17th Brazilian Congress of Thermal Sciences and Engineering
November 25th-28th, 2018, Águas de Lindóia, SP, Brazil

ENCIT-2018-0703
TEMPERATURE PROFILE AND EFFICIENCY OF STRAIGHT FINS WITH TEMPERATURE DEPENDENT INTERNAL HEAT GENERATION AND RADIATION EFFECTS

L.F.S. Ferreira
L.C. Chaves
Y. L. Souza
M.F. Curi
Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, Department of Mechanical Engineering, Rio de Janeiro, Rodovia Mário Covas, lote J2, quadra J, Rio de Janeiro, 23812-101, Brazil
marcos.curi@cefet-rj.br

Abstract. *Fins are of great importance in many applications of mechanical engineering, and are widely studied for optimizes the heat transfer between a surface and the ambient. In this paper, we analyze with a routine that employs the Method of Lines from Wolfram Mathematica software the temperature distribution within the fin, to estimate the efficiency of convective straight fins with temperature dependent internal heat effects of radiation,* which increases the partial differential equation degree of non-linearity. *In order to show the effectiveness of the computational method, the results obtained from a direct study of the effects of some physical parameters are demonstrated in this work, presenting the temperature distribution for a range of parameters values appeared in the mathematical formulation. The proposed method in this work shows that the implemented procedure is very effective and convenient for convective fin problems with an excellent agreement when compared with literature, achieving suitable results of such problems.*

Keywords: *Extended Surface, Heat Transfer, Fin Efficiency, Variable Heat generation, Radiation, Computational Methods*

1. INTRODUCTION

In order to increase the heat exchange between the primary surface and the environment, it is common to use extended surfaces. Such surfaces are used extensively in common applications in industries such as refrigeration, internal combustion engines and heat exchangers.

The mechanism of fin heat transfer is the heat conduction of the base of the fin with the dissipation of the same by means of the convection. In real situations, there is also the effect of thermal exchange by radiation, which increases the degree of non-linearity of the equations that model the problem. The influence of such thermal changes is due, among other parameters, to the temperature difference between the surface of the fin and its vicinity, the surface area and its geometric shape. Fins with rectangular section are widely used and their study is simplified by the non-variation of such area along the fin.

According to Kern and Kraus (1972), there are several techniques to optimize the performance and minimize production costs. For example, it is a consensus that the straight fins with a parabolic concave profile achieves the maximum heat dissipation for a given area, but producing fins with this format is relatively expensive (Kern and Kraus, 1972; Yu and Chen, 1998). The influence of the heat generation as well as the thermal conductivity variation was studied by Ghasemi *et al*. (2014), through the TMD. In Ferreira *et al*. (2017), the same solution was obtained by means of an analytical method for the case of constant thermal conductivity. Fins with influence of radiation, convection and generation were extensively studied by Mabood et al. (2013) and Mosayebidorcheh et al. (2015), without considering the study of their efficiency.

In this work we study the temperature distribution with properties that vary with temperature and the effects of radiation, Wolfram Mathematica software is used to solve such problem. The default routine for getting the results is the Method of Lines. The temperature distribution inside the fin estimates the efficiency of the fin under the effect of radiation and convection and through this the effectiveness of the method used is demonstrated.

2. MATHEMATICAL PROCEDURE

The mathematical procedure follows the standard available in Mabood et al. (2013), to write the code that uses the line method to solve the final differential equation. It is considered a longitudinal flap with area section rectangular, length L, perimeter P, section area A_c, thermal conductivity k_0 and heat generation $\dot{q}$ constants.

The fin is fixed on basis with temperature T_b and loses heat to ambient with temperature T_∞ and convective coefficient h. The surface emissivity is ε. We can write the differential equation and the boundary conditions as:

$$\frac{d^2T}{dx^2} - \frac{hP}{A_c k_0}(T - T_\infty) - \frac{\varepsilon\sigma P}{A_c k_0}(T^4 - T_\infty^4) + \frac{\dot{q}}{k_0} = 0 \quad (1)$$

$$T(L) = T_b \quad (2)$$

$$\frac{dT}{dx}(0) = 0 \quad (3)$$

For dimensionless variables, the following expressions are used:

