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A NEW CONSTITUTIVE EQUATION TO REPRESENT DRILLING FLUIDS

Tainan Gabardo Miranda dos Santos
Cezar Otaviano Ribeiro Negrão
Federal University of Technology – Parana, Rua Deputado Heitor Alencar Furtado, 5000, Curitiba.
tainansantos@utfpr.edu.br
negrao@utfpr.edu.br

Abstract. *Drilling fluids present thixotropic behavior and normally gel at rest. The sol-gel transition is important in order to prevent the deposition of rock cuts generated by the drill bit in eventual stops during the process. When the gel is formed, a pressure higher than the usual is required to restart the flow. It is important to control the pressure level, as a high pressure may damage the formation and a low pressure can cause a kick. In the first case, drilling fluid can be lost to the formation. In the second case, if the kick is not controlled, a blowout may occur. Both cases represent environmental and financial problems. Therefore it is imperative to understand better the behavior of these fluids. So the objective of this paper is to propose a new constitutive equation for time-dependent materials and use the rheological data of a drilling fluid to fit the model. In order to show the model potential, a modified Jeffreys model coupled with a regularized Herschel-Bulkley equation and an unbalance equation for the viscosity are going to be used. Next the model is fitted to rheological data of a drilling fluid. The preliminary results show agreement with the literature regarding shear stress and shear rate controlled tests.*

Keywords: *Drilling fluid, Constitutive equation, Mathematical model, Thixotropy.*

1. INTRODUCTION

During the drilling process, the drill bit generates rock cuts that need to be removed from the well in order to continue the process. So drilling fluid is injected into the drill pipe and returns through the annular space formed between the drill pipe and the wall of the well. During eventual shutdowns, the drilling fluid must prevent the rock cuts from returning to the bottom of the well. So there is a change in the behavior of the fluid from sol to gel. This change due to the thixotropic nature of the fluid. Thixotropy is defined as the continuous decrease of viscosity with time when flow is applied to a sample that has been previously at rest and the subsequent recovery of viscosity in time when the flow is discontinued (Mewis and Wagner 2009).

As the gel is formed, a pressure higher than the usual is required to restart the flow. The pressure needs to be strictly controlled because a high pressure may fracture the formation, which causes loss of fluid to the formation, and a low pressure may cause a kick (invasion of gas and oil from the formation to the well). If the kick is not managed, it can cause a blowout. Both situations represent financial and environmental problems. Therefore is essential to understand and to model the behavior of thixotropic fluids in order to prevent these issues.

Several works have developed constitutive equations to predict the behavior of thixotropic materials (Acierno et al. 1976; Huang and Lu 2005; de Souza Mendes and Thompson 2013). Most of them are based on a structure parameter that describes the level of material structure and use a kinetic equation to predict the change of the material structure with time (Mewis 1979). Usually, the structure parameter varies from 0 to 1, meaning the fluid is totally unstructured and completely structured, respectively. Some thixotropy models do not include elasticity (Houska 1981; Šesták et al. 1983; Roussel et al. 2004) and some others consider it in the material structure (Coussot et al. 1993; Dullaert and Mewis 2006; de Souza Mendes 2011; de Souza Mendes and Thompson 2013; Azikri de Deus et al. 2016). Despite the large amount of models available, a general model has not yet been able to describe appropriately the thixotropic behavior.

In this work, a general model for time-dependent materials is proposed. This model, however, is not based on a structure parameter but rather, on time-dependent properties computed from a kinetic equation derived from the equilibrium condition. In order to show the model potential, the modified Jeffreys constitutive equation is coupled with the regularized Herschel-Bulkley equation and an unbalance equation for the viscosity. Next, the model is submitted to shear stress and shear rate controlled tests. In the final paper, the rheological data of a drilling fluid will be used to fit the proposed model.

