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### Three-Dimensional instability analysis in a porous medium with inclined temperature gradient and horizontal and vertical throughflow

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**Abstract.** *In this article, the three-dimensional convective and absolute instabilities applying the approach of linear instability analysis in an extended horizontal layer of a saturated porous medium with inclined temperature gradient and horizontal and vertical throughflow are analyzed for a variety of values of horizontal Rayleigh number,  $R_h$ , and the Péclet number,  $Q_v$ . The control parameter is the vertical Rayleigh number,  $R_v$ , which represent the temperature difference between the contours. The computations are performed by using the shooting method for both convective and absolute analysis, where it can guarantee a minimum and a saddle point, respectively. The convective results are compared with the results found in the literature. Due the difficulty in obtaining the critical values of the onset to absolute instability, mainly in three-dimensional analysis, there is not a complete mapping about the absolute instability. The absolute results in the literature are found indirectly through the convective analysis calculating the group velocities. All convective results compare well with the results found in the literature, where there is a stabilization effect with the increase of both  $R_h$  and  $Q_v$ . The absolute results are in agreement with the few results found in literature. Although, there are competition between the transverse and longitudinal modes to be the critical, the control parameter increases when both  $R_h$  and  $Q_v$  are function. When the transverse modes are the critical, the disturbance is oscillatory, when it is longitudinal, it is non-oscillatory.*

**Keywords:** *Saturated porous Layer, Darcy flow, Inclined temperature gradient, Stability of transverse modes, Absolute and convective instability.*

## 1. INTRODUCTION

The problem of convection in a porous medium with inclined temperature gradient is of a considerable practical importance due to a large of environmental, geophysical, biological and industrial applications. Whether or not a flow is stable is also of practical importance because when the instability occurs, the rate of transfer of heat is increased.

The first study of stability of convection in a porous medium with inclined thermal gradient was Weber (1974). This work considered the ratio of height and length small and it was limited just for the case of small horizontal temperature gradient. Years later, Nield (1991) and Nield (1994) using a Galerkin approximation of second and eighth order, respectively, solved the resulting differential equation removing this limitation. Their conclusion was that the increase of the horizontal component of the thermal gradient stabilize and after destabilize the flow. In an extended work, Nield (1998) considered a vertical throughflow, being of relevance to the performance of packed bed reactors. The critical values were computed using a Galerkin approximation of 12-th order, and his analysis of longitudinal modes concluded that the increase of both vertical throughflow and horizontal temperature stabilize the problem. In recent works, Brevdo and Ruderman (2009a) and Brevdo and Ruderman (2009b) using a high-precision pseudo-spectral Chebyshev-collocation method studied the transition to convective and absolute instability of transversal modes, respectively. No previous work related to this topic studied the onset of the convection through the three-dimensional linear instability analysis until Brevdo (2009). His main goal was to study the nature of the destabilization analyzing the group velocity of the wave packet. Whether it is equal to zero, the transition is absolute, otherwise, it is convective. Although this methodology obtained some critical values for the absolute transition, it can not obtain critical values when the onset of the convection is convective, not being very appropriate for absolute analysis.

The present work analyze the three-dimensional convective and absolute instability using the shooting method. The computations are used to find a global minimum and a saddle point, which are features of the transition to convective and absolute instability, respectively. In the absolute analysis, the zero group velocity condition is imposed, however, it is just necessary, but not sufficient condition, so, the collision criterium imposed by Brevdo (1991) must be verify. Both convective and absolute results are compared with those present in Brevdo (2009).

## 2. MATHEMATICAL MODEL

An analysis of a flow in a saturated homogeneous porous medium of height  $H$  with an inclined thermal gradient and horizontal and vertical throughflow bounded by two permeable surfaces is carried out. The origin of the Cartesian system is in the middle of the height between the contours, where the  $z$ - and the  $x$ -axis are vertically upward and horizontally to the right. The temperature difference between the contours is the  $\Delta T$ , the horizontal temperature gradient  $\sigma$  is constant with the opposite direction of the  $x$ -axis and the vertical flow is also constant and upwards. All dimensional analysis can be viewed in (Brevdo and Ruderman, 2009a). A sketch of the dimensionless problem can be showed in the Fig. (1).