$$\theta = T/T_b \quad (4)$$

$$\theta_\infty = T_\infty/T_b \quad (5)$$

$$X = x/L \quad (6)$$

$$h_b = h(T_b - T_\infty) \quad (7)$$

$$M_c = h_b P L^2 / k_0 A_c \quad (8)$$

$$M_r = \varepsilon\sigma P T_\infty^3 L^2 / k_0 A_c \quad (9)$$

$$Q_g = \dot{q} L^2 / k_0 T_b \quad (10)$$

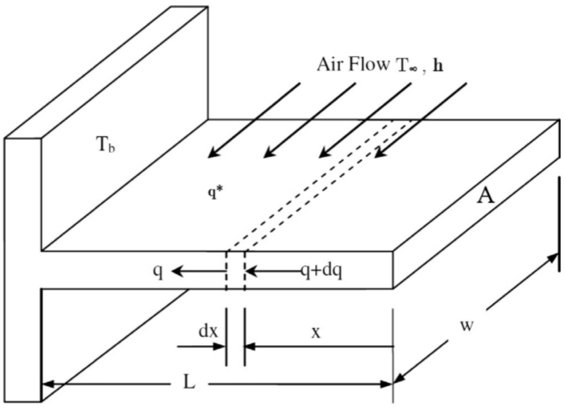

Figure1 - Rectangular fin with internal heat generation and convective air flow.

Equation (1) can be rewritten, with their respective boundary conditions. Once the variables involved vary only with x and T, these mathematical simplifications can be disregarded for the obtained solution.

$$\frac{d^2\theta}{dX^2} - \frac{M_c}{1-\theta_\infty}(\theta - \theta_\infty)^2 - M_r(\theta^4 - \theta_\infty^4) + Q_g = 0 \quad (11)$$

$$X = 0 \rightarrow \frac{d\theta}{dX} = 0 \quad (12)$$

$$X = 1 \rightarrow \theta = 1 \quad (13)$$

Fin efficiency η_a, which will also be studied for various cases, is given by the expression:

$$\eta_a = \frac{Q}{Q_{ideal}} = \frac{Q_c + Q_r}{Q_{c,T_b} + Q_{r,T_b}} \quad (14)$$

$$\eta_a = \frac{\frac{M_c}{(1-\theta_s)} \int_0^1 (\theta - \theta_s)^2 dX + M_r \int_0^1 (\theta^4 - \theta_s^4) dX}{M_c(1-\theta_s) + M_r(1-\theta_s^4)} \quad (15)$$

To get results consistent with the literature, you must specify the values of the variables before use the computational routines. In the case of constant thermal conductivity.

Mathematica uses an NDSolve subroutine, which implements the methods of the lines as numerical solution for nonlinear equations, the method can present more or less accuracy according to the discretization parameter chosen in the implementation. The results obtained are shown in the next topic.

3. RESULTS

In this topic, we show the results obtained by the computational implementation of the proposed problem, according to the parameters given by Equations (4) to (10), applied to Equation (11) with its boundary conditions. With these expressions, cases were simulated in which the parameters Q_g, M_c, M_r and θ_∞varies and generated graphs that summarize the thermal behavior.

A temperature profile is shown in Figure 2, where the parameter of the heat generation Q_g varies with the other parameters set. It is observed that with increasing the value of $Q_g$there is a gradual increase in the values of the fin temperature gradient. Note that in the base region this increase is more accentuated in the interval 1 <Q_g<1.5, the other increases in the generation of heat follow the same trend.

As it approaches the tip of the fin, the temperature values tend to equalize, since this region is considered to be thermally isolated. The form of decay of the temperature can be justified taking into account that due to a greater generation of heat the temperature of the surface of the fin also raises its temperature.

With a greater difference of temperature of the surface the exchange of heat by radiation and convection if becomes more intense due to the fourth-order dependence of the temperature on the radiation term and the square of the temperature in the case of convection.

