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ANALYTICAL TRANSIENT HEAT CONDUCTION THROUGH AN COMPOSITE REGION OF AN INFINITE SOLID CYLINDER WITH A PERFECT THERMAL CONTACT TO A SEMI-INFINITE MEDIUM

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Abstract. Aiming the development of new technologies for the petrol field operations, an innovative method to tamponing oils wells is being studied due to significant advantages over the traditional cementing procedures. In a general context, a cylinder positioned at the well hole will work, initially, as a vessel for a fluid composed of chemical species capable to generate such an intense heat after an exothermic reaction, so that a strong heat source will flow by conduction throughout the composite region from the inner wall of the cylinder to the outer surface that has a perfect thermal contact with the surroundings geological formation, which will be exposed to high values of temperatures after a certain period of time. The magnitude of the temperature will be sufficient to reach the melting temperature of the cap rock, causing the vitrification and so resulting in the complete well sealing. Carrying out an unprecedent study, the present methodology will investigate which analytical approach will provide the most realistic and bests results in the existing literature by a comparison made between two mathematical techniques for the transient conduction flow through composite regions: The generalized orthogonal expansion technique utilized in deriving the solution and the Laplace transform for solving semi-infinite and infinite medium problems.

Keywords: Tamponing Oil Wells, Conduction Flow, Heat Source, Composite Region, Orthogonal Expansion Technique, Laplace Transform

1. INTRODUCTION

After the production oil well becomes exhausted or has no more economic potential, it is provided by the law the responsibility of the owner's oil well to seal up the entire hole in order to prevent water contamination by some residual retained oil on the reservoir (Abshire *et.al.,* 2012).

The usual procedures for permanent abandonment wells is done by cementing the hole drilled on deep ocean. This tamponing method has relevant disadvantages when referring to the huge amount of time and money spent by doing so (Barclay *et.al.,* 2001). Due to this concerns raised an innovative proposal of a device that will overcome those adversities, consisting by a vertical pipe containing a mixture of different chemical species placed on the production oil well that, with an exothermic reaction, will work as strong source of energy generation that will flow by conduction through the cylinder thickness, which outer surface is at perfect contact to the geological formation interface, until the heat reaches the cap rock verified at the geological formation after an amount of time, which temperature will achieve such great value of temperature that will be sufficient to reach the melting temperature of the rock.

Two mathematical techniques will be used in order to develop the analytical approach, the generalized orthogonal expansion technique (Carslaw and Jaeger, 1959) utilizing a nondimensional analysis (Gu and O'Neal, 1995) in order to provide gain of generality for the case study and the Laplacian transform (Özisik, 1980). Both methods will be investigated to check if the use of one provide realistic results for the phenomena. The former scheme allows a solution of multi-region problems by a direct expansion without resort to integral transformation or Green's functions and it is an extension of the method of separation of variables heretofore beyond its scope (Tittle, 1965), whereas the latter is a convenient technique for the solution of composite medium problems involving regions of semi-infinite or infinite in extent (Özisik, 1968).

As a preliminary study around the subject, some assumptions would be necessary to be considered since the embryonic knowledge does not provide recommendations about optimal boundary and initial conditions. Those hypotheses will be fixed as close as possible to the existing configurations present in the literature. Then, after a tangible conclusion based on future results will allow to specify the correctness conditions that describe the phenomenon.

2. METHODOLOGY

Once a mathematical scheme should be selected (or both as a complement for a more effective solution), some thermal properties (table 1) and geometrical measures would be considered in order to gather useful information that describes the behavior for each equation that compose each mathematical method here presented.

Table 1. Thermal Properties of each existing material of the composite region

Material	$k \left[\frac{W}{m\,K}\right]$	Melting Point Temperature [ºC]
Steel AISI 304L	81	1465
Shale Rock	1,7	1500

