Proceedings of the XVIII International Symposium on Dynamic Problems of Mechanics (DINAME 2019),
M.A. Savi, T.G. Ritto and W.M. Bessa (editors), ABCM, Buzios, RJ, Brazil, March 10th to 15th, 2019

Kriging/FORM-Based Reliability Analysis of Rotor-Bearing Systems

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Abstract: In this work, it is proposed a Kriging/FORM-based reliability assessment of rotor-bearing systems. The motivation is to significantly reduce computation effort in limit-state function evaluation by using surrogate models for rotordynamics. Usually, the analysis of rotor-bearing systems dynamic features relies on the Finite Element (FE) method, which may lead to complex and time consuming simulation procedures. Approximations of the input/output relationships created by the FE simulations can be obtained using the Kriging interpolating method. The derived models work as fast-running surrogates for the underlying computer model. Comparison of the results from Kriging surrogates generated using different training samples shows that these low-dimension mathematical relationships can accurately predict rotor-bearing system outputs with considerably low training effort. Subsequently, these surrogates are used to approximate and replace implicit limit-state functions for a Kriging-based reliability assessment by means of the first-order reliability method (FORM). The proposed methodology shows impressive gains in reducing the computation cost with minimal loss in terms of reliability prediction when compared with the classical Monte Carlo simulation approach.

Keywords: Rotor-bearing system, Surrogate model, Kriging, Structural reliability, FORM

INTRODUCTION

With the advent of the digital computer, the use of numerical simulation models has become a valuable engineering design tool when physical experiments are either not feasible or overly expensive and time-consuming. Following this trend, the understanding of the dynamic behavior of complex rotor-bearing systems has definitely improved to a much higher level of sophistication and coverage, all thanks to the advances in computer methods such as the Finite Element (FEM) (Friswell et al., 2010).

Unfortunately, although FEM has proved to be a primary tool in rotordynamics prediction for many industrial applications, simulations can be extremely time-consuming and the method may become impracticable when uncertainty propagation or parametric analysis are required for real engineering design problems. Specially in reliability-based design, in order to evaluate the so-called limit-state functions by means of the Monte Carlo sampling (MCS) technique and the first/second-order reliability methods (FORM/SORM) approximations, several runs of the computational model are required, which may imply a prohibitively large computational effort even with high-performance computing architectures.

Within this scenario, it may be worthwhile to work towards methods based on surrogate models. Surrogates are simpler and explicit mathematical relationships that approximate the complicated and implicit function defined by a high-fidelity simulation model, becoming an efficient and low-dimension tool for understanding the behavior of a physical system, predicting its response, optimizing its design, and performing verification and validation.

In this contribution, we approach Kriging interpolation models as surrogates for rotordynamics. Kriging surrogate modeling (also known as Gaussian process regression) was one of the techniques that has drawn a lot of attention in the past decades, becoming a popular method for approximating deterministic computer models (Viana et al., 2014). In the field of structural dynamics and rotordynamics, many researches have been developed exploring the possibilities of applying Kriging technique to alleviate the computational burden of expensive simulations (Stocki et al., 2010; Nechak et al., 2015; Sinou, Nechak and Besset, 2018). Nevertheless, the application of Kriging method to structural reliability problems (Kaymaz, 2005; Gaspar, Teixeira and Soares, 2014) is rather limited, specially for rotordynamics problems.

The general objective of this research work is to investigate the capabilities of a Kriging/FORM-based method as a suitable alternative for rotor-bearing systems reliability assessment in the presence of parameter uncertainties.

THEORETICAL BACKGROUND

Rotor-Bearing Model

The finite element method in rotordynamics provides a systematic approach for the discretization of the rotor system into a set of the so-called finite elements. These elements are connected at discrete points called nodes, located at the ends of each element. The governing equations of motion for the rotor-bearing system are written in terms of the translation and rotation at the nodes, where these deformations represent the independent generalized coordinates $\mathbf{q}$ in the finite element formalism. In steady-state condition, with a constant speed Ω, we have a set of second-order ordinary differential equations given by:

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{C}_s + \mathbf{C}_b + \Omega\mathbf{C}_g)\dot{\mathbf{q}} + (\mathbf{K}_s + \mathbf{K}_b)\mathbf{q} = \mathbf{Q}, \quad (1)$$

where, $\mathbf{M}$ is the positive definite real symmetric inertia matrix, $\mathbf{C}_s$ and $\mathbf{K}_s$ are the structural damping and stiffness matrices resulting from the shaft strain energy, $\mathbf{C}_g$ is the real skew-symmetric gyroscopic matrix derived from the rotational kinetic energy caused by the gyroscopic moments, $\mathbf{C}_b$ is the linearized bearing dynamic damping matrix and $\mathbf{K}_b$ is the linearized bearing stiffness matrix, both non-symmetric, and the forcing vector is represent by $\mathbf{Q}$ and includes the synchronous excitation from residual mass unbalance.

The speed-dependent linearized stiffness and damping coefficients for the hydrodynamic bearing used in this work are calculated as a function of the journal eccentricity and the modified Sommerfeld number for a particular speed, load, radial clearance and fluid dynamic viscosity, assuming the short-bearing approximation (Ocvirk, 1952).

The resulting system is then solved for two different cases: first for eigenvalues and eigenvectors (i.e., damped natural frequencies, damping factors and mode shapes) using eigenanalysis, and lately for frequency response to harmonic excitation forces (i.e., unbalance forces) (Vance, Murphy and Zeidan, 2010).

Kriging-Based Surrogate

Many researchers have used Kriging metamodeling for approximating deterministic noise-free simulation models. According to Forrester, Sobester and Keane (2008), the goal of the surrogate training is to learn a mapping $\mathbf{y} = f(\mathbf{X})$ that, in a black box fashion, converts a set of sample data $\mathbf{X} = \left(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \cdots, \mathbf{x}^{(n)}\right)^t$ into outputs $\mathbf{y} = \left\{y^{(1)}, y^{(2)}, \cdots, y^{(n)}\right\}^t$.

In Kriging formalism, we consider the observed responses as if they are from the realization of a stochastic process, i.e. $\mathbf{Y} = \left\{Y(\mathbf{x}^{(1)}), Y(\mathbf{x}^{(2)}), \cdots, Y(\mathbf{x}^{(n)})\right\}^t$ (Jones, 2001). The Kriging correlation matrix is given by:

$$\mathbf{R} = \begin{pmatrix} \mathrm{cor}\left[Y(\mathbf{x}^{(1)}), Y(\mathbf{x}^{(1)})\right] & \cdots & \mathrm{cor}\left[Y(\mathbf{x}^{(1)}), Y(\mathbf{x}^{(n)})\right] \\ \vdots & \ddots & \vdots \\ \mathrm{cor}\left[Y(\mathbf{x}^{(n)}), Y(\mathbf{x}^{(1)})\right] & \cdots & \mathrm{cor}\left[Y(\mathbf{x}^{(n)}), Y(\mathbf{x}^{(n)})\right] \end{pmatrix}, \quad (2)$$

where the design variables are correlated with each other using, for example, the basis function with Gaussian form, which depends upon the parameters and θ_j and p_j $(j = 1, ..., k)$:

$$\mathrm{cor}\left[Y(\mathbf{x}^{(i)}), Y(\mathbf{x}^{(l)})\right] = \exp\left(-\sum_{j=1}^{k} \theta_j \left|x_j^{(i)} - x_j^{(l)}\right|^{p_j}\right). \quad (3)$$

To minimize the error of the surrogate model, these parameters must be estimated to maximize the likelihood of the observed data $\mathbf{y}$. Forrester, Sobester and Keane (2008) suggest the use of a metaheuristic global search method such as genetic algorithm or simulated annealing.

Predictions at new points are made based on the computed correlation. As it can be noticed, the Kriging surrogate is built in such a way that the prediction pass through all the data points, interpolating them.

Structural Reliability Analysis

According to Lemaire (2009), the overall goal of the reliability analysis is to determine the probability that a system will fail in service given that its behavior is dependent on random inputs. This behavior is defined by a limit-state or performance function $g(\mathbf{X})$, where $\mathbf{X} = \left\{X_1, X_2, \cdots, X_n\right\}^t$ denotes the n-dimensional vector of random variables represented by well-suited probability distribution functions (PDFs).

