

## ENCIT-2018-0084

### THERMAL ANALYSIS OF HEAT SINKS IN SOLAR PANELS

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**Abstract.** *This work proposes an analytical approach of the heat transfer analysis in heat sinks applied on cooling electronic components of solar panels. On the bottom of the base of the heat sink, the presence of the oncoming heat flux from a heated solid-state electronic needs to have its heat dissipated, in order to ensure a profitable performance of the solar panel. Since the thickness of the base is small compared to other dimensions of the heat sink, a partial lumping approach in z-direction is performed and the final mathematical formulation is two dimensional, considering also the heat flux source term. In order to obtain the final solution, the Classical Integral Transform Technique is applied and four different cases are presented: the heat sink without any fin, with one, two fins and with four fins. The achieved results are compared by the increase of the heat dissipation affecting the final temperature of the heat sink.*

**Keywords:** *Thermal Analysis, Heat Sink, Classical Integral Transform Technique*

#### 1. INTRODUCTION

The solar energy has been used since ancient civilizations for heating and light purposes. Archeological evidence has shown that many ancient cultures built their houses according the principles of passive solar design, Chen (2011). In the last centuries, however, the sun energy has been applied directly to make electricity. In 1839, Alexandre Becquerel discovered that certain materials produced small amounts of electric current when exposed to light, Shah *et al.* (1999). In 1954, D.M. Chapin, C.S. Fuller and G.L. Pearson, of Bell Laboratory, patented a way of making electricity directly from sunlight using silicon-based solar cells, Gotzberger *et al.* (2002). The solar energy and specially photovoltaics (PV) is a promising alternative source due to its advantages: abundance, pollution free and renewability and has been more and more implemented all over the world, Singh (2013).

Several researches have been studying how to enhance photovoltaic systems. It is known that high temperatures decrease the efficiency of PV systems, (Cuce and Cuce, 2012). The electronic components of solar panels system present high temperatures when in operation and this heat may affect the system's performance. Cooling them with cost-effective modifications such as heat sinks may be a considered key point to minimize the electronic components temperature.

The thermal dissipation promoted by heat sinks has motivated several works about analysis and optimization of fins in heat sinks. The work of Teertstra *et al.* (2000) presented an analytical model to approach the average heat transfer rate for forced convection, air cooled, plate fin heat sinks. The work of Lehtinen (2005) analyzed both heat conduction and convection in fins applying well-known analytical and experimental results for convective heat transfer. The geometry of the fin was also studied for maximizing the heat transfer. The work of Ong *et al.* (2005), for instance, analysed different geometries of rectangular and cylindrical fins optimizations for maximum heat dissipation on electronic components. The behavior of different geometric parameter heat sinks with rectangular fins was also analyzed by Anselmo (2016). The work of Azarkish *et al.* (2010) investigated the geometry of the longitudinal fins with variable cross sectional area achieving its optimum fin profile using genetic algorithm. On the other hand, Cuce and Cuce (2014) tested different rectangular fins configurations to produce the maximum heat loss in a specific volume and length of fin numerically exposed to convection and radiation heat transfer.

The Integral Transform Technique is a powerful method to solve differential equations based on separation of variables method and is classified as Classical (CITT) or Generalized (GITT). The CITT is an all analytical method and is most applied in linear problems, Chalhub *et al.* (2014). The GITT is a hybrid analytical-numerical technique and transforms nonlinear partial differential equation models to a coupled nonlinear system of ordinary differential equations (ODEs) to be solved numerically. Sphaier and Cotta (2000) applied the Integral Transform Technique on the solution of a multidimensional partial differential models within irregularly shaped domains. Braga Junior (2015) also applied the GITT in order to obtain the heat transfer solution in heterogeneous mediums such as Functionally Graded Materials with variable properties. The Integral Transform Technique has been previously applied on electronic problems. Dantas (1996) have applied the GITT for the solution of heat transfer in microchips. She considered an encapsulated microchip and also

different thermal conductivity layers over the chip thickness. Recently, Corrêa and Chalhub (2017) presented the solution of Solid State Electronics with one heat generation on its domain and solved by Classical Integral Transform Technique.