The sketch of the case study is illustrated on figure 1, where the geometric quantities are given by the radius of each concentric layer that compose the region of study interest, where all radius measurements are taken from the center line.
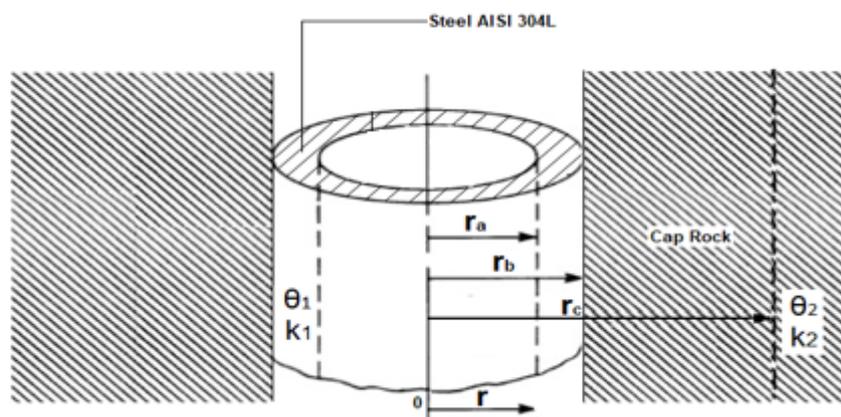

Figure 1. Schematic of the composite region with a hollow cylinder in perfect contact with the Cap Rock

Some simplifying assumptions are stablished in order to provide an initial approach of the case study, being them:

Hypothesis 1: The ratio between the cylinder length and their respective radius, L/r must be greater than 10 (Incropera, *et al.*, 2003), so that the cylinder in considered as an infinite solid, which allows a unidimensional approach considering the radial direction.

Hypothesis 2: Cap Rocks is a sedimentary rock, so that is a material with an anisotropic behavior of the flow of heat conduction, which means that the thermal conductivity of the material varies with direction (Özisik, 1980). In order to validate which mathematical tool fits better for the study, the Cap Rock will be considered as an isotropic material with a thermal conductivity that does not depends on direction.

Hypothesis 3: Geological formation is an extremely nonhomogeneous media. The moisture level embedded on the interstice of the media verified throughout the composite ground in deep sea regions, causing the necessity to consider the latent temperature in the mathematical procedures (Ingersoll, 1986). Different layers of Cap Rock, soil and other miscellaneous ground compound are also verified. The techniques of generalized orthogonal expansion and Laplace transform will be applied for a composite media limited by three homogeneous layers.

Hypothesis 4: An intense heat source generated by an exothermic reaction of a mixture made of different species. The magnitude of the temperature generated must be sufficient to melt the Cap Rock, whose melting point is unknown, but significantly superior than the Steel AISI 304 L (T=1465ºC) and, due to this fact, this source will melt the cylinder surface forming one solid-liquid interface which several relevant studies was already developed. The effect of phase-change problem will be neglected by considering that the two solid cylinders composite region are only treated as cylindrical heat sources.

Hypothesis 5: The mixture of species will be replaced for a cooper inner cylinder due to his greatness thermal conductivity, and a value of heat generation will be given. The intensity of this source value will be set aiming that the conductive flow heat in a given time is sufficient to melt the geological formation in contact with the composite concentric cylinders and can be deducted by using the technique of the Inverse Heat Conduction Problems (Özisik, 1968)

Hypothesis 6: It will be assumed that the contact between all neighbor surfaces are perfect, i.e. thermal contact resistance is neglected at any interface.

Hypothesis 7: The initial temperature is uniform throughout the whole domain at an arbitrary value. These provide initial conditions for the solution. With a system that has been operating for a period of time, the surrounding temperatures are not uniform (Gu and O'Neal, 1995).

Hypothesis 8: An imaginary far-field boundary is set at $r > r_c$ where the temperature may be taken as a constant (far-field temperature) equal to the initial value. If this point is far enough away from the pipe surface, then it's temperature will not change unless exceptionally long periods of time are considered (Coogan, 1949; Dobson, 1991).

The analytical equations about to be introduced will be solved by using the software MATLAB.

2.1 Generalized Orthogonal Expansion Equations

The following form of the Orthogonal Expansion shown below aims to derivate the solutions for a multilayer composite region. The Orthogonal expansion presented by Özisik (1980) solve the heat conduction equation, subject to certain boundary conditions, by transforming nonhomogeneous boundary conditions into homogeneous character proceeding with a superposition of three simpler problems: steady-state problem for the same region, with no heat generation, but with one non-homogeneous boundary condition at $x = x_1$; steady-state problem for the same region, with no heat generation, but with one non-homogeneous boundary condition at $x = x_{M+1}$; and time-dependent heat conduction problem for the same region, with heat generation, but subject to homogeneous boundary conditions. By splitting the problem, the Green's function approach should be used for solving nonhomogeneous problems, making an extense mathematical formulation.

In order to represent a direct method of the study case with nondimensional approach, Gu and O'Neal (1995) derived a solution giving origin of a system of equations that denies the use of Green's function, making the method applied more direct.

With the assumptions made before and applying the mathematical method by analyzing the Figure 1, the heat conduction equation in cylindrical coordinates is:

$$\frac{\partial^2 \theta_i}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_i}{\partial r} = \frac{1}{\alpha_i}\frac{\partial \theta_i}{\partial t}, \qquad r_a \leq r \leq r_c, \qquad t \geq 0 \quad (i = 1, 2) \tag{1}$$

Where θ is the temperature in excess of the initial value, r is the radial coordinate (r_a is the cooper cylinder radius and r_c is the radius that the heat flow should reach in order to melt the Cap Rock), $\propto$ is the thermic diffusivity, the i index is an integer number which identifies the current layer on analyses (for $i = 1$, stands for the Steel AISI 304L and for $i = 2$ refers to the geological formation that surrounds the cylinder.

Equation (1) is subject to the following boundary and initial conditions:

$$-k_1\frac{\partial \theta_i}{\partial r} = \frac{Q'}{2\pi r_a} ; \qquad r = r_a, \qquad t > 0 \tag{2a}$$

$$\theta_1 = \theta_{2,} \quad \frac{\partial \theta_1}{\partial r} = \frac{k_2}{k_1}\frac{\partial \theta_2}{\partial r} ; \qquad r = r_b, \qquad t > 0 \tag{2b}$$

$$\theta_2 = 0, \qquad r = r_c, \qquad t > 0 \tag{2c}$$

$$\theta_{1,2} = 0, \qquad t = 0, \quad r_a \leq r_b \leq r_c \tag{2d}$$

Where k_i is the thermal conductivity of the cylinder material and Q' is the heat generation source.

Assuming some dimensionless correlations, it will be possible to write Eq. (1) in a nondimensional approach:

$$Fo = \frac{\propto_2 t}{r^2{}_a} \tag{3a}$$

$$T^* = \frac{k_2 \theta}{Q'} \tag{3b}$$

$$r^* = \frac{r}{r_a} \tag{3c}$$

The Fourier Number (Fo) it is one of the most important parameters in the study. As explained by Özisik (1980): "is a measure of the rate of heat conduction compared with the rate of heat storage in a given volume element. Therefore, the larger the Fourier number, the deeper the penetration of heat into a solid over a given time.", T^* is a non-dimensional temperature and r^* the dimensionless radius.

In addition, for the raised correlations, a group of definitions are stablished, supporting the former considerations in order to write the final version of the solution on a nondimensional basis:

$$H = \frac{k_1}{k_2} \tag{4a}$$

$$G = \frac{\propto_1}{\propto_2} \tag{4b}$$

$$F = \frac{H}{\sqrt{G}} \tag{4c}$$

$$S_b = \frac{r_b}{r_a} \tag{4d}$$

$$S_c = \frac{r_c}{r_a} \tag{4e}$$

By means of Eq. (3) and Eq. (4), Eq. (1) can be rewritten on a dimensionless form:

$$\frac{\partial^2 T_i^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial T_i^*}{\partial r^*} = \frac{1}{G}\frac{\partial T_i^*}{\partial Fo} \qquad 1 \le r^* \le S_b, \qquad Fo \ge 0 \tag{5a}$$

$$\frac{\partial^2 T_i^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial T_i^*}{\partial r^*} = \frac{\partial T_i^*}{\partial Fo} \qquad S_b \le r^* \le S_c, \qquad Fo \ge 0 \tag{5b}$$

