

ENCIT-2018-XXXX

A COMPUTATIONAL STUDY OF THE HYDRODYNAMIC DEVELOPING REGION IN SINGLE-PHASE MINICHANNELS

A. P. C. Sarmiento, G. Maccari, F. H. Milanese, M. B. H. Mantelli.

Heat Pipe Laboratory, Department of Mechanical Engineering, Federal University of Santa Catarina, Florianópolis, 88040-900, SC, Brazil

e-mail: {andres,marcia}@labtucal.ufsc.br

Abstract. *In many engineering systems, such as compact heat exchangers, system performance depends on laminar flow development in minichannels. This study investigates effects of Reynolds number, hydraulic diameter, and channel aspect ratio on the entrance length in rectangular minichannels. Numerical investigations were performed for minichannels with hydraulic diameters between 1 and 4 mm, Reynolds numbers between 100 and 1000, and channel aspect ratios between 0.5 and 2. The results show good agreement when compared to other correlations for microscale and macroscale.*

Keywords: *Minichannels, pressure drop, hydrodynamic developing region, CFD*

1. INTRODUCTION

Although thermal system technologies have advanced significantly lately, the search for efficient solutions is an important subject that continues to deserve large research effort. Investigations on fundamental aspects actually lead to deeper understanding of thermo-hydraulic phenomena, allowing the optimization of equipment, including their size reduction, high heat transfer coefficients of cooling fluid fluxes and low-pressure gradients.

Several works (Shah and London, 1978; Tuckerman and Pease, 1981) have shown that the size reduction (mainly the hydraulic diameter reduction), actually resulted in a significant increase in heat transfer rates, but, in contrast, lead to an increase of the pressure gradients. The first researchers in this area observed a large divergence between the classical theory and the experimental values, for single-phase flow in microchannels (Morini, 2004; Garimella and Sobhan, 2003). This discrepancy lead them to conclude that the classical theory could not be applied, and so, a new theory to explain this behavior was necessary.

Many authors investigated and compared the classical Hagen - Poiseuille theory with experimental evaluations of fluid flowing in miniscale channels. They found a good agreement between classic theory and experiments for $f Re$ numbers, in horizontal flow (Dutkowski, 2008; Sahar *et al.*, 2016; Agostini *et al.*, 2002, 2004). In particular, Agostini *et al.* (2002, 2004) showed that the laminar to turbulent transition occurs for $Re \approx 2000$ and, in this Re region, the value of the friction factor is well predicted by correlations established for conventional tubes.

Several papers address reviews of pressure drop and friction factor experimental evaluations of microchannel flows (Asadi *et al.*, 2014; Morini, 2004; Steinke and Kandlikar, 2006). Comparative analysis of results from several studies shows that $f Re$ numbers at the microscale level could be variable: similar or very different from those predicted by conventional macro Hagen - Poiseuille theory. The most usual discrepancy noted by researchers is the early departure from the laminar theory, that suggests a lower critical Reynolds number.

More recently, Judy *et al.* (2002); Niklas and Favre-Marinet (2003); Bucci *et al.* (2003); Celata *et al.* (2002); Baviere *et al.* (2004), reported new experimental and numerical results for $f Re$ numbers, and concluded that classical theory is applicable to modeling the fluid flow in microchannels, as long as the scale effects are properly modeled. As pointed out by Judy *et al.* (2002) and Steinke and Kandlikar (2006), large experimental errors area associated with microscale experiments, explaining the large discrepancy of data reported in literature by some of the earlier researchers. Uncertainties are associated to measurements of channel geometry and of flow rates, as the developing regions are strongly influenced by the entrance and exit loss effects.

As the field of microfluidics continues to grow, it is increasingly important to understand the physical phenomena governing fluid flow and heat transfer in mini and microchannels, as well as the limitations associated to the application of correlations developed for macroscale flows in microfluidics. In thermofluid systems, such as heat exchangers, laminar flow in noncircular ducts are frequently encountered, for example, in automotive coolers, compact heat exchangers, and microchannel heat sinks. As mentioned above, scaling effects are very important phenomena and needs to be emphasized. Yun *et al.* (2009) used numerical simulations to investigate fluid flow and heat transfer in smooth rectangular microchannels. Their results indicate that the entrance effects can be neglected for ratios of channel length to hydraulic diameter (L/d_h) greater than 70. However, many mini and microfluidic devices, such as sensors, actuators, and compact heat exchangers,

involve transport phenomena in relatively short channels, whose length is often not sufficient to give rise to fully developed flows (Kandlikar *et al.*, 2006). Thus, it is important to accurately estimate the entrance length in microchannels.