The probability of an undesired or unsafe state is expressed in terms of the continuous joint probability density function of the random variables ($f_{\mathbf{X}}$), by integrating it over the failure region, as indicated in Eq. (4).

$$P_f = \int_{g(\mathbf{X})\leq 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}. \tag{4}$$

The classical Monte Carlo simulation (MCS) is the simplest random sampling method used to numerically integrate this equation. However, MCS method usually requires a very large number of function evaluations to ensure an accurate estimate of the probability of failure, which can be impractical whenever the limit-state function is expensive to evaluate.

Approximated methods that ease the computational difficulties by simplifying the integrand $f_{\mathbf{X}}$ and approximating the limit-state function have also been applied. In this work we approach the first-order reliability method (FORM), in which the performance surface $g(\mathbf{X}) = 0$ is approximated by a hyperplane.

The FORM method consists in locating the point on the limit-state surface that has the greatest probability of occurring, the so-called most probable failure point (MPP). In order to simplify the integrand $f_{\mathbf{X}}(\mathbf{X})$ and make the probability integration easy to be computed, the search for the MPP is performed in the standardized normal space (U-space). The distance from the origin of the reduced coordinate system to the MPP is equivalent to the Hasofer-Lind reliability index β (Hasofer and Lind, 1974), used to estimate the probability of failure. Figure 1 depicts the geometric sense of the reliability index in the centered standardized space for two random variables. The first-order approximation of the failure probability is then given by $P_{f_{FORM}} = \Phi(-\beta)$, where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

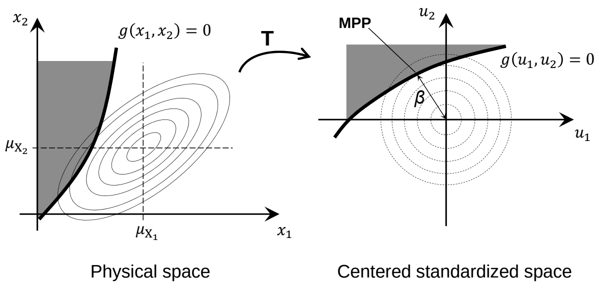


Figure 1: Representation of the Hasofer-Lind index - adapted from Lemaire (2009).

RELIABILITY ASSESSMENT: KRIGING/FORM-BASED METHODOLOGY

The proposed Kriging/FORM-based methodology consists of assessing rotor-bearing system reliability by means of the first-order approximation method, using the Kriging interpolation to approximate and replace implicit performance functions, which are originally based on the finite element model. The ultimate goal of this approach is the reduction of the computational effort without degradation of the accuracy of the reliability results.

The stochastic nature of the rotor system residual unbalance as well as randomness of hydrodynamic bearing stiffness and damping have been taken into account in the framework of the system reliability analysis. Table 1 presents the random variables and corresponding probability distributions used in this approach. Previous researches have demonstrated that in the case of rotating machinery supported by hydrodynamic bearing, the basic design variables possessing stochastic characteristics are the oil-film temperature and the bearing radial clearance.

Table 1: Basic random variables and stochastic models.

Variable	Units	Mean	Std. dev.	Prob. dist.
Oil temperature at the left bearing	°C	35	10%	Normal
Radial clearance at the left bearing	μm	60	10%	Normal
Mass unbalance at the left disk	g·mm	1,500	10%	Normal
Oil temperature at the right bearing	°C	35	10%	Normal
Radial clearance at the right bearing	μm	60	10%	Normal

A computer code was implemented in Matlab® environment, aiming to carry out Kriging metamodeling and reliability analysis. This new tool has capacity to perform Monte Carlo simulations and FORM-based analysis using implicit limit-state functions associated with the rotor-bearing system problem.