Equations (5a) and (5b) are subject to the following nonhomogeneous boundary and initial conditions:

$$\frac{\partial T_i^*}{\partial r^*} = -\frac{1}{2\pi H} \qquad r^* = 1, \qquad Fo > 0 \tag{6a}$$

$$\frac{\partial T_i^*}{\partial r^*} = \frac{1}{H}\frac{\partial T_2^*}{\partial r^*} \qquad r^* = S_b, \qquad Fo > 0 \tag{6b}$$

$$T_2^* = 0 \qquad r^* = S_c, \qquad Fo > 0 \tag{6c}$$

$$T_{1,2}^* = 0 \qquad Fo = 0, \qquad 1 < r^* < S_c \tag{6d}$$

Thus, the nondimensional form of the derived solution results in:

$$T_i^*(r^*, Fo) = \frac{1}{2\pi H}\left(\sum_{n}^{\infty} C_n\varphi_{in}(r^*)\exp(-\beta_n^2 Fo) - V_i(r^*)\right) \tag{7}$$

Where φ_{in} are the eigenfunctions dependent of the Bessel function, tabulated at Incropera,*et al.*,(2003) of the first and second kind (J_0 and J_1), β_n are the eigenvalues, C_n are the coefficients of the series. The eigenfunctions is given by:

$$\varphi_{1n}(r^*) = A_{1n}J_0\left(\frac{\beta_n r^*}{\sqrt{G}}\right) + B_{1n}Y_0\left(\frac{\beta_n r^*}{\sqrt{G}}\right) \tag{8a}$$

$$\varphi_{2n}(r^*) = A_{2n}J_0(\beta_n r^*) + B_{2n}Y_0(\beta_n r^*) \tag{8b}$$

The coefficients A_{1n}, B_{1n}, A_{2n} and B_{2n} are given as:

$$A_{1n} = 1 \tag{9a}$$

$$B_{1n} = \frac{-J_1\left(\frac{\beta_n}{\sqrt{G}}\right)}{Y_1\left(\frac{\beta_n}{\sqrt{G}}\right)} \tag{9b}$$

$$A_{2n} = \frac{\dfrac{J_0\left(\frac{\beta_n S_b}{\sqrt{G}}\right)}{B_{1n}Y_0\left(\frac{\beta_n S_b}{\sqrt{G}}\right)}}{J_0\left(\beta_n S_b\right) - Y_0(\beta_n S_b)\left[\frac{J_0\left(\beta_n S_c\right)}{Y_0(\beta_n S_c)}\right]} \tag{9c}$$

$$B_{2n} = -A_{2n}\frac{J_0\left(\beta_n S_c\right)}{Y_0(\beta_n S_c)} \tag{9d}$$

$$V_1(r^*) = \ln\left(\frac{S_b^{H-1}r^*}{S_c^H}\right), V_2(r^*) = H\,\ln\left(\frac{r^*}{S_c}\right) \tag{10}$$

$$C_n = \left[\frac{H}{G}\int_1^{S_b} r^*\varphi_{1n}(r^*)\,V_1(r^*)dr^* + \int_{S_b}^{S_c} r^*\varphi_{2n}(r^*)\,V_2(r^*)dr^*\right]\Bigg/N_n \tag{11}$$

Where

$$N_n = \frac{H}{G}\int_1^{S_b} r^*\varphi_{1n}^2(r^*)dr^* + \int_{S_b}^{S_c} r^*\varphi_{2n}^2(r^*)\,dr^* \tag{12}$$

The eigenvalues β_n are estimated solving the transcendental equation given below, and it is verified that exists a infinite number of roots for the eigenvalues in the form $\beta_1 < \beta_2 < \beta_3 < \cdots < \beta_n < \cdots$, and for each root exists magnitudes of A_{in}, B_{in} and φ_{in}.

$$\begin{bmatrix} J_1\left(\frac{\beta_n}{\sqrt{G}}\right) & Y_1\left(\frac{\beta_n}{\sqrt{G}}\right) & 0 & 0 \\ J_0\left(\frac{\beta_n S_b}{\sqrt{G}}\right) & Y_0\left(\frac{\beta_n S_b}{\sqrt{G}}\right) & -J_0(\beta_n S_b) & -Y_0(\beta_n S_b) \\ FJ_1\left(\frac{\beta_n S_b}{\sqrt{G}}\right) & FY_1\left(\frac{\beta_n S_b}{\sqrt{G}}\right) & -J_0(\beta_n S_b) & -Y_1(\beta_n S_b) \\ 0 & 0 & J_0(\beta_n S_C) & Y_0(\beta_n S_C) \end{bmatrix} = 0 \tag{13}$$

2.2 Laplace Transform Equations

A recommended solution when considered an interaction between composite regions with a semi-infinite solid. As was explained by Özisik (1980) "In this approach, the partial derivatives with respect to time are removed by the application of the Laplace transform, the resulting system of ordinary differential equations is solved and the transforms of temperatures arc inverted; but the principal difficulty lies in the inversion of the resulting transform.".