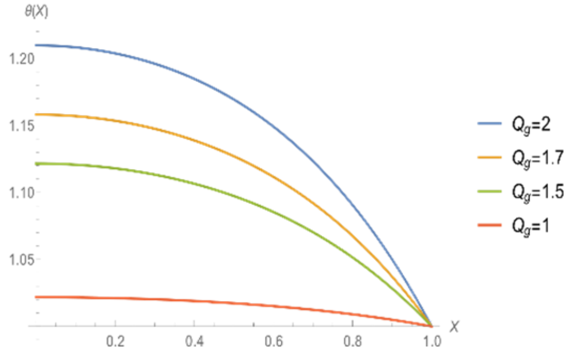

Figure 2 - Temperature distributions for four different cases of heat generation, with $\boldsymbol{M_c = M_r = 0.5}$ and $\boldsymbol{\theta_\infty = 0.2}$

In Figure 3, the temperature distributions for the variation of the radiation parameter M_r, with non-zero heat generation, are shown. As the values of M_r increase the temperature along the fin is reduced. This is a direct relation of the composition of the parameter M_r, which depends directly on the emissivity of the surface and inversely of the conductivity, for example. One of the possible configurations for an increase in the radiation term may be a higher emissivity, which already implies a heat loss accentuated by the fin. This result is corroborated in the case of a lower conductivity, with the same principle of analysis made in the previous items for a lower heat conduction along the fin.

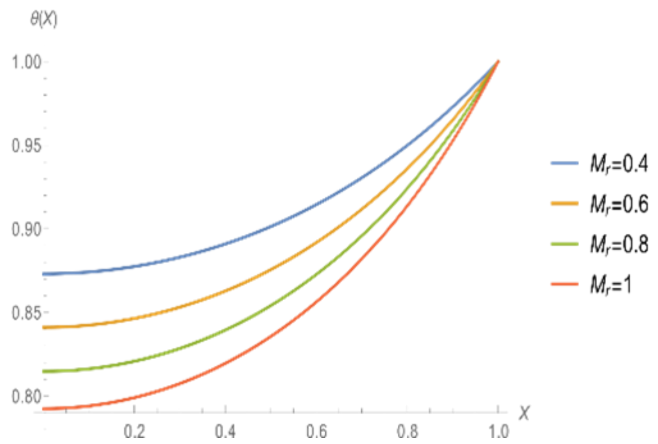

Figure 3 - Temperature distributions for four different cases of radiation parameter, with $M_c = 0.4$, $Q_g = 0.3$ and $\theta_\infty = 0.05$.

Varying the value of M_c, Figure 4 was generated while keeping the other parameters of the differential equation fixed. The heat generation was kept null, for an analysis of the convection with less influence of a power source in the fin. The analysis for convection is similar qualitatively to the study for radiation. For with the increase in the value of M_c, the temperature along the fin has its value decreased throughout its length.

However, considering that different parameters of the previous case of variation in the radiation were assumed, in this study of the influence of the convection a slightly different temperature distribution is observed, but with similar profiles. An important item is that in Figure 4 a greater ambient temperature (neighborhood) was adopted than the case studied in Figure 3, and this already produces a lower temperature gradient between the fin and the neighborhood, since $\theta_\infty = T_\infty / T_b$, consequently the heat exchange is less intense between the surface and the neighborhood for the convection heat loss variation.

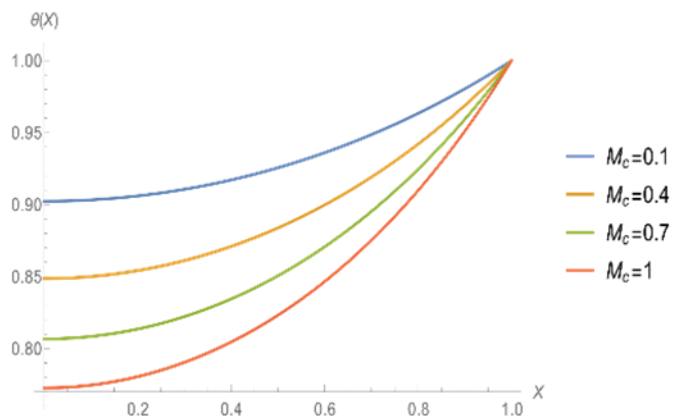

Figure 4 - Temperature distributions for four different cases of radiation parameter, with $M_r = 0.2$, $Q_g = 0$ and $\theta_\infty = 0.3$.

In Figure 5 the temperature distribution for four different temperature values of the dimensionless neighborhood as a function of the position of the fin. To study the behavior of the extended surfaces with variation of this external temperature, it can be observed, according to Figure 5, that maintaining the heat generation and the influence of constant convection and radiation, for higher values of relative ambient temperature θ_∞, we obtain a temperature distribution with higher values, that is, a greater gradient along the fin with less value of θ_∞.