2. MATHEMATICAL MODEL

Constitutive equations normally correlate the shear stress tensor as a function of the material response and can generally be written as:

$$\tau_{ij} = F_1\left(\dot{\gamma}_{ij}, \ddot{\gamma}_{ij}, \dot{\tau}_{ij}, \theta_1, \theta_2, \ldots, \theta_n\right) \quad (1)$$

where τ_{ij} is the shear stress tensor that is a function of the shear rate, $\dot{\gamma}_{ij}$, of the rate of the shear rate, $\ddot{\gamma}_{ij}$, and of the shear stress rate, $\dot{\tau}_{ij}$, tensors. θ_1, θ_2 and θ_n are time dependent properties of the material, such as, viscosity, elastic modulus, etc. Those material properties can be dependent only on the load (shear stress or shear rate), for time-independent properties, or dependent on both load and time, for time-dependent properties. A general form of a time-dependent property can be written as a function of the load and of time:

$$\theta_k = \theta_k\left(L, t\right) \quad (2)$$

where L is an invariant of the load - either shear stress, τ_{ij}, or shear rate, $\dot{\gamma}_{ij}$, tensors - and t is the time. For time-independent property fluids, a change on the load (shear rate for instance) causes an immediate change on the property that, of course, depends on the time scale of the change.

If a constant load (shear rate or shear stress) is imposed to the material, its properties tend to the equilibrium. According to Eq. (2), the equilibrium properties can then be written as:

$$\theta_{k,e}\left(L\right) = \theta_k\left(L, t \to \infty\right) \quad (3)$$

where $\theta_{k,e}$ is the equilibrium counterpart of θ_k that depends only on the load.

Many have devised models for the time dependent properties based on a structure parameter(Mujumdar et al. 2002; Dullaert and Mewis 2006; de Souza Mendes and Thompson 2013) that is derived from a kinetic equation. The proposed model uses the difference between the equilibrium and the instantaneous value of a property as the driving force for a property change. A rate equation for any material property, based on the product of a load function and on a function of the property unbalance from the equilibrium, is thus proposed:

$$\frac{d\theta_k\left(L, t\right)}{dt} = F_2\left(L\right) F_3\left[\theta_{k,e}\left(L\right) - \theta_k\left(L, t\right)\right] \quad (4)$$

where $F_2\left(L\right)$ is a positive function of an invariant of an imposed load and F_3 is a function of the property unbalance. As noted, the difference in brackets is the unbalance between the equilibrium value and the instantaneous value of θ_k, which is positive if the equilibrium value is larger than the instantaneous counterpart and negative if the opposite happens. If a constant load, L, is maintained for a long period, $t \to \infty$, the variation rate tends to zero, as θ_k approaches $\theta_{k,e}$:

$$\frac{d\theta_k\left(L, t \to \infty\right)}{dt} = 0 \quad (5)$$

Considering that the breakdown is usually faster than the buildup (Barnes 1997), a possible function for F_2 can be defined as:

$$F_2\left(L\right) = 1 + \alpha L^{\beta} \quad (6)$$

where α and β are fitting parameters. As noted, if the load is removed, the material builds up to the equilibrium at zero load condition, $\theta_{k,e}\left(L = 0\right)$. This function also allows the material break-up to be load dependent.

Many structure parameter models base the calculation on the equilibrium. Although this is equivalent to the Eq. (4), the approach proposed here is much easier to understand and perhaps to fit. It is also worth mentioning that the current model states that the driving force can be either the shear stress or the shear rate.

In order to illustrate the model potential, a thixotropy model based on the modified Jeffreys equation is used as an example:

$$\tau+\theta_1(\tau,t)\frac{d\tau}{dt}=\left[\eta_1+\eta_2(\tau,t)\right]\left[\dot{\gamma}+\theta_2(\tau,t)\frac{d\dot{\gamma}}{dt}\right] \tag{7}$$

where θ_1 and θ_2 are the time-dependent relaxation and retardation times given, respectively, by:

$$\theta_1(\tau,t)=\frac{\eta_2(\tau,t)}{G} \tag{8}$$

$$\theta_2(\tau,t)=\frac{\eta_1\eta_2(\tau,t)}{\left[\eta_1+\eta_2(\tau,t)\right]G} \tag{9}$$

where $\dot{\gamma}$ is the second invariant of $\dot{\gamma}_{ij}$, η_2 is the instantaneous viscosity that depends on τ, η_1 is the completely unstructured state viscosity, and G is the shear modulus. The properties are assumed to be shear stress dependent rather than shear rate dependent, as suggested by Souza Mendes and Thompson (2013) and Larson (2015). In order to simplify the model, G and η_1 are assumed load and time-independent. Based on Eq. (4), a simple equation rate for the viscosity is proposed:

$$\frac{d\eta_2(\tau,t)}{dt}=\frac{\eta_{2,e}(\tau)-\eta_2(\tau,t)}{t_{eq}} \tag{10}$$

where t_{eq} is the material structure time constant. As noted in Eq. (4), F_2 is assumed to be 1.0 and F_3 as $\left[\eta_{2,e}(\tau)-\eta_2(\tau,t)\right]/t_{eq}$. According to Eq. (10), the instantaneous viscosity approaches exponentially the equilibrium viscosity for an applied constant shear stress. t_{eq} represents the time for the viscosity to reach exponentially 67% of its final value if a constant shear stress is applied to the material.

The equilibrium viscosity is based on the Herschel-Bulkley model with the Papanastasiou (1987) regularization summed with a finite viscosity, η_1:

$$\eta_{2,e}(\dot{\gamma})=\left[1-\exp\left(-\frac{\eta_0\dot{\gamma}}{\tau_0}\right)\right]\left(\frac{\tau_0}{\dot{\gamma}}+k\dot{\gamma}^{n-1}\right)+\eta_1 \tag{11}$$

where η_0 is the viscosity of the completely structured material, τ_0 is the yield stress, k is the consistency index and n is the power-law index. As noted, the equilibrium viscosity changes from $\eta_0+\eta_1$ (maximum) to η_1 (minimum) as the shear rates varies from 0 to ∞, respectively.

In order to evaluate the model, the above equations are written in the dimensionless form as:

$$\tau^*+\eta_2^*\frac{d\tau^*}{dt^*}=\left[1+\eta_2^*\right]\left[\dot{\gamma}^*+\frac{\eta_2^*}{1+\eta_2^*}\frac{d\dot{\gamma}^*}{dt^*}\right] \tag{12}$$

$$\frac{d\eta_2^*(\tau^*,t^*)}{dt^*}=\frac{\eta_{2,e}^*(\tau^*)-\eta_2^*(\tau^*,t^*)}{t_{eq}^*} \tag{13}$$

$$\eta_{2,e}^*(\dot{\gamma}^*)=\left[1-\exp\left(-\frac{\eta_0^*\dot{\gamma}^*}{\tau_0^*}\right)\right]\left(\frac{\tau_0^*}{\dot{\gamma}^*}+k^*\dot{\gamma}^{*n-1}\right)+1 \tag{14}$$

where $\tau^*=\tau/(\eta_1\dot{\gamma}_{ref})$, $\eta_2^*=\eta_2/\eta_1$, $\eta_0^*=\eta_0/\eta_1$, $t^*=t(G/\eta_1)$, $t_{eq}^*=t_{eq}(G/\eta_1)$, $k^*=k(\dot{\gamma}_{ref}^{n-1}/\eta_1)$, $\dot{\gamma}^*=\dot{\gamma}/\dot{\gamma}_{ref}$ and $\tau_0^*=\tau_0/(\eta_1\dot{\gamma}_{ref})$. Notably, the relaxation time η_1/G is used as a reference for the dimensionless time, so that t_{eq}^* is the ratio of the structured material time constant, t_{eq} and of η_1/G. If the structured material time constant tends to zero, the properties change immediately and consequently, a viscoelastic response is observed. As t^*_{eq} increases, the variation of the properties is not instantaneous, and the response is the one of a viscoelastic-thixotropic fluid.

It is worth mentioning that the proposed model is represented by only five dimensionless parameters, η^*_0, k^*, τ^*_0, n and t^*_{eq}.

3. PRELIMINARY RESULTS

In order to demonstrate the model potential, this section presents results obtained from the solution of Eqs. (12), (13) and (14) for shear rate and shear stress controlled tests. The initial condition for these flows is of a fully relaxed fluid, $\tau^* = 0$, at rest, $\dot{\gamma}^* = 0$. Consequently, the initial viscosity is $\eta^*_2(t=0) = 1 + \eta^*_0$. All the simulations were performed with the following set of parameters: η^*_0, k^*, τ^*_0, n and t^*_{eq}.

3.1 Shear rate controlled test

For these tests, a constant shear rate, $\dot{\gamma}^*_f$, is imposed to the material at $t^* = 0$. Figure 1 presents the shear stress as a function of time for different imposed shear rates.

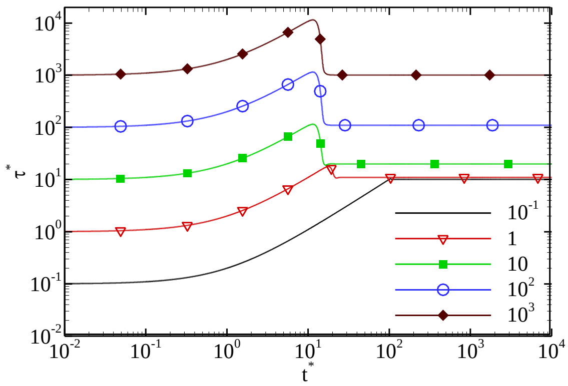

Figure 1 – Shear stress as a function of time for the constant shear rates of 10^{-1}, 1, 10, 10^2 and 10^3.

As shown by Souza Mendes and Thompson (2013), the initial shear stress is $\tau^*_i = \dot{\gamma}^*_f$. After the shear rate is imposed, $t^* > 0$, there is a linear increase in the shear stress as a result of an elastic response. An overshoot is then noted, indicating a transition from an elastic dominant region to a viscous predominant region. After that, the material relaxes and the shear stress decreases to the equilibrium. The smaller the shear rates the smaller is the overshoot and the higher is the time to reach it, as the experiment time scale reduces with the shear rate increase. For a shear rate of 10^{-1}, the overshoot is not observed, indicating that the material structure time constant is small in comparison to the experiment time scale and consequently showing only the material viscoelastic behavior.

3.2 Shear stress controlled test

In this test, a constant shear stress, τ^*_f, is imposed at $t^* = 0$. Figure 2 presents the results of the shear rate as a function of time for different shear stresses.

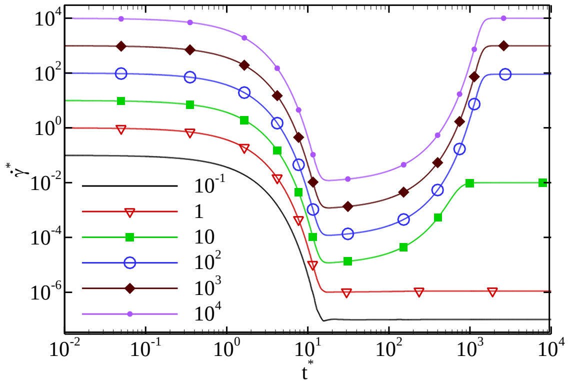

Figure 2 – Shear rate as a function of time for the constant shear stress of 10^{-1}, 1, 10, 10^2, 10^3 and 10^4.

As shown by Souza Mendes and Thompson (2013), the initial shear rate is $\dot{\gamma}_i^* = \tau_f^*$. After the shear stress is imposed, the initial shear rate reduces slightly and then decreases significantly. After that, the shear rate is maintained at very slow values or increases rapidly to a steady-state value. This fast increase is well documented in the literature (Coussot et al. 2002; Moller et al. 2009; de Souza Mendes and Thompson 2013) and is known as avalanche effect. As noted for the lower shear stress imposed (10^{-1} and 1), there is no avalanche effect, because the equilibrium viscosity for the imposed stress is significantly high. Since the viscosity almost does not change under these conditions, it can be assumed that the material does not flow.

4. CONCLUSIONS

In this work, a general model to predict the behavior of time-dependent materials such as drilling fluids was devised. The time-dependent properties are computed from a rate equation that is based on the equilibrium property condition.

The modified Jeffreys model and an equilibrium viscosity equation based on the Herschel-Bulkley model were employed to show the model potential. The final model has only five dimensionless parameters, a number significantly smaller than that observed in literature (Houska 1981; Šesták et al. 1983; Coussot et al. 1993; Dullaert and Mewis 2006; de Souza Mendes and Thompson 2013).

The results for shear rate and shear stress controlled tests presented the key features of the model. In the constant shear rate test, the stress response showed a linear elastic increase and a stress overshoot which were expected for viscoelastic-thixotropic materials. As for the constant shear stress test, the avalanche effect was observed for high-imposed stresses and the material did not yield for low shear stresses.

5. ACKNOWLEDGEMENTS

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