The following solution was developed for solutions which the inversion of the transformers can be performed by using conventional Laplace Transform tables.

Considering the sketch given by figure 1 and a semi-infinite approach for the region far from the reference center line of the composite region,

$$\theta_i(r,t) = \frac{T_i(r,t)}{T_0}, \qquad (i = 1,2) \tag{14}$$

The mathematical formulation of the Eq. (14) in terms of $\theta_i(r,t)$ is as follows:

$$\frac{\partial^2 \theta_1}{\partial r^2} = \frac{1}{\alpha_1} \frac{\partial \theta_1(r,t)}{\partial t}, \qquad 0 < r < r_b, \qquad t > 0 \tag{15a}$$

$$\frac{\partial^2 \theta_2}{\partial r^2} = \frac{1}{\alpha_2} \frac{\partial \theta_2(r,t)}{\partial t}, \qquad r > r_b, \qquad t > 0 \tag{15b}$$

Equations (15a) and (15b) are subjected to the boundary conditions below:

$$\frac{\partial \theta_1}{\partial r} = 0, \qquad r = 0, \qquad t > 0 \tag{16a}$$

$$\theta_1(r,t) = \theta_1(r,t), \qquad r = r_b, \qquad t > 0 \tag{16b}$$

$$k_1 \frac{\partial \theta_1}{\partial r} = k_2 \frac{\partial \theta_2}{\partial r}, \qquad r = r_b, \qquad t > 0 \tag{16c}$$

$$\theta_2(r,t) \to 0, \qquad r \to \infty, \qquad t > 0 \tag{16d}$$

Initial conditions:

$$\theta_1(r,t) = 1, \qquad 0 < r < r_b, \qquad t = 0 \tag{17a}$$

$$\theta_2(r,t) = 0, \qquad r > r_b, \qquad t = 0 \tag{17b}$$

Laplace transform of Eq. (15a) and (15b) is stated as:

$$\frac{\partial^2 \overline{\theta_1}(r,s)}{\partial r^2} = \frac{1}{\alpha_1} [s\overline{\theta_1}(r,s) - 1], \qquad 0 < r < r_b \tag{18a}$$

$$\frac{\partial^2 \overline{\theta_2}(r,s)}{\partial r^2} = \frac{1}{\alpha_2} [s\overline{\theta_2}(r,s)], \qquad r > r_b \tag{18b}$$

Laplace transform of the boundary conditions gives:

$$\frac{d\overline{\theta_1}}{dr} = 0, \qquad r = 0 \tag{19a}$$

$$\overline{\theta_1} = \overline{\theta_2}, \qquad r = r_b \tag{19b}$$

$$k_1 \frac{\partial \overline{\theta_1}}{\partial r} = k_2 \frac{\partial \overline{\theta_2}}{\partial r}, \qquad r = r_b \tag{19c}$$

$$\overline{\theta_2} \to 0, \qquad r \to \infty \tag{19d}$$

The solution of the Eq. (18a) that satisfies the boundary condition (19a) is shown as:

$$\overline{\theta_1}(r,s) = \frac{1}{s} + A \cosh\left(x \sqrt{\frac{s}{\alpha_1}}\right), \qquad 0 < r < r_b \tag{20a}$$

And the solution of Eq. (18b) that satisfies the boundary condition (19b) is that:

$$\overline{\theta_2}(r,s) = B\, e^{-x(\sqrt{s/\alpha_2})}, \qquad r > r_b \tag{20b}$$

The coefficients A and B are given as:

$$A = -\frac{1-\gamma}{s} \frac{e^{-\sigma r}}{1 - \gamma e^{-2\sigma r}} \tag{21a}$$

$$B = \frac{1+\gamma}{2s} e^{\sigma\mu r} \frac{1-e^{-2\sigma r}}{1-\gamma e^{-2\sigma r}} \qquad (21b)$$

Where:

$$\gamma = \frac{\beta-1}{\beta+1} \; ; \; \beta = \frac{k_1}{k_2}\frac{1}{\mu} \; ; \; \sigma = \sqrt{\frac{s}{\alpha_1}} ; \; \mu = \sqrt{\frac{\alpha_1}{\alpha_2}} \qquad (21c)$$

Replacing the Eq. (21) into Eq. (20), comes that:

$$\overline{\theta_1}(r,s) = \frac{1}{s} - \frac{1-\gamma}{2s}\frac{e^{-\sigma(r-x)}+e^{-\sigma(r+x)}}{1-\gamma e^{-2\sigma r}}, \qquad 0 < r < r_b \qquad (22a)$$

$$\overline{\theta_2}(r,s) = \frac{1+\gamma}{2s}\frac{e^{-\sigma\mu(x-r)}-e^{-\sigma(2r+\mu x-\mu r)}}{1-\gamma e^{-2\sigma r}}, \qquad r > r_b \qquad (22b)$$

It is verified that $|\gamma| < 1$ and it is possible to expand in binomial series the term $[1-\gamma e^{-2\sigma r}]^{-1}$, making the Eq. (22) become:

$$\overline{\theta_1}(r,s) = \frac{1}{s} - \frac{1-\gamma}{2s}\sum_{n=0}^{\infty}\gamma^n\left[\frac{e^{-\sigma[(2n+1)r-x]}}{s}+\frac{e^{-\sigma[(2n+1)r+x]}}{s}\right], \qquad 0 < r < r_b \qquad (23a)$$

$$\overline{\theta_2}(r,s) = \frac{1+\gamma}{2s}\sum_{n=0}^{\infty}\gamma^n\left[\frac{e^{-\sigma[2nr+\mu(x-r)]}}{s}-\frac{e^{-\sigma[(2n+2)r+\mu(x-r)]}}{s}\right], \qquad r > r_b \qquad (23b)$$

Inverting the Laplace transform by means of tables listed on the references (Özisik, 1980), the temperature distribution throughout the media can me measured by the relations below:

$$\theta_1(r,t) = \frac{T_1(r,t)}{T_0} = 1 - \frac{1-\gamma}{2}\sum_{n=0}^{\infty}\gamma^n\left\{erfc\left[\frac{(n+1)r-x}{2\sqrt{\alpha_1 t}}\right]+erfc\left[\frac{(2n+1)r+x}{2\sqrt{\alpha_1 t}}\right]\right\} \quad 0 < r < r_b \qquad (24a)$$

$$\theta_2(r,t) = \frac{T_2(r,t)}{T_0} = \frac{1+\gamma}{2}\sum_{n=0}^{\infty}\gamma^n\left\{erfc\left[\frac{2nr+\mu(x-r)}{2\sqrt{\alpha_1 t}}\right]-erfc\left[\frac{(2n+2)r+\mu(x-r)}{2\sqrt{\alpha_1 t}}\right]\right\} \quad r > r_b \qquad (24b)$$

The $erfc$ is a complementary error function, that is dependent of the Gauss error function, which can be found tabled in Incropera, *et al.,* (2003)

3. EXPECTED RESULTS

As Gu and O'Neal (1995) proposed, the main focus would be around the behavior of the results when changed the homogeneous condition to a heterogeneous analyses application. Other leverage that will sustain the study in related to the technique of the Inverse Heat Conduction Problems (IHCP) that will provide magnitudes to be considered in the study as a first approach to achieve the desired temperature at a specific distance of the heat source generator after a given period of time.

It is extremally important for the technique of Orthogonal Expansion Equations to set a far-field temperature, since this mathematical scheme is available only for finite solids. This arbitrated temperature has magnitude of 20ºC in a great distant point of the heat generation source, which allows the approach of the theorem for infinite and semi-infinite solids.

It is expected that both analytical results can provide greater correctness when used concomitantly. The Laplacian Transform method predicts a better appliance for the conductive heat flow when considered the semi-infinite geological formation but, on the other hand, the generalized Orthogonal Expansion scheme tends to describe with more accuracy the phenomena that occurs on the cylinders layer regions. So, within that scope, binding one method to another might approximate the obtained results with the numerical approach.

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