The effect of Reynolds numbers on the entrance length was first experimentally investigated for macroscale flows in circular pipes and between parallel plates by Atkinson *et al.* (1969) and Chen (1973). The correlations from these two studies were obtained through a linear combination of the creeping flow and boundary-layer type solutions.

Muzychka and Yovanovich (2009) present a detailed review and analysis of the hydrodynamic characteristics of developing and fully developed laminar flows in noncircular ducts for a several Reynolds numbers. They proposed new analytical models, which simplify the prediction of the Poiseuille number and entrance length, for developing and fully developed flows in most noncircular duct geometries found in heat exchangers. The model proposed for square channels showed a good agreement with the well-known correlation for the entrance length proposed by Shah and London (1978).

Limited results are available in literature for microchannels. Ahmad and Hassan (2010) studied experimentally the hydrodynamical length entrance in rectangular microchannels by using PIV. A comparison with conventional entrance length correlations showed a good agreement only for a limited Reynolds number range. They proposed new empirical correlations for predicting the entrance length in rectangular channels. Muzychka *et al.* (2012) investigated slip flow and continuum flow in circular and noncircular microchannels. They developed a model for predicting the Poiseuille number at the entrance for developing slip and continuum flows. The accuracy of the model was estimated to be approximately 10 % for most common duct shapes. Galvis *et al.* (2012) developed a criterion to estimate the entrance length in rectangular microchannels by numerical simulations, proposing new correlations.

Considering minichannels, one can expect that the length entrance on the hydrodynamic developing region would be between Ahmad and Hassan and Langhaar criteria. There is no criterion for the entrance length in rectangular minichannels available in the literature. Therefore, the present work is focused on evaluating, using numerical simulations, the entrance length in rectangular minichannels of various aspect ratios, for a wide range of Reynolds numbers, frequently used in compact heat exchangers.

2. METHODOLOGY

The numerical modeling was developed for single-phase flow in rectangular straight minichannels. The following assumptions are made to model the fluid flow in rectangular channels: steady state, incompressible fluid, laminar flow and constant fluid properties. As showed by Rosa *et al.* (2009), the Navier-Stokes equations can be used for modeling single-phase fluid flow in mini or microchannels for Knudsen numbers less than 0.1. The fluid properties are assumed to be constant, neglecting viscous dissipation term (a valid hypothesis for water in microchannels with $d_h \geq 100 \mu\text{m}$, as proposed by Celata *et al.*, 2006). The governing equations for mass and momentum balances are, respectively:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0. \quad (1)$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j u_i) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right) + \mathbf{F} = 0, \quad (2)$$

where x_j are the cartesian components (x, y, z), x_i is the velocity component (u, v, w) and \mathbf{F} are the field forces.

2.1 Computational domain and boundary conditions

Figure 1 shows a schematic of the minichannel under study, with its geometric parameters. For hydrodynamic simulations, a half-domain of the minichannels, due to symmetry, is considered.

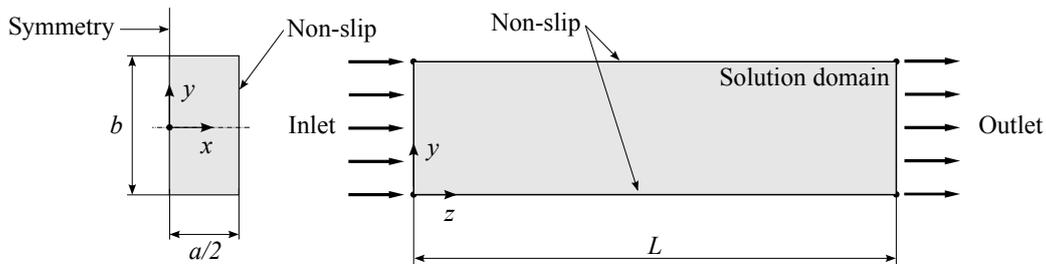


Figure 1: Computational domain for hydrodynamics simulations

The aspect ratio of the rectangular channel, and its hydraulic diameter are defined as:

$$\beta = \frac{b}{a}; \quad d_h = \frac{4 A_s}{P_s} = \frac{2 a b}{a + b}. \quad (3)$$

