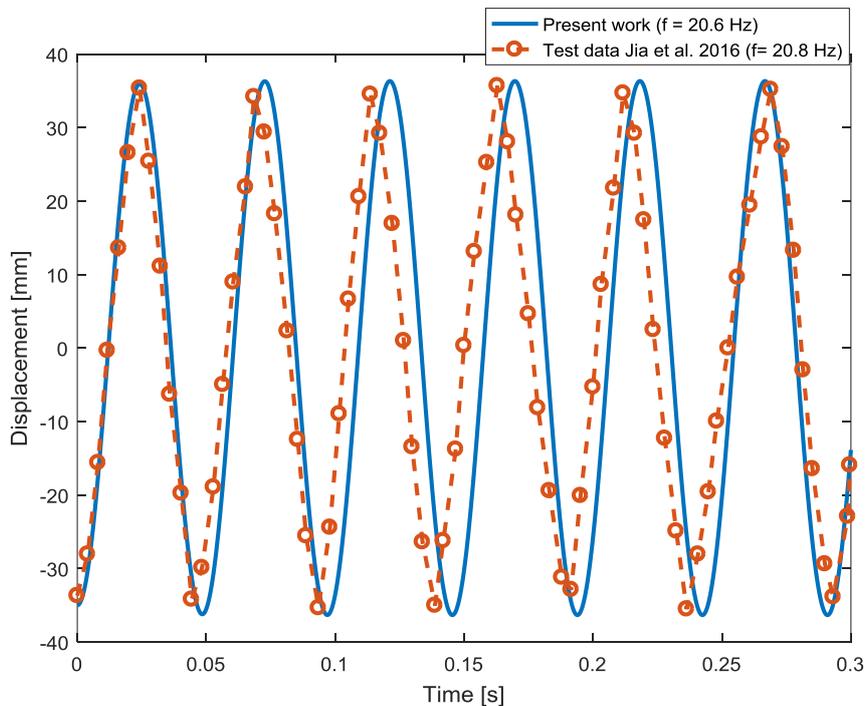

Figure 2 - Comparison between the present work (f = 20.6 Hz) and test data Jia et al. 2016d ,(f= 20.8).

The simulation frequency was around 18 Hz, whereas the cycle data was around 20 Hz. A factor of 1.15 was used in the results to compensate the frequency between the simulated and the prototype results.

The comparison between the computational models is shown in Fig. (3). Although there is a difference between the amplitudes of the curve, it can be seen that the simulation frequencies coincide.
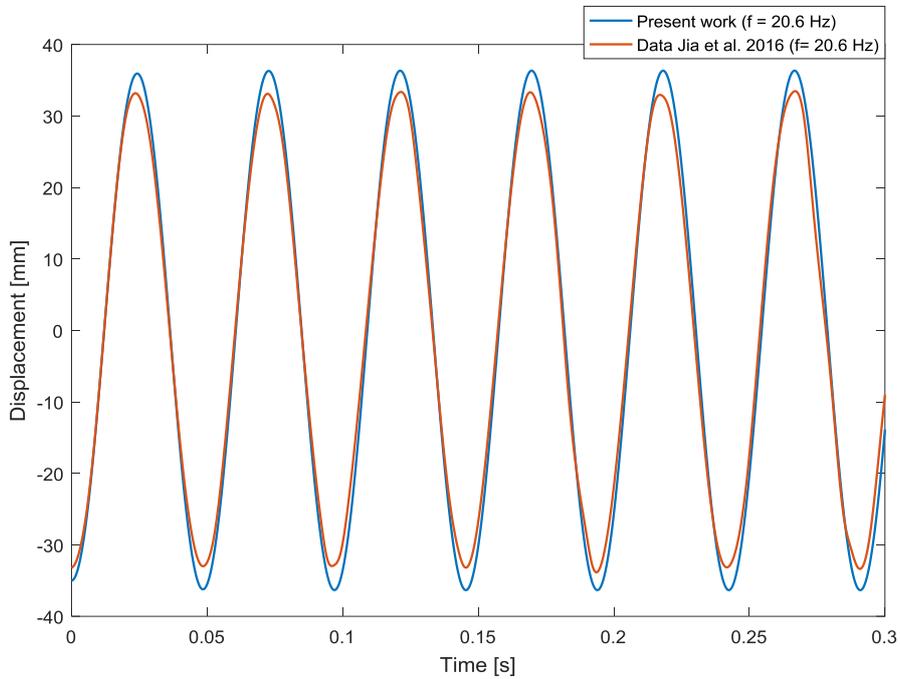

Figure 3 - Comparison between the present work (f = 20.6 Hz) and test data Jia et al. 2016d ,(f= 20.8).
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The Figure. (4) plots the response of the system under the excitation of 20.6 Hz. Fig. 4a describes the velocity as a function of time while (b) is the phase plane figure. For this instance, the compression ratio is about 13.