In general, there are several factors to be considered in fins with radiation and convection, because due to the influence of several physical mechanisms, the simulation becomes computationally more demanding. This is also due to the relationship present in the radiative interaction because the temperature influences with a fourth order factor and this increases the computational effort.

When analyzing the fin efficiency, one must take into account some limitations inherent in the problem, for example, when the fin has heat generation, the influence of this parameter is implicitly described in the value of θ. It is thus expected that a relatively high value of heat generation Q_g leads to an inversion in the direction of heat dissipation, causing the fin base to receive the heat generated by the fin. Therefore a description of the efficiency as a function of such parameter would be limited to a very low range of such influence, and moderate heat generation values should be considered for a study of the other factors, such as M_r and M_c.

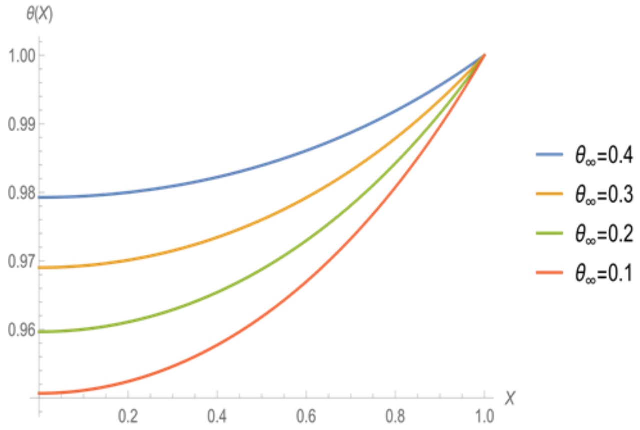

Figure 5 – Temperature distributions for four neighborhood temperature cases, with $M_r = M_c = 0.3$ and $Q_g = 0.4$.

In Figure 6 the parameters M_c and Q_g are changed in each of the efficiencies as a function of M_r. By setting the value of Q_g to 0.3, and increasing the parameter M_c by 0.2 dimensionless units, there is a general drop in efficiency which does not show great effect for values of M_r greater than 3. Decreasing the heat generation, the efficiency becomes even smaller. The temperature distribution seen in Figure 3 confirms the fact that low values of M_r imply greater efficiencies.

A decrease in efficiency is reasonable with the increase in the value of M_c, this parameter increase may be directly linked to an increase in the convective coefficient or a decrease in the thermal conductivity. With these changes, a smaller flap would be more feasible from a certain point.

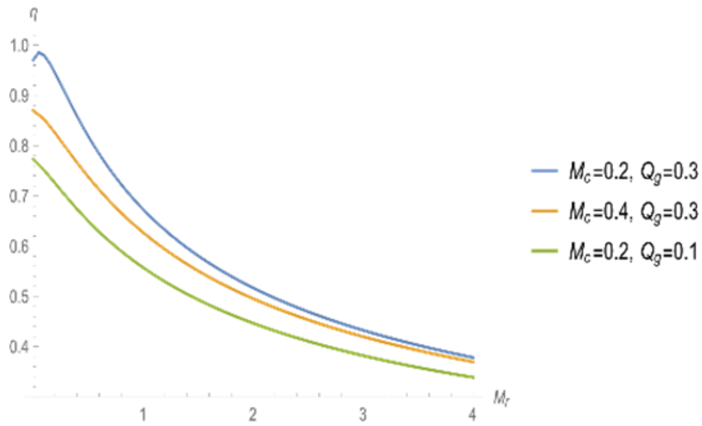

Figure 6 - Efficiency of the fin as a function of the geometric thermal parameter of radiation M_r varying M_c and Q_g.

Figure 7 shows the efficiency as a function of the geometric thermal parameter of convection M_c and the analysis is quite similar, except that in each curve besides the generation of heat the radiation parameter varies. This parameter depends on the thermal conductivity and the emissivity of the surface, for surfaces with higher emissivity and lower conductivity imply higher values of M_r , which decreases the efficiency. Similarly to the previous case, the temperature distribution seen in Figure 4, confirms the fact that low values M_c to have a higher efficiency. In the two parameter variations shown in the calculated efficiencies, a greater influence of such parameters on the efficiency variation is observed for low values of M_r and M_c, with a trend of similar values efficiencies with their growth.