The governing equations are solved with the following boundary conditions: zero velocity over all wall boundaries (non-slip condition), and, at the channel inlet, the mass flow is assumed to be constant, i.e., $(u, v, w) = (0, 0, U_m)$. Also, the static pressure was set as zero at the channel outlet. Symmetry conditions are used to reduce the number of elements in the computational model, and, therefore, convective fluxes across any symmetry plane are considered to be zero (the normal velocity component at the symmetry plane is zero). There is also no diffusion flux across the symmetry plane (normal gradients of all flow variables are zero). The dimensions of the minichannels considered in this work are listed in Tab. 1.

Table 1: Dimensions of investigated minichannels

Case	β	$a \times 10^3$ [m]	$b \times 10^3$ [m]	$d_h \times 10^3$ [m]	Re
1	1	4	4	4	100 - 1000
2	1	3	3	3	100 - 1000
3	1	2	2	2	100 - 1000
4	1	1	1	1	100 - 1000
5	1.25	2	2.5	2.22	100 - 1000
6	1.5	2	3	2.40	100 - 1000
7	1.75	2	3.5	2.55	100 - 1000
8	2	2	4	2.67	100 - 1000
9	0.5	2	1	1.33	100 - 1000

2.2 Solution method

The governing equations in their steady state and incompressible form, along with the mentioned boundary conditions, were solved using commercial finite-volume element software package FLUENT / ANSYS. Convergence criteria for Root Mean Square error (RMS) for discretized Equations 1 and 2 was established in 10^{-7} , selected to ensure suitable convergence of conservation equations. The SIMPLE algorithm was used for the velocity - pressure coupling in the solution procedure. The momentum equations were solved with a second-order upwind scheme. The Reynolds number, Re, in the present study is defined as:

$$\text{Re} = \frac{\rho U_m d_h}{\mu}; \quad U_m = \frac{\dot{m}}{\rho A_s}, \quad (4)$$

where ρ is the fluid density, U_m is the mean flow velocity, μ is the fluid dynamic viscosity, \dot{m} is the mass flux and A_s is the cross-sectional area of the channel. The cross-sectional area-averaged pressure at a given axial location s is defined as

$$\bar{p}_s = \frac{1}{A_s} \int_{A_s} p \, dA_s. \quad (5)$$

The fully developed velocity profile was calculated based on the exact solution for laminar pressure driven flow in a rectangular duct, as showed in Eq. 6 (Shah and London, 1978).

$$\begin{aligned} u(x, y, 0) &= 0 \\ v(x, y, 0) &= 0 \\ w(x, y, 0) &= -\frac{16}{\pi^3} \left(\frac{dp}{dz} \right) \frac{(b/2)^2}{\mu} \sum_{n=1,3,\dots}^{\infty} \frac{(-1)^{(n-1)/2}}{n^3} \left[1 - \frac{\cosh(n\pi y/b)}{\cosh(n\pi a/2b)} \right] \cos\left(\frac{n\pi x}{b}\right) \end{aligned} \quad (6)$$

where the pressure gradient dp/dz is given in terms of the mean fluid velocity, U_m , by

$$U_m = -\frac{1}{3} \left(\frac{dp}{dz} \right) \frac{(b/2)^2}{\mu} \left[1 - \frac{192}{\pi^5} \left(\frac{b}{a} \right) \sum_{n=1,3,\dots}^{\infty} \frac{1}{n^5} \tanh\left(\frac{n\pi a}{2b}\right) \right] \quad (7)$$

Combining Eqs. 6, 7, is obtained velocity profile $w(x, y, 0)$, independent of pressure gradient.

$$\begin{aligned} w(x, y, 0) &= \frac{48}{\rho \pi^3 A_c} \left[\frac{\dot{m}}{1 - \frac{192}{\pi^5} \left(\frac{b}{a} \right) \sum_{n=1,3,\dots}^{\infty} \frac{1}{n^5} \tanh\left(\frac{n\pi a}{2b}\right)} \right] \\ &\quad \sum_{n=1,3,\dots}^{\infty} \frac{(-1)^{(n-1)/2}}{n^3} \left[1 - \frac{\cosh(n\pi y/b)}{\cosh(n\pi a/2b)} \right] \cos\left(\frac{n\pi x}{b}\right) \end{aligned} \quad (8)$$

In this investigation, numerical velocity profiles in the fully developed flow region were verified to match the analytical solution. Traditionally, the hydrodynamic entrance length is defined as the length from the inlet of a channel to a location where the velocity profile has attained 99 % of the fully developed velocity profile (Shah and London, 1978). In order to compare the results from numerical simulation with values provided by the correlations, the maximum percent difference, $\Delta_{L_{d_h}}$, was used as well as the total average absolute difference, $\Delta\Phi_{L_{d_h}}$, calculated as follows, respectively:

$$\Delta_{L_{d_h}} = \max \left| \frac{L_{d_h, \text{model}} - L_{d_h, \text{simulation}}}{L_{d_h, \text{model}}} \right| \times 100 \quad (9)$$

$$\Delta_{L_{d_h}} = \sum \left| \frac{L_{d_h, \text{model}} - L_{d_h, \text{simulation}}}{L_{d_h, \text{model}}} \right| \times 100 \quad (10)$$

2.2.1 Grid independence

The entire domain was meshed using hexahedral elements. To establish mesh size influence on the simulation results, Roache's criteria was used. Roache (1994) proposed the grid convergence index (GCI), to evaluate the discretization error in mesh convergence analysis. The GCI can be estimated by:

$$\text{GCI}_i = \epsilon_i^d \approx 100 \left| \frac{\phi_i - \phi_{i-1}}{\phi_{i-1}} \right| \frac{\xi}{r^m - 1}; \quad r = \frac{n_i}{n_{i-1}}, \quad (11)$$

where where ϵ_i^d [%] is the discretization error, ϕ_i [-] is the solution value of the variable under analysis with fine mesh, ϕ_{i-1} [-] is the solution value of the variable under analysis with coarse mesh, ξ [-] is the uncertainty coefficient factor, r [-] is the relation between mesh elements and m [-] is the order of accuracy. According to Roache, ξ depends on type problem and vary from 1 to 3. For "nearby-type"¹ problems, $\xi = 3$. When $\xi = 1$ one has Richardson extrapolation.

Table 2 shows GCI for a minichannel with $\beta = 1$, $d_h = 2 \times 10^{-3}$ [m] and $\text{Re} = 1000$. For every simulation case showed in Table 1 it was done a grid independence analysis by using GCI.

Table 2: Grid convergence index for $\xi = 1$. Minichannel with $\beta = 1$, $d_h = 2 \times 10^{-3}$ [m] and $\text{Re} = 1000$

Mesh number	Nodes	Elements	Δp [Pa]	r	ϵ^d [%]
1	22428	14400	776,68	–	–
2	36045	25600	825,71	1,78	2,92
3	72891	57600	865,17	2,25	1,18
4	122550	102400	880,20	1,78	0,80
5	185030	160000	887,60	1,56	0,58
6	260330	230400	891,91	1,44	0,45
7	348440	313600	894,71	1,36	0,37
9	449360	409600	896,67	1,31	0,31
10	504630	462400	897,43	1,13	0,31
11	689660	640000	899,12	1,38	0,20
12	1062126	1000000	900,86	1,56	0,13
13	1514691	1440000	901,89	1,44	0,11
14	2047356	1960000	902,57	1,36	0,09
15	2660121	2560000	903,07	1,31	0,08
16	3240000	3352986	903,07	1,31	0,08

3. RESULTS

Figure 2a shows the variation of the normalized entrance length with the Reynolds number using the numerical results obtained in this study. Also presented in this figure are the results obtained from the experimental correlations described before. As already mentioned, the correlations of Galvis *et al.* (2012) and Langhaar (1942) are the limiting cases for micro and macroscale hydrodynamic developing region in channels, respectively. In Muzychka and Yovanovich's work, the correlation ranges for hydraulic diameter are not specified. For cases 1 and 2, as the hydraulic diameter becomes larger, i.e., towards macrochannel, the Muzychka and Yovanovich correlation presents a better performance, with a maximum