This work proposes an analytical approach to analyze the heat transfer occurrence in heat sinks applied on electronic components of solar panels. On the bottom of the base of the heat sink, there is the presence of the oncoming heat flux from the heated solid-state electronic, which needs its heat to be dissipated. The top of the base of the heat sink presents fins to increase the heat dissipation to the environment and, consequently, cool the chip. The mathematical formulation for the fins is developed and the heat transfer coefficient for convection depends on the position of the fins. The final formulation for the problem is applied on the base of the heat sink. Since the thickness of the base is small compared to other dimensions of the heat sink, a partial lumping approach in z-direction is performed and the final mathematical formulation is two dimensional, considering also the heat flux source term. In order to obtain the final solution, the Classical Integral Transform Technique is applied. In this case, however, the transformed equation cannot be solved analytically and requires a numerical discretization. A truncation error is involved since the infinite summation must be truncated. This error decreases as the number of summation terms (truncation order) is increased, and the solution converges to a final value. The results show an analysis of the proposed approach for comparison purposes of the final temperature of the heat sink without any fin, with one, with two fins and with four fins.

## 2. MATHEMATICAL FORMULATION OF PARALLEL PLATE FINS

For this approach, rectangular fins were modelled on the heat sink working on steady state. The boundary conditions applied on the fin are fixed temperature at the base of the heat sink (isothermal base) and convection at the fin longitudinal end.

$$\frac{d}{dx_a} \left( k_a A_a \frac{dT}{dx_a} \right) - P_a h (T - T_f) = 0 \quad \text{for } 0 \leq x_a \leq L_a \quad (1a)$$

$$T|_{x_a=0} = T_b; \quad -k_a \frac{dT}{dx_a} \Big|_{x=L_a} = h(T(L_a) - T_f) \quad (1b)$$

where  $k_a$  is the thermal conductivity of the fin,  $A_a$  is the transversal area of the fin,  $P_a$  is the perimeter of the fin,  $h$  is the heat transfer coefficient by convection and  $L_a$  is the length of the fin.

The nondimensionalization leads to the following mathematical formulation for the fin:

$$\frac{d^2\Theta}{d\xi_a^2} - m^2\Theta = 0 \quad \text{for } 0 \leq \xi_a \leq 1 \quad (2a)$$

$$\Theta(0) = 1 \quad \frac{d\Theta}{d\xi_a} \Big|_{\xi_a=1} = -Bi_a\Theta(1) \quad (2b)$$

The non-dimensional groups are defined as:

$$\xi_a = \frac{x_a}{L_a}; \quad \Theta = \frac{T - T_f}{T_b - T_f}; \quad Bi_a = \frac{hL_a}{k_a}; \quad m^2 = \frac{Bi_a P_a L_a}{A_a} \quad (3)$$

The solution for (2a) is:

$$\Theta(\xi_a) = \frac{m \cosh(m(1 - \xi_a)) + Bi_a \sinh(m(1 - \xi_a))}{m \cosh(m) + Bi_a \sinh(m)} \quad (4)$$

The heat flux in each fin of the heat sink is:

$$q_a'' = -k_a(T - T_f) \frac{d\Theta}{d\xi_a} \Big|_{\xi_a=0} \rightarrow q_a'' = k_a(T - T_f) \frac{m (Bi_a \cosh(m) + m \sinh(m))}{m \cosh(m) + Bi_a \sinh(m)} = h_{fin}(T - T_f) \quad (5)$$

where:

$$h_{fin} = k_a \frac{m (Bi_a \cosh(m) + m \sinh(m))}{m \cosh(m) + Bi_a \sinh(m)} \quad (6)$$

## 3. PROBLEM FORMULATION

The mathematical formulation of the heat conduction at the base of the heat sink is given by the energy equation in steady-state after applying a partial lumping approach in z-direction.

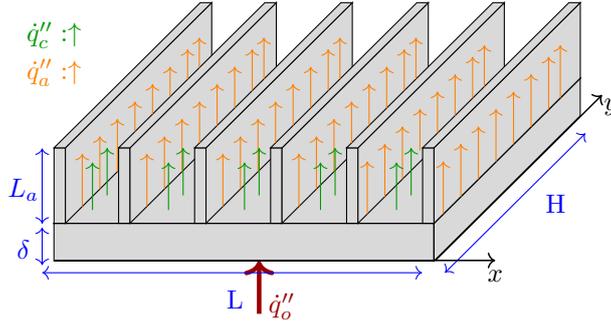


Figure 1: Schematic view of heat sink

For this work, it is considered a heat flux from the chip ( $\dot{q}_o''$ ) acting over the heat sink. The convection flux ( $\dot{q}_c''$ ) happens in the top surface of the heat sink, however, where the fin is located, the heat flux is the heat flux from the fin ( $\dot{q}_a''$ ) of the heat sink:

$$\dot{q}_h''(x) = \begin{cases} \dot{q}_a'' = h_{fin}(T - T_f) & \text{if } x_{aiK} \leq x \leq x_{afK} \\ \dot{q}_c'' = h_{conv}(T - T_f) & \text{if } x < x_{aiK} \text{ or } x > x_{afK} \end{cases}$$

For this reason, is possible define the heat transfer coefficient for convection ( $h$ ) dependent of its location on the base of the heat sink. On the region where the fins are located,  $h$  assume  $h_{fin}$  value and where there isn't fin  $h$  assume  $h_{conv}$  value:

$$h(x) = \begin{cases} h_{fin} & \text{if } x_{aiK} \leq x \leq x_{afK} \\ h_{conv} & \text{if } x < x_{aiK} \text{ or } x > x_{afK} \end{cases}$$

where  $x_{aiK}$  and  $x_{afK}$  indicates the boundaries of the  $K$  fins.

The formulation is showed bellow with its respective boundary conditions.

$$k \left( \frac{\partial^2 T(x, y)}{\partial x^2} + \frac{\partial^2 T(x, y)}{\partial y^2} \right) = \frac{\dot{q}_h''(x)}{\delta} - \frac{\dot{q}_o''(x, y)}{\delta} \quad \text{for } 0 \leq x \leq L \quad \text{and} \quad 0 \leq y \leq H \quad (7)$$

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0; \quad \left. \frac{\partial T}{\partial x} \right|_{x=L} = 0; \quad \left. \frac{\partial T}{\partial y} \right|_{y=0} = 0; \quad \left. \frac{\partial T}{\partial y} \right|_{y=H} = 0; \quad (8)$$

where  $T$  is the temperature,  $k$  is the thermal conductivity of the plate,  $\dot{q}_o''$  is the heat flux from the chip to the heat sink,  $T_f$  is the environment air temperature,  $h$  is the convection heat transfer coefficient and  $L$ ,  $H$  and  $\delta$  are the dimensions of the chip in  $x$ ,  $y$  and  $z$  directions respectively.

The nondimensionalization of the problem leads to the following mathematical formulation:

$$\frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\beta^2 \partial^2 \Theta}{\partial \eta^2} - \text{Bi}(\xi) \gamma \Theta = -Q(\xi, \eta) \quad \text{for } 0 \leq \xi \leq 1 \quad \text{and} \quad 0 \leq \eta \leq 1 \quad (9)$$

$$\left. \frac{\partial \Theta}{\partial \xi} \right|_{\xi=0} = 0; \quad \left. \frac{\partial \Theta}{\partial \eta} \right|_{\eta=0} = 0; \quad \left. \frac{\partial \Theta}{\partial \xi} \right|_{\xi=1} = 0; \quad \left. \frac{\partial \Theta}{\partial \eta} \right|_{\eta=1} = 0; \quad (10)$$

The non-dimensional groups are defined as:

$$\xi = \frac{x}{L}; \quad \eta = \frac{y}{H}; \quad \Theta = \frac{T - T_f}{T_b - T_f}; \quad \beta = \frac{L}{H}; \quad \gamma = \frac{L}{\delta}; \quad \text{Bi}(\xi) = \frac{h(\xi)L}{k}; \quad Q(\xi, \eta) = \frac{\dot{q}_o'' L^2}{k \delta \Delta T}. \quad (11)$$

where  $\beta$  and  $\gamma$  are aspect ratios,  $\text{Bi}(\xi)$  is the Biot number and depends of  $\xi$ ,  $\Theta$  is the dimensionless temperature,  $\xi$  and  $\eta$  are the dimensionless versions of  $x$  and  $y$ ; and  $Q$  is the heat flux acting over the domain from the chip to the base of the heat sink. The chip is located at the center of the base.

#### 4. SOLUTION BY CLASSICAL INTEGRAL TRANSFORM TECHNIQUE

In order to solve the proposed problem, the Classical Integral Transform Technique (CITT) is applied. This is an analytical technique that uses expansions of the sought solution in terms of an infinite orthogonal basis of eigenfunctions, keeping the solution process always within a continuous domain. In order to establish the transformation pair, the

temperature field is written as function of an orthogonal eigenfunctions obtained from the following auxiliary eigenvalue problem known as the Helmholtz classic problem in cartesian coordinates, where  $\Psi(\eta)$  are the eigenfunctions and  $\lambda_n$  are the eigenvalues. For this particular problem, the case where  $\lambda=0$  also exists.

The solution of the equation (9) is defined as:

$$\Theta = \sum_{n=0}^{\infty} \frac{\bar{\Theta}(\xi)\Psi_n(\eta)}{N_n} \quad (12)$$

where  $\bar{\Theta}_n(\xi)$  and  $\Psi_n(\eta)$  are the functions to be solved separately in order to find the temperature field and are eigenfunctions.  $\bar{\Theta}_n(\xi)$  is also the transformed version of  $\Theta$ .  $N_n$  is the norm and is defined as:

$$N_n = \int_0^1 \Psi_n^2 d\eta \quad (13)$$

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This step is solved just as separation of variables and the objective is to find the values of the eigenfunction  $\Psi_n(\eta)$ .

$$\Psi_n''(\eta) + \lambda_n^2 \Psi_n(\eta) = 0 \quad (14a)$$

$$\Psi_n'(0) = 0; \quad \Psi_n'(1) = 0. \quad (14b)$$

Solving the differential equation, the solution shows that the eigenfunction is formed by sines and cosines. Applying the boundary conditions, the term formed by sines is eliminated from the solution and the values of the eigenvalues  $\lambda_n$  are found.

For  $\lambda=0$ , the solution of the eigenvalue problem is given by:

$$\Psi_0(\eta) = 1; \quad \lambda_0 = 0; \quad (15)$$

and for  $\lambda > 0$ :

$$\Psi_n(\eta) = \cos(\lambda_n \eta); \quad \lambda_n = n\pi, \quad \text{for } n = 1, 2, 3, \dots \quad (16)$$

To apply the CITT, the transformation pair is defined.

$$\text{Transformation} \Rightarrow \bar{\Theta}_n(\xi) = \int_0^1 \Theta \Psi_n(\eta) d\eta \quad (17)$$

$$\text{Inversion} \Rightarrow \Theta = \sum_{n=0}^{\infty} \frac{\bar{\Theta}_n(\xi)\Psi_n(\eta)}{N_n} \quad (18)$$

The equation (9) is written again, multiplied by  $\Psi_n$  and integrated in the domain for  $\eta$ . The objective in this step is obtain the transformed equation by the replacement of the terms with the transformation input for the transformation term.

$$\int_0^1 \frac{\partial^2 \Theta}{\partial \xi^2} \Psi_n d\eta + \beta^2 \int_0^1 \frac{\partial^2 \Theta}{\partial \eta^2} \Psi_n d\eta - \text{Bi}(\xi)\gamma \int_0^1 \Theta \Psi_n d\eta = - \int_0^1 Q \Psi_n d\eta \quad (19a)$$

Finally, the transformed equation is obtained:

- For  $\lambda > 0$ :

$$\bar{\Theta}_n'' - (\beta^2 \lambda_n^2 + \text{Bi}(\xi)\gamma)\bar{\Theta}_n = -\bar{Q}_n(\xi) \quad (20)$$

where  $\bar{Q}_n$  is given by:

$$\bar{Q}_n(\xi) = \int_0^1 Q(\xi, \eta) \Psi_n(\eta) d\eta \quad (21)$$

The transformed boundary conditions are:

$$\bar{\Theta}_n'(0) = 0; \quad \bar{\Theta}_n'(1) = 0 \quad (22)$$

This equation cannot be solved analytically because of the dependence of  $\xi$  on Biot number. For this reason, the equation (20) is solved numerically using partial differential equation discretization with Finite Element Method.

- For  $\lambda = 0$ :

$$\bar{\Theta}_0'' - (\text{Bi}(\xi)\gamma)\bar{\Theta}_0 = -\bar{Q}_0(\xi) \quad (23)$$

where  $\bar{Q}_0$  is given by:

$$\bar{Q}_0(\xi) = \int_0^1 Q(\xi, \eta)\Psi_0(\eta)d\eta = \int_0^1 Q(\xi, \eta)d\eta \quad (24)$$

The transformed boundary conditions are:

$$\bar{\Theta}'_0(0) = 0; \quad \bar{\Theta}'_0(1) = 0 \quad (25)$$

Again, the equation cannot be solved analytically for the same previous reason and the equation (23) is solved numerically using Finite Element Method.

Finally, in order to obtain the final temperature field, the inversion formula (18) must be utilized and the summation must be truncated to a finite value ( $n_{\max}$ ).

## 5. RESULTS AND DISCUSSION

After describing the problem, the parallel plate fins formulation and the solution methodology, in this section the results are shown. The heat sink (HS) presents an square dimension and, consequently,  $\beta$  values 1. The value of  $\text{Bi}\gamma$  for convection is 0.1 and on the regions where the fins are located  $\text{Bi}\gamma$  is 3. The heat flux from the chip is shown in figure 2a.

In the current work, four different cases were tested. The first one was the heat sink without fins, under the effects of heat flux from the chip and convection. The second case has its heat dissipation increased by one fin with width of 0.1 of the length of the heat sink. The 2D view of the heat sink is shown on figure 2b. Third case introduces two fins on the heat sink. It must be noticed that the width of each sink must be same as the previous case for heat dissipation comparison. The width of the fin affects the value of Biot and, for this reason, the same width is applied in all the fins of this work. Figure 2c shows the two-fin heat sink. Finally, a four-fin layout heat sink was tested, which is a more common heat-sink layout and is shown on figure 2d. The edges of the chip and the fins are indicated on table 1.

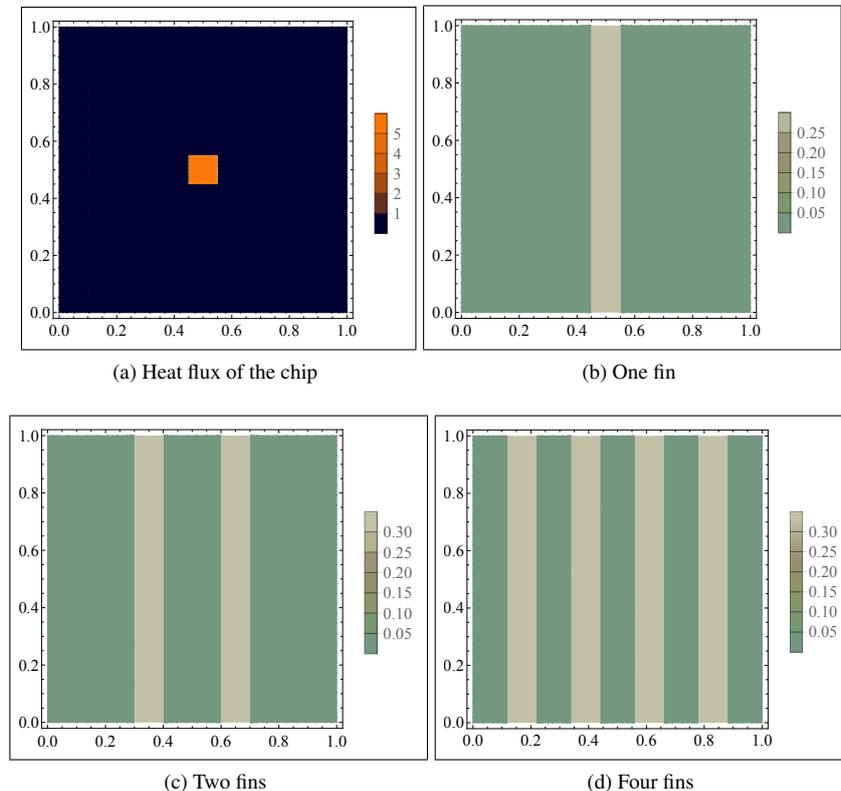


Figure 2: Contour plot of the heat flux and the fin layout for the considered cases.

Table 1: Edges of chip and fins

Components of HS	$\xi_i$	$\xi_f$	$\eta_i$	$\eta_f$	Components of HS	$\xi_i$	$\xi_f$	$\eta_i$	$\eta_f$
heat flux	0.45	0.55	0.45	0.55	4-fin case	0.12	0.22	0	1
1-fin case	0.45	0.55	0	1	4-fin case	0.34	0.44	0	1
2-fin case	0.3	0.4	0	1	4-fin case	0.56	0.66	0	1
2-fin case	0.6	0.7	0	1	4-fin case	0.78	0.88	0	1

The results of convergence for the four different cases solved by CITT are presented on table 2. For analyzing the convergence of the solution six different positions of the heat sink were selected. One position is at the center of the heat sink and at the region where the chip is located, (0.5,0.5). This position (0.5,0.5) is also where is found the maximum temperature of the heat sink. Also, two positions which are not at the chip location very close, though, were selected to be analyzed, which are positions (0.42,0.58) and (0.56,0.44). The position (0.6,0.7) is next to the chip, not as close as the previous positions. And, finally, two positions far from the chip, which were (0.2,0.7) and (0.8,0.3).  $n_{\max}$  refers to the number of terms which are summed before truncated.

The first part of table 2 shows the convergence obtained for no-fins case. It can be noticed that the solution converged very fast on the far-from-chip locations, in contrast, the center of the heat sink location, which is where the chip is located, the temperature is higher. At the locations near the chip, the temperature was also higher and took more terms to converge. While 10 terms were sufficient for fully convergence at (0.2,0.7), 200 terms were necessary for the convergence at (0.56,0.44).

The convergence results for one-fin case is shown on the second part of the table 2. First, it can be noticed a decreased on the temperature at all the selected positions of the heat sink, justifying the application of fins in order to increase the heat dissipation. As can be seen, a truncation order of  $n_{\max} = 150$  was necessary for the convergence of position (0.5,0.5) while only 10 terms were necessary for the convergence for position (0.8,0.3). At (0.6,0.7), 20 terms were required for its fully convergence.

The convergence results for the two-fin heat sink layout are presented on the third part of table 2. The addition of the second fin increased the heat dissipation and, consequently, the temperature of the heat sink was lower in this case. In this case, also, at the center of the heat sink, the fully convergence required 250 terms instead of 150 of the previous cases. The complexity of this case may be a reason which justify more terms to be summed. This increase of terms to be summed also happens at positions (0.42,0.58) and (0.56,0.44), which required also 250 terms for these positions solution convergence.

Finally, the last part of table 2 shows the results for the four-fin layout heat sink. Similarly from the two-fin layout, more terms were necessary for the fully convergence at (0.5,0.5), 350 terms in this position. In contrast, in this case, the position (0.42,0.58) had converged requiring less terms, 150, and (0.6,0.7) converged summing 100 terms, the same as the previous case. The temperature along the heat sink had reduced about half from the two-fin layout, which states that the efficiency of increasing fins to heat sinks in order to increase the heat dissipation and reduce the temperature.

After analyzing the CITT convergence table 2, now it is shown the thermal profile of the solution by CITT in all the four different layouts of heat sink. The figure 3a shows the solution for the heat sink without fins, the one-fin heat sink solution is shown on figure 3b and the two-fin and four-fin layout solution are figures 3c and 3d, respectively.

Analyzing figure 3a, it can be noticed at first the isotherms curves that bounds the region where the chip is located. One important detail to be noticed is also the size of the inner dark red isotherm, which presents similar dimensions as the chip. The temperature in the no-fin heat sink varies between a little more than 0.6175 and a little less than 0.5975. Figure 3b presents one fin at the center of the heat sink and the increase of the heat dissipation on the region of the fin is noticed by the darker blue stains exactly where the fin is found. The size of the inner dark red isotherm is smaller in this case, indicating again a more intense heat dissipation. Also, all the heat sink presents lower temperatures in comparison with the previous case without fins.

Figure 3c shows the CITT solution for the two-fin case, which has a visual expressive reduction of temperature where the fins are located, at  $0.3 \leq \xi \leq 0.4$  and  $0.6 \leq \xi \leq 0.7$ . Also, the inner dark red isotherm is smaller from the one-fin case and all the heat sink presents lower temperatures from the previous cases, the maximum temperature on this heat sink does not achieve 0.1. Finally, the four-fin layout heat sink is shown in figure 3d. Because, this layout present equally spaced fins in all the extension of the heat sink, the thermal profile of this case resemble with the no-fin case. However, the inner dark red isotherm is reduced to a small point on the profile and maximum temperature at the heat sink is reduced from 0.619608, of the no-fin case, to 0.0576527 with this current layout. This four-fin case is, then, the most efficient heat sink layout shown in this work, dissipating more heat and reducing the temperature for the lower values.

Table 2: Temperature  $\Theta(\xi, \eta)$  convergence for different layouts of heat sink solved by CITT

No fin						
$n_{\max}$	$\Theta(0.2, 0.7)$	$\Theta(0.42, 0.58)$	$\Theta(0.5, 0.5)$	$\Theta(0.56, 0.44)$	$\Theta(0.6, 0.7)$	$\Theta(0.8, 0.3)$
10	0.599113	0.608526	0.619605	0.611228	0.602554	0.599113
20	0.599113	0.608530	0.619775	0.611252	0.602550	0.599113
50	0.599113	0.608526	0.619605	0.611228	0.602551	0.599113
100	0.599113	0.608526	0.619610	0.611224	0.602551	0.599113
150	0.599113	0.608526	0.619608	0.611224	0.602551	0.599113
200	0.599113	0.608526	0.619608	0.611225	0.602551	0.599113
250	0.599113	0.608526	0.619608	0.611225	0.602551	0.599113
300	0.599113	0.608526	0.619608	0.611225	0.602551	0.599113
350	0.599113	0.608526	0.619608	0.611225	0.602551	0.599113
One fin						
$n_{\max}$	$\Theta(0.2, 0.7)$	$\Theta(0.42, 0.58)$	$\Theta(0.5, 0.5)$	$\Theta(0.56, 0.44)$	$\Theta(0.6, 0.7)$	$\Theta(0.8, 0.3)$
10	0.153132	0.159512	0.168785	0.161935	0.153887	0.153132
20	0.153132	0.159414	0.169485	0.161726	0.153883	0.153132
50	0.153132	0.159411	0.169315	0.161702	0.153883	0.153132
100	0.153132	0.159411	0.169320	0.161698	0.153883	0.153132
150	0.153132	0.159411	0.169318	0.161698	0.153883	0.153132
200	0.153132	0.159411	0.169318	0.161699	0.153883	0.153132
250	0.153132	0.159411	0.169318	0.161699	0.153883	0.153132
300	0.153132	0.159411	0.169318	0.161699	0.153883	0.153132
Two fins						
$n_{\max}$	$\Theta(0.2, 0.7)$	$\Theta(0.42, 0.58)$	$\Theta(0.5, 0.5)$	$\Theta(0.56, 0.44)$	$\Theta(0.6, 0.7)$	$\Theta(0.8, 0.3)$
10	0.0763385	0.0842956	0.0948784	0.0871884	0.0782018	0.0763385
20	0.0763385	0.0841974	0.0955795	0.0869797	0.0781976	0.0763385
50	0.0763385	0.0841937	0.0954094	0.0869557	0.0781982	0.0763385
100	0.0763385	0.0841937	0.0954142	0.0869515	0.0781981	0.0763385
150	0.0763385	0.0841936	0.0954121	0.0869519	0.0781981	0.0763385
200	0.0763385	0.0841935	0.0954119	0.0869521	0.0781981	0.0763385
250	0.0763385	0.0841936	0.0954122	0.0869520	0.0781981	0.0763385
300	0.0763385	0.0841936	0.0954122	0.0869520	0.0781981	0.0763385
350	0.0763385	0.0841936	0.0954122	0.0869520	0.0781981	0.0763385
Four fins						
$n_{\max}$	$\Theta(0.2, 0.7)$	$\Theta(0.42, 0.58)$	$\Theta(0.5, 0.5)$	$\Theta(0.56, 0.44)$	$\Theta(0.6, 0.7)$	$\Theta(0.8, 0.3)$
10	0.0374997	0.0465666	0.057119	0.0494305	0.0405881	0.0374997
20	0.0374997	0.0464686	0.0578201	0.0492219	0.0405839	0.0374997
50	0.0374997	0.0464649	0.0576499	0.0491979	0.0405845	0.0374997
100	0.0374997	0.0464648	0.0576547	0.0491937	0.0405844	0.0374997
150	0.0374997	0.0464647	0.0576527	0.0491941	0.0405844	0.0374997
200	0.0374997	0.0464647	0.0576525	0.0491943	0.0405844	0.0374997
250	0.0374997	0.0464647	0.0576527	0.0491943	0.0405844	0.0374997
300	0.0374997	0.0464647	0.0576528	0.0491942	0.0405844	0.0374997
350	0.0374997	0.0464647	0.0576527	0.0491942	0.0405844	0.0374997
400	0.0374997	0.0464647	0.0576527	0.0491942	0.0405844	0.0374997
450	0.0374997	0.0464647	0.0576527	0.0491942	0.0405844	0.0374997

## 6. CONCLUSION

This paper presented the thermal analysis of a heat sink dissipating heat from a solid-state electronic utilized on a solar panel solved by Classical Integral Transform Technique. The mathematical formulation of parallel plate fins was described and applied on the problem formulation. The  $\xi$  dependence on the Biot number made unworkable the achievement of an analytical solution for the problem. However, the numerical discretization by Finite Element Method made possible to solve the differential equations (20) and (23). To obtain the final value of  $\theta$ , the  $n_{\max}$  terms needed to be summed until its fully convergence.

The convergence analysis showed that CITT has a great performance having no difficulties to obtain high accuracy with very few terms in the solution summation far from the heat flux, and required more terms near the chip. The Classical Integral Transform Technique has shown to be a good alternative method for this kind of problem. Finally, the inclusion of fins performed the expected solution, which was the progressive reduction of the temperature as the number of fins with

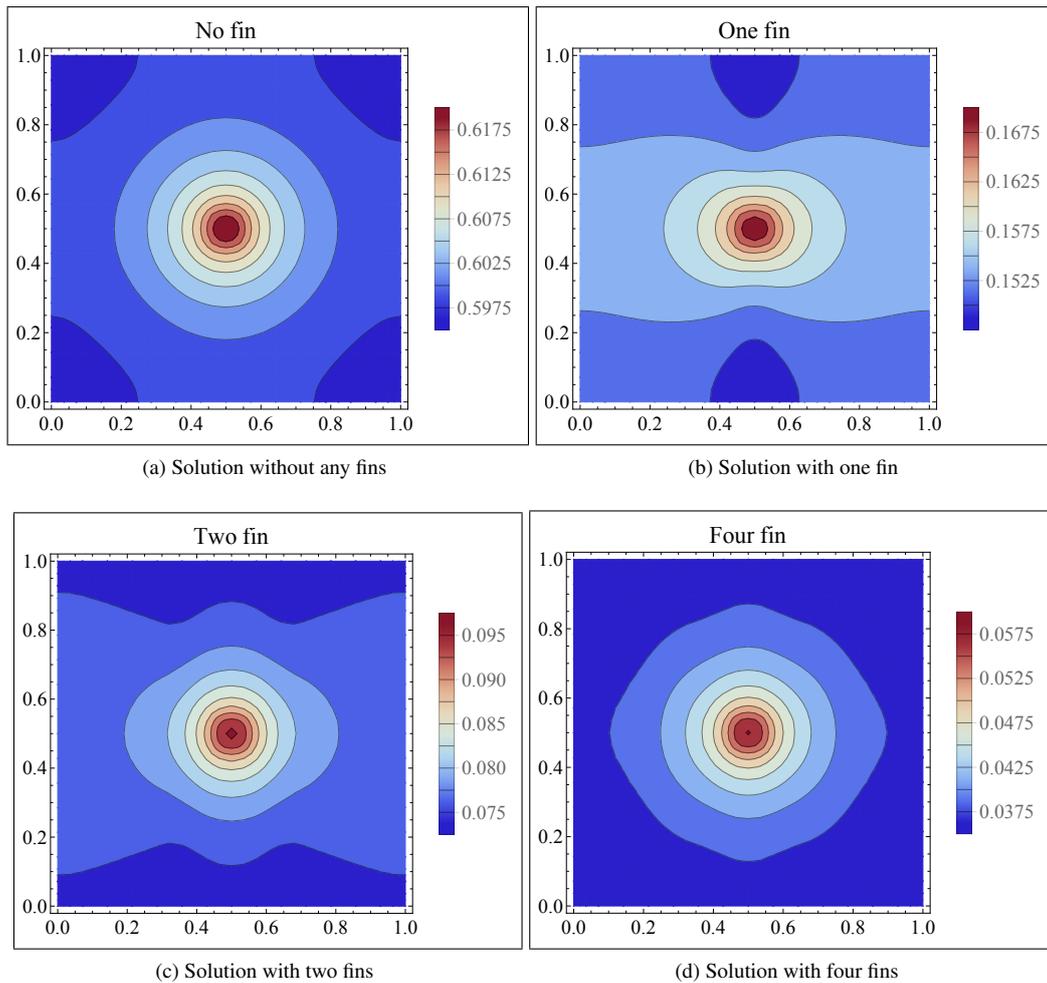


Figure 3: Contour plot of the CITT solutions for the considered cases.

same width was increased. The four-fin layout was the most efficient for dissipating the oncoming heat flux and reducing the temperature.

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## 8. RESPONSIBILITY NOTICE

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