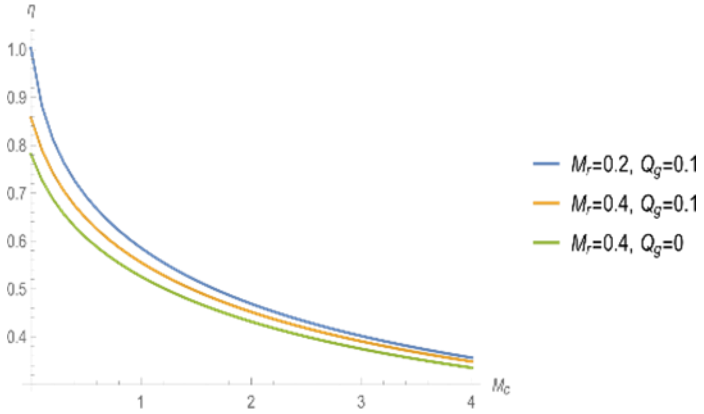

Figure 7 – Efficiency of the fin as a function of the geometric thermal parameter of radiation M_c varying M_r and Q_g.

4. CONCLUSIONS

In this work commands of *Wolfram Mathematica* software to solve the problem of temperature distribution in a fin with temperature-dependent convection, radiation and both constant thermal conductivity and heat generation.

Higher heat generation values imply a greater temperature difference between the fin and the environment and consequent heat exchange with higher performance. On the other hand, a much higher generation value than the fin cooling factors such as M_r and M_c, modify the direction of the heat flow and can cause the extended surfaces to lose their most common applicability, which is the removal of heat.

The results found that it is possible to model with a minimum amount of computational requirements. Future works, the aim is to analyze the temperature distribution in other fin formats, as well as other forms of thermal conductivity variation due to various effects, such as soot, corrosion and aging of finned structures, where both thermal conductivity and heat generation parameters vary with temperature.

5. REFERENCES

Kern, Q.D., Kraus, D.A., 1972. *Extended Surface Heat Transfer*, McGraw-Hill, New York.

Yu, L.T., Chen, C.K., 1998. “Application of Taylor transformation to optimize rectangular fins with variable thermal parameters”, *Applied Mathematical Modelling*, Vol. 22, p. 11-21.

Ghasemi, S.E., Hatami, M., Ganji, D.D., 2014. “Thermal analysis of convective fin with temperature-dependent thermal conductivity and heat generation”, *Case Studies in Thermal Engineering*, Vol. 4, p. 1-8.

Ferreira, L. F. S., Curi, M., Oliveira, L. C. C., 2017. “Analysis of temperature profile and efficiency of convective straight fins with temperature dependent thermal conductivity and internal heat generation”. In *24th ABCM International Congress of Mechanical Engineering*.

Mabood, F.; Khan, W. A.; Ismail, A. I. M., 2013. “Series Solution for Steady Heat Transfer in a Heat-Generating Fin with Convection and Radiation”. *Mathematical Problems in Engineering*, Vol. 2013, p. 7 pages.

Mosayebidorcheh, S. et al., 2015. “Optimization analysis of convective–radiative longitudinal fins with temperature-dependent properties and different section shapes and materials”. *Energy Conversion and Management*, Vol. 106, p. 1286-1294.

Khan, W. A., Aziz, A., 2012. “Transient Heat Transfer in a Heat-Generating Fin with Radiation and Convection with Temperature-Dependent Heat Transfer Coefficient”. *Heat Transfer - Asian Research*, Vol. 41 (5), p. 402-417.

Singh, S., Kumar, D., Rai, K. N., 2014. “Convective-radiative fin with temperature dependent thermal conductivity, heat transfer coefficient and wavelength dependent surface emissivity”. *Propulsion and Power Research*, Vol. 3, p. 207-221.

Bergman, T. L. et al., 2015. *Fundamentos de Transferência de Calor e Massa*. 7ª. ed. Rio de Janeiro: LTC.

Wolfram, S., 2003. *The Mathematica Book*. 5th. ed. Cambridge: Wolfram Media.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper