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ANALYSIS OF HEAT AND MASS SIMULTANEOUS TRANSFER IN AQUEOUS SOLUTION DESCENDENT FLOW IN THE VAPOUR ABSORPTION PROCESS

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Abstract*. The present work aims at analyzing the simultaneous transfer of heat and mass present in absorbers in the systems of refrigeration by vapor absorption. The Generalized Integral Transformation Technique (GITT) is used to solve the coupled equations of the thermal and mass fields in the flow of the descending film formed by the solution around the outer wall of the absorber tube. The results presented, in the form of tables and graphs, allow to analyze the influence that the coupling factor and the number of Lewis exerts on the temperature field and the concentration field. Parameters of practical interest are obtained, such as the Sherwood number, the Nusselt number and the mass flow in the interface. The results obtained are compared with values found in the specialized literature, with good agreement between them.*

Keywords: *simultaneous transfer of heat and mass, GITT, temperature field, concentration field.*

1. INTRODUCTION

One of the main components of an absorption refrigeration machine is the absorber, and this component often dictates the performance of the system. The absorption process occurring within this component is extremely complex and involves not only the simultaneous transfer of heat and mass associated with absorption but also the flow of a liquid film over the absorber tube (Xavier, 2010).

The heat transfer in the descendent flow of the LiBr-H_2O aqueous solution in the vapor absorption process has been studied both theoretically and experimentally (Grigor'Eva and Nakoryakov,1977; Grossman,1983; Xavier, 2010; KaramiandFarhanied, 2011; Giannetti *et al*., 2017). A fairly comprehensive bibliographic review can be found in the (Wassenaar, 1994; Killion and Garimella, 2001).

Even with the large number of studies already done in the analysis of absorption refrigeration, the theme still continues to arouse interest of researchers mainly dealing with absorbers which is an important component of the system. The interface is the region that requires a more complex analysis, due to the bounded boundary conditions, represented by complex formulations and greater mathematical difficulties associated with obtaining the temperature and concentration fields.

With the evolution of state-of-the-art computers that increasingly use sophisticated numerical models capable of representing real behavior, complex and highly relevant problems can be simulated with a high degree of accuracy and estimated error control. In the present study we use the GITT, of an analytical-numerical hybrid nature, which presents itself as a consecrated methodology having been successfully used in several classes of heat and mass diffusion problems (Mikhailov and Özisik,1984).

The present study has great relevance, since, from it, it is possible to develop parameters for optimization of the process of heat transfer and mass, as well as it becomes possible to develop a more systematic and precise design of vapor absorbers in absorption refrigeration systems.

2. MATHEMATICAL MODELING

The problem to be studied is a horizontal tube in which the absorbent solution flows over it, as shown in figure 1.

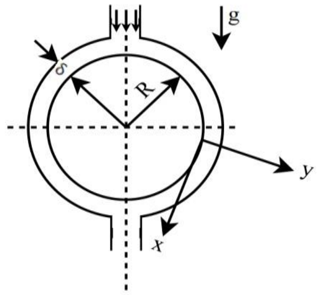

Figure 1. Illustration of the problem

For the mathematical modeling of the proposed physical problem, the following considerations were made:

- The system is in steady state;
- The liquid is Newtonian and has constant physical properties. The value of the properties depends on the liquid;
- The movement of the liquid is affected only by gravity and friction in the wall of the tube;
- The increase in mass flow of the mixture along the tube due to the absorption of vapor is negligible;
- At the interface the vapor and liquid are in steady state. The relationship between interface temperature and concentration is linear with constant coefficients;
- All absorption heat is released at the interface;
- The liquid is a binary mixture and only one component is present in the vapor phase;
- There is no heat transfer from the liquid to the vapor, nor heat transfer by radiation, viscous dissipation, pressure gradients, concentration gradients (Dufour effect), or gravitational effects;
- There is no diffusion because of pressure gradients, temperature gradients (Soret effect), or chemical reactions;
- The diffusion of heat and mass in the direction of flow is negligible in relation to diffusion perpendicular to the direction of flow;
- The thickness of the film is extremely small relative to the radius of the tube. This allows the formulation of the problem in rectangular coordinates.

Taking into account the simplifying hypotheses presented, the governing equations take the form:

Energy equation

$$u(y)\frac{\partial T(x,y)}{\partial x} = \alpha\frac{\partial^2 T(x,y)}{\partial y^2}; \quad 0 < y < \delta; \quad x > 0 \tag{1}$$

Where α represents the thermal diffusivity, δ represents the film thickness and u (y) is the velocity field, given by:

$$u(y) = \frac{3}{2}\bar{u}\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] \tag{2}$$

Concentration equation

$$u(y)\frac{\partial C(x,y)}{\partial x} = D\frac{\partial^2 C(x,y)}{\partial y^2}; \quad 0 < y < \delta; \quad x > 0 \tag{3}$$

Boundary conditions of energy and concentration

$$y = 0 \quad \rightarrow \quad \left.\frac{\partial T(x,y)}{\partial y}\right|_{y=0} = 0 \tag{4}$$

$$y = \delta \quad \rightarrow \quad -k\left.\frac{\partial T(x,y)}{\partial y}\right|_{y=\delta} = H_{abs}\rho\frac{D}{C_p}\left.\frac{\partial C(x,y)}{\partial y}\right|_{y=\delta} \tag{5}$$

$$y = 0 \quad \rightarrow \quad \left.\frac{\partial C(x,y)}{\partial y}\right|_{y=0} = 0 \tag{6}$$

$$y = \delta \quad \rightarrow \quad T(x, y) = AC(x, y) + B \tag{7}$$

Inlet conditions

$$x = 0 \quad \rightarrow \quad T(x, y) = T_0 \tag{8}$$

$$x = 0 \quad \rightarrow \quad C(x, y) = C_0 \tag{9}$$

2.1 Dimensionless of the problem

For the analysis of the problem, the following dimensionless parameters were defined, given by equations (10a-i), with the objective of solving not only a particular problem but a class of problems that are defined by the same proposed model.

$$X = \frac{1}{Pe}\frac{x}{\delta}, \quad Y = \frac{y}{\delta}, \quad U(Y) = \frac{u(y)}{\bar{u}}, \quad Re = \frac{\bar{u}.\delta}{\nu}$$

$$\theta(X, Y) = \frac{T(x, y) - T_0}{T_e - T_0}, \qquad W(X, Y) = \frac{C(x, y) - C_0}{C_e - C_0}, \quad Le = \frac{\alpha}{D} \tag{10a-i}$$

$$T_e = AC_0 + B, \quad T_0 = AC_e + B$$

Where ν is the kinematic viscosity, $\bar{u}$ is the average flow velocity, D is the mass diffusivity, T_0 is the initial temperature, T_e is the temperature of a solution of mass fraction C_0 in equilibrium with the vapor, C_0 is the initial concentration, C_e is the mass fraction of a solution of temperature T_0 in equilibrium with the vapor, and Re, Le and Pe are, respectively, the Reynolds, Lewis and Peclet numbers.

Applying the dimensionless parameters in equations (1), (3), (4), (5), (6), (7), (8) and (9), we find the main equations, the inlet conditions and the conditions of contour in the dimensionless form:

Energy equation

$$U(Y)\frac{\partial\theta(X, Y)}{\partial X} = \frac{\partial^2\theta(X, Y)}{\partial Y^2}; \quad 0 < Y < 1 \ ; \quad X > 0 \tag{11}$$

Concentration equation

$$U(Y)\frac{\partial W(X, Y)}{\partial X} = \frac{1}{Le}\frac{\partial^2 W(X, Y)}{\partial Y^2}; \quad 0 < Y < 1; \quad X > 0 \tag{12}$$

Dimensionless boundary conditions

$$Y = 0 \quad \rightarrow \quad \left.\frac{\partial\theta(X, Y)}{\partial Y}\right|_{Y=0} = 0 \tag{13}$$

$$Y = 1 \quad \rightarrow \quad \left.\frac{\partial\theta(X, Y)}{\partial Y}\right|_{Y=1} = f_{ac}\left.\frac{\partial W(X, Y)}{\partial Y}\right|_{Y=1} \tag{14}$$

$$Y = 0 \quad \rightarrow \quad \left.\frac{\partial W(X, Y)}{\partial Y}\right|_{Y=0} = 0 \tag{15}$$

$$Y = 1 \quad \rightarrow \quad \theta(X, 1) + W(X, 1) = 1 \tag{16}$$

Dimensionless inlet conditions

$$X = 0 \quad \rightarrow \quad \theta(0, Y) = 0 \tag{17}$$

$$X = 0 \quad \rightarrow \quad W(0, Y) = 0 \tag{18}$$

Where *fac* is a coupling factor at the liquid-vapor interface determined by the expression: $f_{ac} = -\frac{H_{abs}}{C_p}\frac{D}{\alpha}\frac{C_e - C_0}{T_e - T_0}$.