¹Rapidly varying coefficients or problems with high gradients.

difference of 9.66%. For the other correlations, the maximum differences are 23.53%, 35.32% and 30% for Ahmad and Hassan (2010), Galvis *et al.* (2012) and Langhaar (1942), respectively. The large discrepancy with the correlations of Ahmad and Hassan (2010) and Galvis *et al.* (2012) can be explained because these correlations were developed for channels with hydraulic diameters between 100 and 500 [μm]. For the third case, with $d_h = 2 \times 10^{-3}$ [m], it is observed a similar behavior to cases 1 and 2. Again, Muzychka and Yovanovich correlation presents a better performance with a maximum difference of 11.18%. Ahmad and Hassan (2010), Galvis *et al.* (2012) and Langhaar (1942) present maximum differences of 20.87%, 33.07% and 31.80%, respectively. It should be noted, that as the hydraulic diameter becomes smaller, the maximum difference for Ahmad and Hassan (2010) and Galvis *et al.* (2012) becomes smaller. For case 4, with the smaller hydraulic diameter, Muzychka and Yovanovich and Ahmad and Hassan (2010) have similar maximum differences of 14.56% and 15.15%, respectively. For Galvis *et al.* (2012) and Langhaar (1942), the maximum differences are 28.579% and 35.80%, respectively.

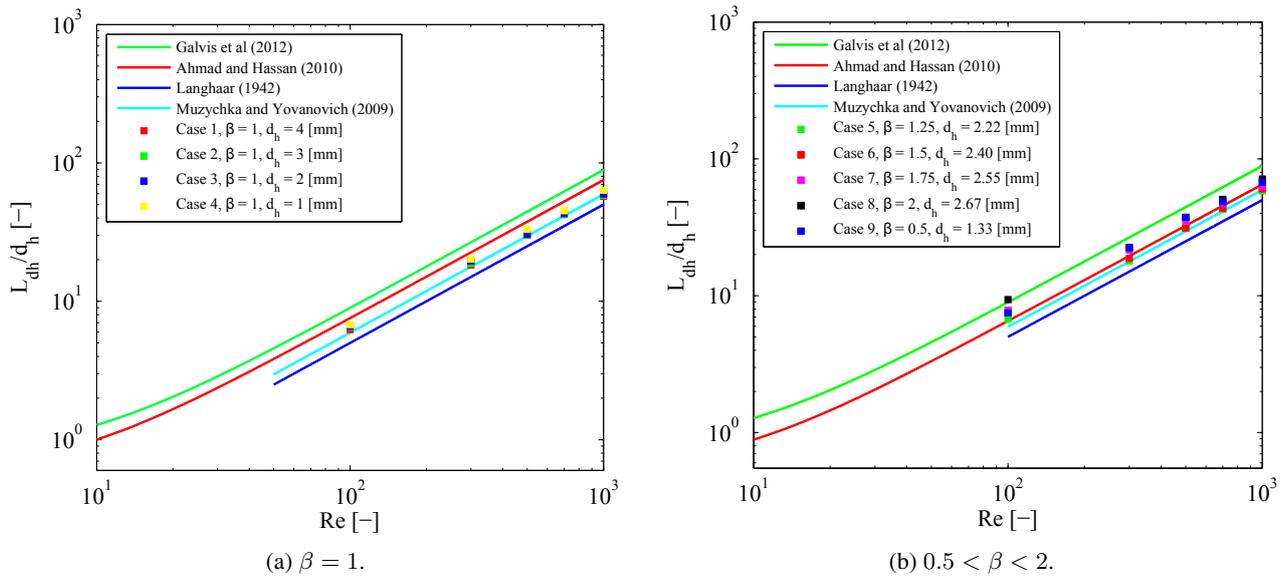


Figure 2: Normalized entrance length L_{hd}/d_h

Figure 2b shows the variation of the normalized entrance length as function of the channel aspect ratio. For cases 5 to 9, the variation of the channel aspect ratio, β is considered. According Ahmad and Hassan (2010), their correlation can be used for channels with aspect ratio less than 3. Muzychka and Yovanovich (2009) consider β ranges from 0.01 to 1. Therefore, for cases 5 to 8 it is considered $\beta > 1$, then for comparison, β in Muzychka and Yovanovich's correlation is considered equal to 1. The channel aspect ratio in the correlation of Galvis *et al.* (2012) is set $\beta = 1.25$ for case 5, and for the cases 6 to 9, the aspect ratio is 1. Table 3 shows the maximum percent difference and the total average absolute difference between the results of numerical simulations and the correlations described above.