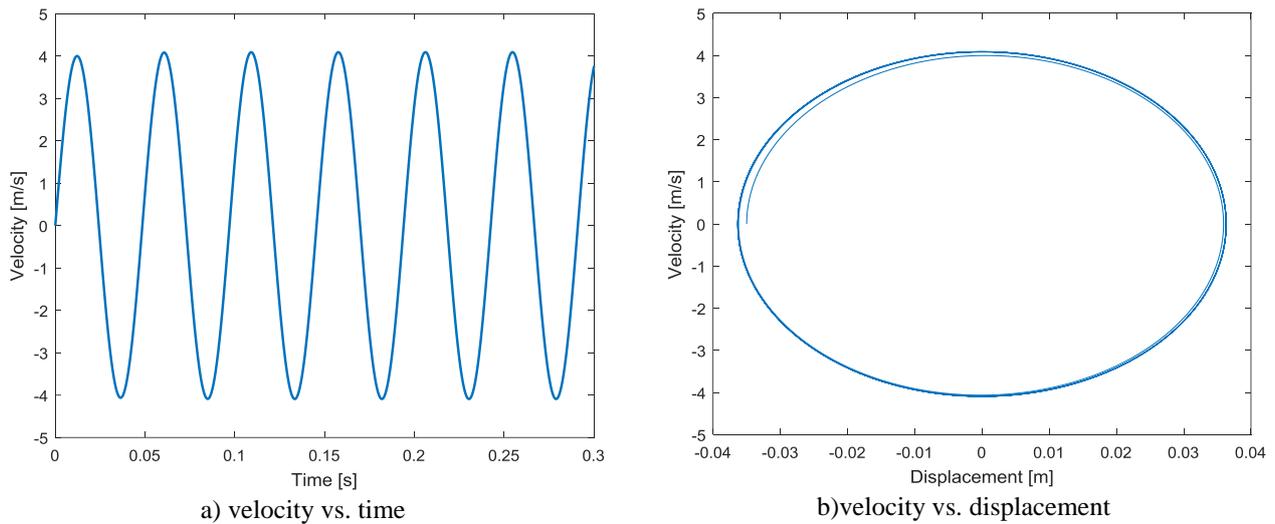a) velocity vs. time

b)velocity vs. displacement

Figure 4 - Oscillation characteristic of FPGE (20.6 Hz).

It was initially considered that the left piston started from the -0.035m position and was in the prime of starting the expansion process so that the velocity at this moment was zero.

6. CONCLUSION

It can be concluded that the combustion process controls the final operation and convergence of the FPEG. According to the mathematical model and based on literature data, the combustion frequency can be adjusted by changing the ignition time, mass of fuel, pressure of admission, etc. Thus, all these methods could be used to change the frequency of operation of the motor. The frequency map shows the possible values of the natural frequency of the system for input data previously established, so it is possible to explore the resonance effect.

The comparison between the data of the literature and those developed in this work presented satisfactory results. The initial discrepancy between the frequency of the simulated model and the prototype is due to simplifications of the thermal model and linearization of the exiting force. The difference in amplitude is due to the values adopted in the simulation, which may be different from those actually used in the work that was taken as reference.

It was assumed that the oblique piston started from -0.035 m and zero volition, thus Fig. (4.b) shows a line at the intersection of the ellipse, which is where the simulation started. As not all adopted parameters are coincident with that of the literature, a greater breadth of displacement was obtained than the reference data. However, it is perceived that the system converges to a value over a short response time. No matter if the initial state, its trajectory always reaches the limit under periodic excitation. Therefore, maintaining an excitation is crucial to this mechanism.

7. ACKNOWLEDGEMENTS

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