2.2 Auxiliary problem of the energy eigenvalue

Auxiliary problem for the thermal field lies in the typical Sturm-Liouville problem. The auxiliary eigenvalue problem for the determination of the temperature field is written as follows:

$$\frac{d^2\psi_i(Y)}{dY^2} + \mu_i^2 U(Y)\psi_i(Y) = 0; \quad 0 < Y < 1; \quad X > 0 \tag{19}$$

$$\frac{d\psi_i(Y)}{dY} = 0 \qquad \rightarrow \qquad Y = 0 \tag{19a}$$

$$\frac{d\psi_i(Y)}{dY} = 0 \qquad \rightarrow \qquad Y = 1 \tag{19b}$$

In the present work, the signal-count method is used to determine the eigenvalues (μ_i) of the temperature field, the autofunctions, $\psi_i(Y)$, and the norms (N_i), as described by (Mikhailov and Vulchanov, 1983). The signal counting method was implemented in the computational code in the FORTRAN platform INTEL for the solution of the associated eigenvalue problem.

2.3 Auxiliary problem of the concentration eigenvalue

The auxiliary eigenvalue problem for the determination of the concentration field is written as follows:

$$\frac{d^2\Phi_i(Y)}{dY^2} + \beta_i^2 U(Y)\Phi_i(Y) = 0; \quad 0 < Y < 1; \; X > 0 \tag{20}$$

$$\frac{d\Phi_i(Y)}{dy} = 0 \qquad Y = 0 \tag{20a}$$

$$\Phi_i(Y) = 0 \qquad Y = 1 \tag{20b}$$

In the same way, the signal-count method is used to determine the eigenvalues (β_i), the eigenfunctions, $\Phi_i(Y)$, and the norms(M_i).

3. TREATMENT OF PARTIAL DIFFERENTIAL EQUATIONS USING GITT

Following GITT is methodology of use, we will define a transformed-inverse pair with the purpose of reducing the original problem, which is two partial differential equations, in an infinite and coupled system of ordinary differential equations. In a second moment, the inverse formula can be used to obtain the solution of the original problem (Cotta, 1993 e 1998).

3.1 Integral Transformation of the Temperature Field

$$\theta(X,Y) = \sum_{i=1}^{\infty} \frac{\psi_i(Y)\bar{\theta}_i(X)}{N_i^{1/2}} \qquad \text{Inverse formula} \tag{21a}$$

$$\bar{\theta}_i(X) = \frac{1}{N_i^{1/2}} \int_0^1 U(Y)\psi_i(Y)\theta(X,Y)\,dY \qquad \text{Potential transformed} \tag{21b}$$

3.2 Integral Transformation of the Concentration Field

$$W(X,Y) = \sum_{i=1}^{\infty} \frac{\Phi_i(Y)\bar{W}_i(X)}{M_i^{1/2}} \qquad \text{Inverse formula} \tag{22a}$$

$$\bar{W}_i(X) = \frac{1}{M_i^{1/2}} \int_0^1 U(Y)\Phi_i(Y)W(X,Y)\,dY \qquad \text{Potential transformed} \tag{22b}$$

The integral transformation process of equations (11) and (12) is developed by applying the integral operators with the help of auxiliary and transform-inverse pairs, transforming the system of partial differential equations into a system of ordinary differential equations. For the solution of the transformed system we used the integral balance resource that was suggested by (Cotta, 1993) and used in the thesis of (Xavier, 2010).

System of transformed equations

$$\begin{cases} \dfrac{d\bar{\theta}_i(X)}{dX} + \mu_i^2 \bar{\theta}_i(X) - \sum_{j=1}^{\infty} CT1_{ij} \dfrac{d\bar{W}_j(X)}{dX} = 0 \\ \dfrac{d\bar{W}_i(X)}{dX} + \dfrac{\beta_i^2}{Le} \bar{W}_i(X) - \sum_{j=1}^{\infty} CC1_{ij}\, \bar{\theta}_j(X) - \sum_{j=1}^{\infty} CC2_{ij} \dfrac{d\bar{\theta}_j(X)}{dX} + CC3_i = 0 \end{cases} \tag{23}$$

Transform inlet conditions

$$\bar{\theta}_i(0) = 0 \quad e \quad \bar{W}_i(0) = 0 \tag{23a-b}$$

The coefficients:

$$CT1_{ij} = f_{ac} Le.\tilde{\psi}_i(1) \int_0^1 U(Y) \tilde{\Phi}_j(Y)\, dY \qquad CC1_{ij} = \frac{1}{Le} \frac{d\tilde{\Phi}_i(1)}{dY} \tilde{\psi}_j(0) \tag{24a-b}$$

$$CC2_{ij} = \frac{1}{Le} \frac{d\tilde{\Phi}_i(1)}{dY} \int_0^1 \left[\int_0^{Y'} U(Y') \tilde{\psi}_j(Y')\, dY' \right] dY \qquad CC3_i = \frac{1}{Le} \frac{d\tilde{\Phi}_i(1)}{dY} \tag{24c-d}$$

4. RESULTS

The transformed problem was solved by computer code written in the Fortran programming language using **Fortran Powerstation 4.0** software. The code was implemented through the DIVPAG subroutine of the IMSL library using a tolerance of $10^{-10}$to determine the automatic error in the evaluation of temperature and concentration fields. The results are presented in the form of graphs and tables. The analysis of the solution is made taking into account the influence that the coupling factor and the number of Lewis exerts on the temperature field and the concentration field.

4.1 Validation of results

For the purposes of benchmarking the results of the present work were confronted with results found in the specialized literature, particularly in (Xavier, 2010). The comparison is made for the case where Le = 1000 and that the coupling factor assumes the values of 0.01 and 0.1. In Figures (2-4), the dimensionless mass flow, the Nusselt number and the Sherwood number as a function of the dimensionless length are compared respectively. In all cases, a good concordance between the results can be verified.

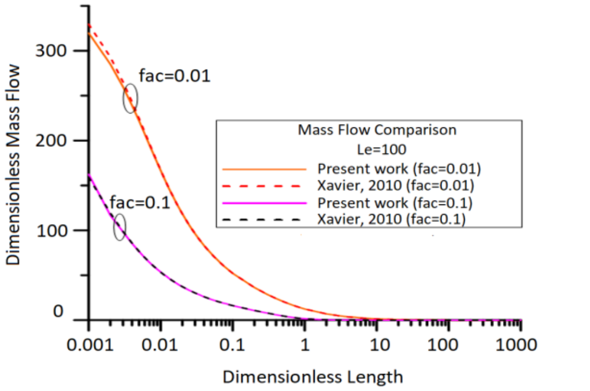

Figure 2: Dimensionless mass flow in function of dimensionless length.

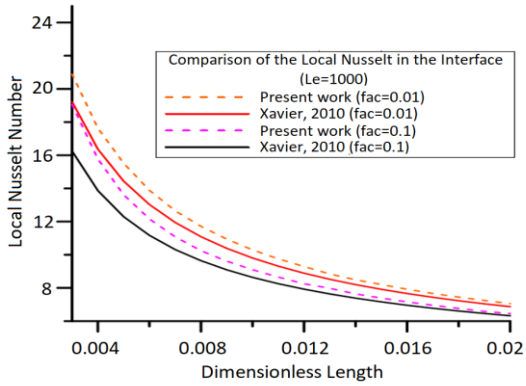

Figure 3: Nusselt number local in function of dimensionless length.

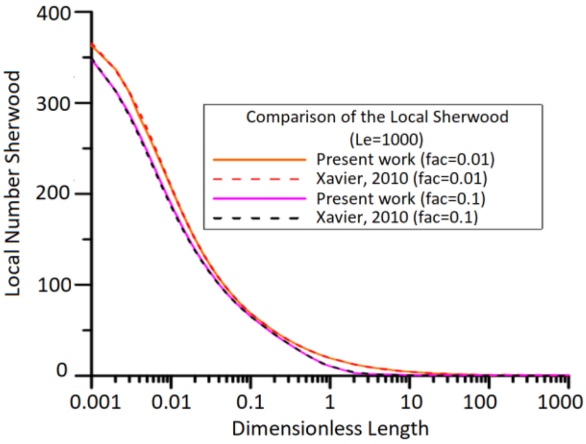

Figure 4: Sherwood number local in function of dimensionless length.

4.2 Effect of coupling factor

The influence of the coupling factor on the temperature and concentration fields is evaluated. Different values are adopted for the coupling factor and a specified Lewis number.

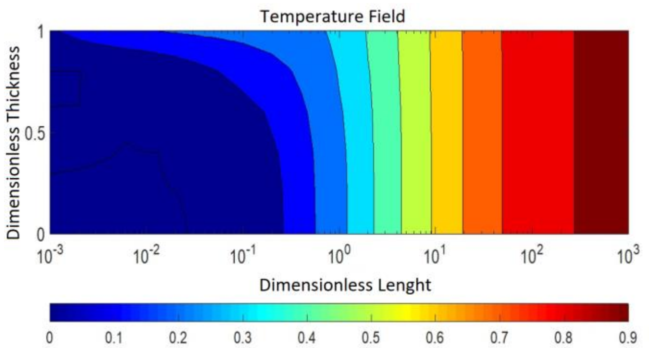

Figure 5: Temperature Field, considering fac= 0.01 and Le=1000.

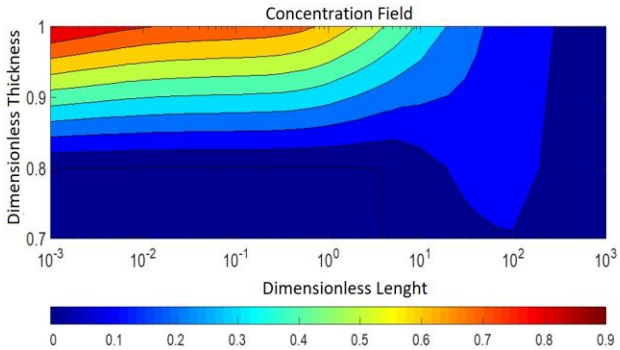

Figure 6: Concentration Field, considering fac= 0.01 and Le=1000.

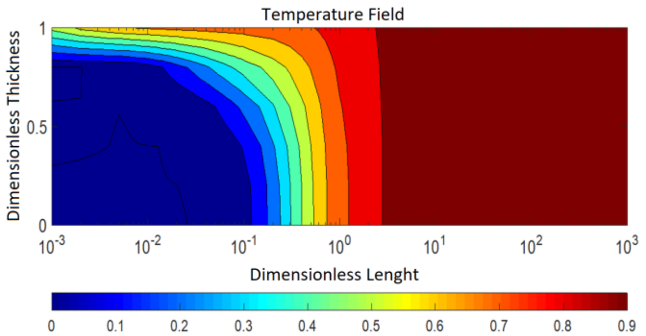

Figure 7: Temperature Field, considering fac= 0.1 and Le=1000.

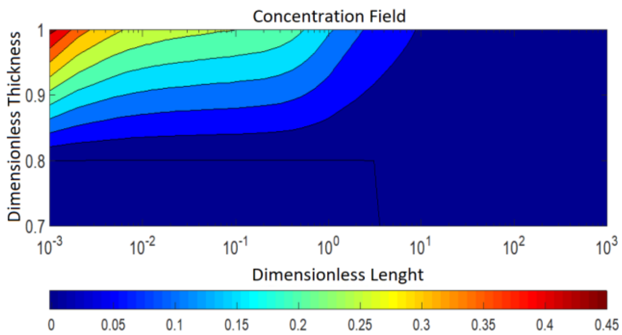

Figure 8: Concentration Field, considering fac= 0.1 and Le=1000.

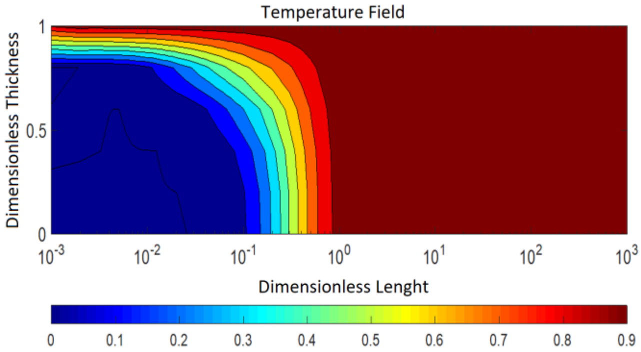

Figure 9: Temperature Field, considering fac= 1 and Le=1000.

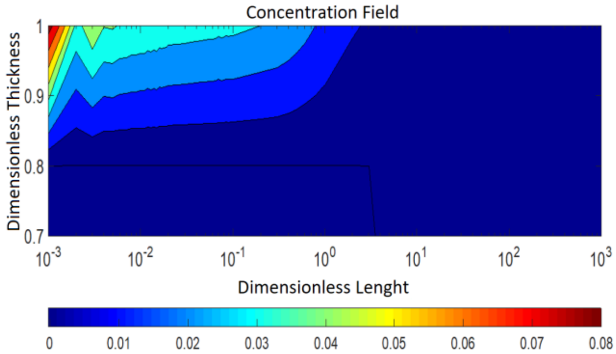

Figure 10: Concentration Field, considering fac= 1 and Le=1000.

By means of the figures (5 - 10) it is noticed that the value of the coupling factor directly influences the distribution of the temperature and concentration fields. It is possible to verify that the increase in the value of the *fac* produces a change in the temperature field, so that it tends to develop completely for smaller values of the dimensionless length. Through these figures it can also be concluded that the increase in the value of the *fac* causes the concentration field to have small variations, so as to modify very little only in the region near the interface.

4.3 Effect of Lewis number

The influence of the number of Lewis on the temperature and concentration fields is evaluated. It adopts, for a specified *fac* different values for the number of Lewis.

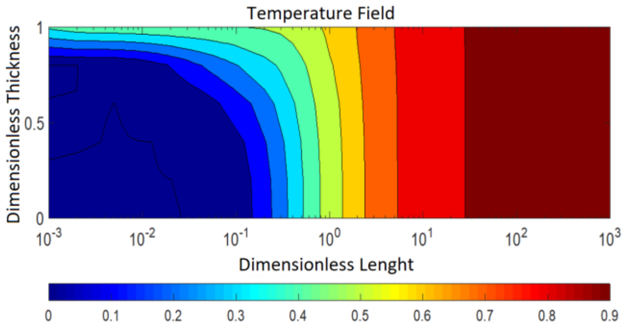

Figure 11: Temperature Field, considering fac= 0.1 and Le=100.

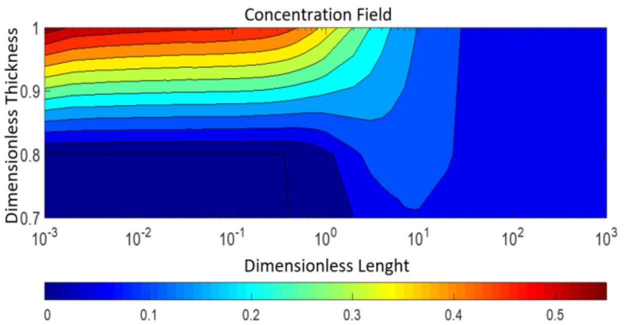

Figure 12: Concentration Field, considering fac= 0.1 and Le=100.

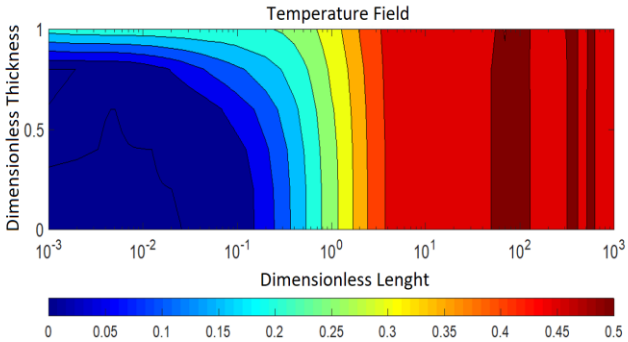

Figure 13: Temperature Field, considering fac= 0.1 and Le=10.

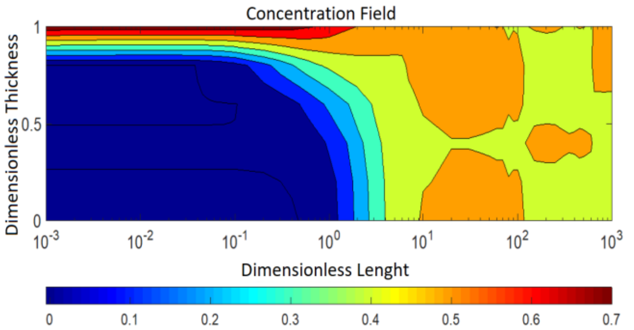

Figure 14: Concentration Field, considering fac= 0.1 and Le=10.

By means of figures (11 - 14) one can verify the influence of the Lewis number on the thermal and mass fields. For the case where Le = 10 it can be seen that the temperature and concentration fields develop completely converging to the value around 0.5, differently to cases where Le = 100 and Le = 1000.

5. CONCLUSIONS

It is concluded from the analysis of the results obtained that the application of GITT is effective in solving the problem proposed, since the presented formulation was validated with the results found in the specialized literature. In this way, the objectives were reached satisfactorily, where the influence of the coupling factor and the number of Lewis on the development of the thermal and mass fields was shown. From the theoretical study carried out, one can optimize the perfomance and the dimensioning of vapor absorbers in absorption refrigeration systems.

6. ACKNOWLEDGEMENTS

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