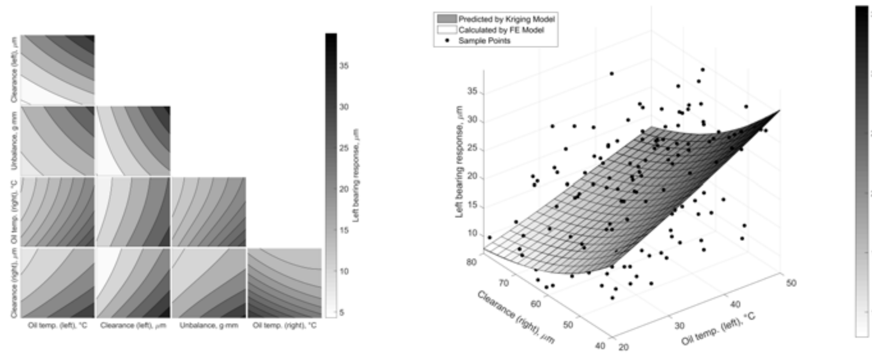
NUMERICAL SIMULATIONS AND RESULTS

The rotor-bearing system simulated is illustrated in Figure 2. It is composed of a steel flexible shaft, two rigid disks, and is supported at each end on an oil-film bearing. The machine operates at 1,200 RPM.

(a) Modeled system. (b) Discretization.

Figure 2: Rotor-bearing system used in the simulations.

Kriging method was used to construct surrogates for the FE model based on different numbers of initial training points sampled by the Latin hypercube technique. Figure 3 depicts the Kriging-based surrogate prediction for the left bearing maximum unbalance response amplitude. The normalized root mean squared errors (NRMSE) were found to be less than 0.25% through the design space, indicating the accuracy of the Kriging predictor when compared with the FEM predictions.

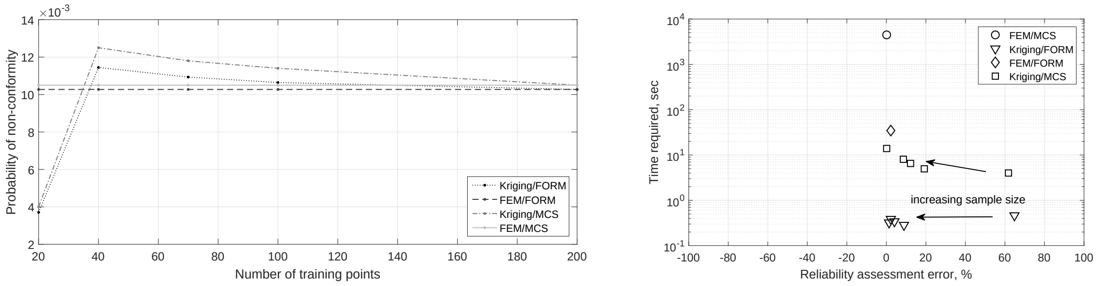


(a) Kriging prediction landscape. (b) Example of a Kriging response surface.

Figure 3: Kriging prediction for the left bearing maximum unbalance response amplitude.

Effectiveness of Kriging/FORM-based reliability analysis

To assess the effectiveness of Kriging/FORM-based reliability analysis, a performance function $g(\mathbf{X}) = Y_{adm} - f(\mathbf{X})$ was defined, where Y_{adm} is the maximum admissible orbit amplitude at the bearings (considered to be 50% of the bearing clearance, i.e. 30μm), and $f(\mathbf{X})$ are the outcomes of the computational model. The probability of failure resulting from the Kriging/FORM-based method was compared with the estimates from the Kriging/MCS, FEM/FORM and FEM/MCS reliability analysis. The computation time to perform the analysis was also assessed. Figure 3 shows the results, with FEM/MCS prediction set as the reference for the true probability of failure. As observed, Kriging estimates converge towards FEM results as the number of training points is increased, and the Kriging/FORM method provides a significant reduction of computational effort.



(a) Kriging-based probability of failure for different sample sizes. (b) Accuracy vs. time efficiency.

Figure 4: Assessment of the effectiveness of Kriging/FORM-based reliability analysis.

CONCLUSIONS

In this work, the capability of Kriging/FORM-based reliability analysis for rotor-bearing systems was assessed. Results indicated that the Kriging surrogates can accurately predict rotor-bearing system outputs with considerably low computational effort. The probability of failure estimated by this method approaches the true value predicted by Monte Carlo simulations as the number of training points used to construct the Kriging model is increased.

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