Table 3: Maximum percent difference and the total average absolute difference.

	Ahmad and Hassan (2010)	Muzychka and Yovanovich (2009)	Galvis <i>et al.</i> (2012)	Langhaar (1942)
Case 5	10.0	13.9	29.1	35.0
Case 6	14.7	26.5	32.0	50.0
Case 7	20.2	32.6	29.3	57.1
Case 8	43.4	32.6	19.9	87.5
Case 9	15.3	26.6	24.1	50.0
$\Delta\Phi$ [%]	8.76	13.89	20.49	39.29

4. CONCLUSIONS

Numerical simulations were used to study the developing region flows in rectangular minichannels, focusing on the entrance length and its dependence on the Reynolds number and the channel aspect ratio. For a given Reynolds number, the dimensionless entrance length increases as the channel aspect ratio increases. For minichannels with $\beta = 1$ and hydraulic diameters between 1 and 4 mm (cases 1 to 4), the correlation proposed by Muzychka and Yovanovich (2009) presented the

smallest difference when compared with the numerical results obtained here. For the cases 5 to 9, the correlation of Ahmad and Hassan (2010) presented the smallest difference of total average absolute difference ($\Delta\Phi = 8.76\%$).

5. REFERENCES

- Agostini, B., Watel, B., Bontemps, A. and Thonon, B., 2004. "Liquid flow friction factor and heat transfer coefficient in small channels: an experimental investigation". *Experimental Thermal and Fluid Science*, Vol. 28, No. 2, pp. 97 – 103. ISSN 0894-1777. The International Symposium on Compact Heat Exchangers.
- Agostini, B., Watel, B., Bontemps, A. and Thonon, B., 2002. "Friction factor and heat transfer coefficient of R134a liquid flow in mini-channels". *Applied Thermal Engineering*, Vol. 22, No. 16, pp. 1821 – 1834. ISSN 1359-4311.
- Ahmad, T. and Hassan, I., 2010. "Experimental analysis of microchannel entrance length characteristics using microparticle image velocimetry". *ASME Journal of Fluids Engineering*, Vol. 132, No. 4, p. 13.
- Asadi, M., Xie, G. and Sunden, B., 2014. "A review of heat transfer and pressure drop characteristics of single and two-phase microchannels". *International Journal of Heat and Mass Transfer*, Vol. 79, pp. 34 – 53. ISSN 0017-9310.
- Atkinson, B., Brocklebank, M.P., Card, C.C.H. and Smith, J.M., 1969. "Low Reynolds number developing flows". *AICHE Journal*, Vol. 15, No. 4, pp. 548–553.
- Baviere, R.R., Ayela, F.F., Le Person, S.S. and M, F.M.M., 2004. "An experimental study of water flow in smooth and rough rectangular micro-channels". In *International Conference on Nanochannels, Microchannels, and Minichannels*. ASME, Rochester, New York, USA, pp. 221–228.
- Bucci, A., Celata, G.P., Cumo, M., Serra, E. and Zummo, G., 2003. "Water single-phase fluid flow and heat transfer in capillary tubes". In *ASME 1st International Conference on Microchannels and Minichannels*. ASME, Rochester, New York, USA, pp. 319–326.
- Celata, G.P., Cumo, M., Guglielmi, M. and Zummo, G., 2002. "Experimental investigation of hydraulic and single-phase heat transfer in 0.130 mm capillary tube". *Microscale Thermophysical Engineering*, Vol. 6, No. 2, pp. 85–97. doi:10.1080/10893950252901240.
- Celata, G., Morini, G., Marconi, V., McPhail, S. and Zummo, G., 2006. "Using viscous heating to determine the friction factor in microchannels - an experimental validation". *Experimental Thermal and Fluid Science*, Vol. 30, No. 8, pp. 725 – 731. ISSN 0894-1777.
- Chen, R.Y., 1973. "Flow in the entrance region at low Reynolds numbers". *ASME Journal of Fluids Engineering*, Vol. 95, No. 1, pp. 153–158.
- Dutkowski, K., 2008. "Single phase pressure drop in minichannels". *Transactions of the Institute of Fluid-flow Machinery*, No. 121, pp. 17–32.
- Galvis, E., Yarusevych, S. and Culham, R., 2012. "Incompressible laminar developing flow in microchannels". *ASME Journal of Fluids Engineering*, Vol. 134, No. 1, p. 1.
- Garimella, S. and Sobhan, C., 2003. "Transport in microchannels - a critical review". In G. Chen, V. Prasad and Y. Jaluria, eds., *Annual Review of Heat Transfer*, Begel House Inc., Vol. 13.
- Judy, J., Maynes, D. and Webb, B., 2002. "Characterization of frictional pressure drop for liquid flows through microchannels". *International Journal of Heat and Mass Transfer*, Vol. 45, No. 17, pp. 3477 – 3489. ISSN 0017-9310.
- Kandlikar, S., Garimella, S., Li, D., Colin, S. and King, M., 2006. *Heat Transfer and Fluid Flow in Minichannels and Microchannels*. Elsevier, USA.
- Langhaar, H.L., 1942. "Steady flow in the transition length of a straight tube". *ASME Journal of Applied Mechanics*, Vol. 64, pp. A55–A58.
- Morini, G.L., 2004. "Single-phase convective heat transfer in microchannels: a review of experimental results". *International Journal of Thermal Sciences*, Vol. 43, No. 7, pp. 631 – 651. ISSN 1290-0729. doi: <http://dx.doi.org/10.1016/j.ijthermalsci.2004.01.003>.
- Muzychka, Y.S., Duan, Z.P. and Yovanovich, M.M., 2012. *Fluid friction and heat transfer in microchannels*, CRC Press, pp. 477–609.
- Muzychka, Y.S. and Yovanovich, M.M., 2009. "Pressure drop in laminar developing flow in noncircular ducts: A scaling and modeling approach". *ASME Journal of Fluids Engineering*, Vol. 131, No. 11, pp. 111105–1 – 111105–11.
- Niklas, M. and Favre-Marinet, M., 2003. "Pressure losses in a network of triangular microchannels". In *ASME 1st International Conference on Microchannels and Minichannels*. ASME, Rochester, New York, USA, pp. 335–342.
- Roache, P.J., 1994. "A method for uniform reporting of grid refinement studies". *ASME Journal of Fluids Engineering*, Vol. 116, pp. 405–413.
- Rosa, P., Karayiannis, T. and Collins, M., 2009. "Single-phase heat transfer in microchannels: The importance of scaling effects". *Applied Thermal Engineering*, Vol. 29, No. 17, pp. 3447 – 3468. ISSN 1359-4311.
- Sahar, A.M., Ozdemir, M.R., Fayyadh, E.M., Wissink, J., Mahmoud, M.M. and Karayiannis, T.G., 2016. "Single phase flow pressure drop and heat transfer in rectangular metallic microchannels". *Applied Thermal Engineering*, Vol. 93, pp. 1324 – 1336. ISSN 1359-4311.

- Schlichting, H., 1979. *Boundary Layer Theory*. McGraw-Hill, New York, 7th edition.
- Shah, R.K. and London, A., 1978. "Laminar flow forced convection in ducts". In R.K. Shah, A.L. London, T.F. Irvine and J.P. Hartnett, eds., *Advances in Heat Transfer*, Academic Press, Advances in Heat Transfer. ISBN 978-0-12-020051-1.
- Steinke, M. and Kandlikar, S., 2006. "Single-phase liquid friction factors in microchannels". *International Journal of Thermal Sciences*, , No. 45, pp. 1073–1083.
- Tuckerman, D.B. and Pease, R.F.W., 1981. "High-performance heat sinking for VLSI". *IEEE Electron Device Letters*, Vol. 2, No. 5, pp. 126–129.
- Yun, H., Chen, B. and Chen, B., 2009. "Numerical simulation of geometrical effects on the liquid flow and heat transfer in smooth rectangular microchannels". In *ASME International Conference on Micro/Nanoscale Heat and Mass Transfer*. Minneapolis, MN, USA, Vol. 3, pp. 271–277.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.