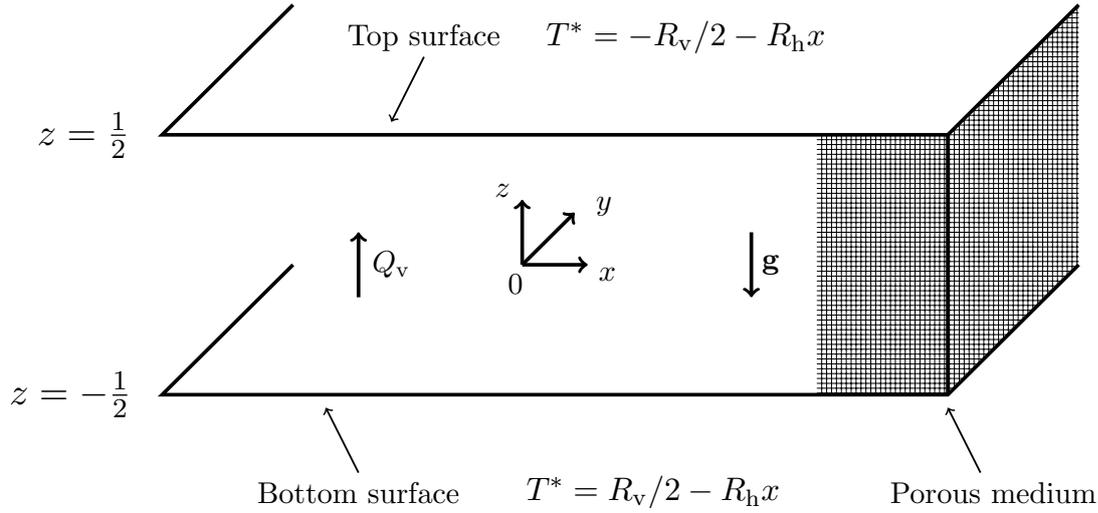


Figure 1. A sketch of the model of the porous medium.

### 2.1 Governing Equation

Assuming that the porous medium is governed by the Darcy's law and the Oberbeck-Boussinesq approximation is valid. Therefore, the dimensionless governing equation can be written as

$$\nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$\nabla P + \mathbf{v} - T\mathbf{k} = 0, \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T, \quad (3)$$

where they are valid in  $-\infty < x, y < \infty$  and  $-1/2 < z < 1/2$ . The boundary condition take the form as following

$$w = Q_v, \quad T = \mp R_v/2 - R_h x \quad \text{at} \quad z = \pm 1/2. \quad (4)$$

The dimensionless parameters that appear are the vertical Rayleigh number  $R_v$ , horizontal Rayleigh number  $R_h$  and the Péclet number  $Q_v$ . They represent the vertical temperature difference between the contours, the horizontal thermal gradient and the vertical throughflow, respectively, and can be shown in Eq. (5).

$$R_v = \frac{\rho_0 g \gamma K H \Delta T}{\mu \alpha_m}, \quad R_h = \frac{\rho_0 g \gamma \bar{K} H^2 \beta}{\mu \alpha_m}, \quad Q_v = \frac{w_v H}{\alpha_m} \quad (5)$$

## 2.2 Base Solution

Looking for a base solution of the Eqn. (1), (2), (3) and (4) of the following form

$$T_s = \tilde{T}(z) - R_h x, \quad u_s = U(z), \quad v_s = 0, \quad w_s = Q_v, \quad P_s = P(x, y, z). \quad (6)$$

Considering that the equation (6) must satisfy the boundary condition for the temperature and the zero net horizontal flow condition, the base solution for horizontal flow  $u_s$  and temperature  $T_s$  is showed in the Eqn. (7) and (8).

$$u_s(z) = R_h z \quad (7)$$

$$T_s(x, z) = \frac{R_h^2}{2Q_v} \left( z^2 - \frac{1}{4} \right) + \frac{R_h^2}{Q_v^2} z - \frac{Q_v^2 R_v + R_h^2}{2Q_v^2 \sinh(Q_v/2)} [e^{Q_v z} - \cosh(Q_v/2)] - R_h x \quad (8)$$

The base solution for the pressure can be obtained integrating the Darcy's equation, Eqn. (2), using the base solution of the velocity and temperature, and can be showed in Eqn. (9).

$$P_s(x, y, z) = \frac{R_h}{2Q_v} \left( \frac{z^3}{3} - \frac{z}{4} \right) + \frac{R_h^2}{2Q_v^2} z^2 - \frac{Q_v^2 R_v + R_h^2}{2Q_v^3 \sinh(Q_v/2)} [e^{Q_v z} - z Q_v \cosh(Q_v/2)] - R_h x z - Q_v z + P_0 \quad (9)$$

## 2.3 Linear Instability Analysis

When a steady state solution is disturbed, the properties can be decomposed in a combination of base solution and perturbation. Considering  $\epsilon \ll 1$  as being the amplitude of the disturbance, the properties are represented by a linear combination of the base solution as showed as following.

$$[\mathbf{v}, P, T] = [\mathbf{v}_s, P_s, T_s] + \epsilon [\mathbf{v}_p, P_p, T_p] \quad (10)$$

where the subscript  $s$  and  $p$  refer the base solution and the perturbation, respectively. Considering that the disturbance behave like a wave, the normal modes expansion is done, as following

$$[u_p, v_p, w_p, T_p, P_p] = [u_n(z), v_n(z), w_n(z), T_n(z), P_n(z)] \times \exp[i(\alpha x + \beta y - \omega t)] \quad (11)$$

where  $i$  is the imaginary number,  $x$  and  $y$  are the streamwise and spanwise coordinates, respectively,  $z$  is the only inhomogeneous coordinate, and  $t$  is time. Furthermore,  $\alpha$ ,  $\beta$  and  $\omega$  are the streamwise and spanwise wavenumber and the frequency, respectively.

Substituting the Eqn. (10) and (11) into Eqn. (1), (2) and (3), collecting the terms of order  $\epsilon$  and vanishing  $u_n$ ,  $v_n$  and  $P_n$  from the resulting equation leads to the differential dispersion equation, which contains both the eigenvalues and eigenvectors. This equation can be showed the Eqn. (12).

$$D : \begin{cases} w_n'' + (\alpha^2 + \beta^2)(T_n - w_n) = 0 \\ (\alpha^2 + \beta^2) \left( T_n'' - Q_v T_n' - (i R_h z \alpha + \alpha^2 + \beta^2 - i \omega) T_n - w_n \tilde{T}' \right) + i R_h \alpha w_n' = 0 \end{cases} \quad (12)$$

where the superscript  $'$  indicate the derivative in relation to  $z$ . The substitution of the Eqn. (10) and (11) into the Eqn. (4) leads the boundary condition for the Eqn. (12).

$$w_n = 0, \quad T_n = 0 \quad \text{at} \quad z = \pm 1/2 \quad (13)$$

## 3. NUMERICAL PROCEDURES

The shooting method consists in transforming a boundary value problem in an initial value problem for then solving it with a marching and a root-finding approaches. To find the convective and absolute instability, some different assumptions are done for each case. According to the linear instability analysis, if  $\omega_i$  is negative, the perturbation decrease along the time, so the problem is considered stable, but if it is positive, the perturbation increase along the time, so the problem is considered unstable. Then, we will consider  $\omega_i$  equal to zero for both of studies, and for the convective analysis,  $\alpha_i$  and  $\beta_i$  will also be zero, however, for the absolute case, they will be considered different to zero.

### 3.1 Convective analysis

The convective results can be obtained plotting marginal surface,  $R_v \times \alpha_R \times \beta_R$  for then derivate it to find a global minimum. Instead of derivating the marginal surface, the procedure used in this work is to derivate the dispersion relation with respect the wavenumbers, considering the derivatives of the dimensionless parameters equal to zero. The following equation show this derivation.

$$\begin{cases} \frac{dD}{d\alpha} = D_\alpha \left( \alpha_R, \beta_R, \omega_R, \frac{\partial \omega}{\partial \alpha}, T_n(z), \frac{\partial T_n(z)}{\partial \alpha}, w_n(z), \frac{\partial w_n(z)}{\partial \alpha}, R_v, R_h, Q_v \right) = 0 \\ \frac{dD}{d\beta} = D_\beta \left( \alpha_R, \beta_R, \omega_R, \frac{\partial \omega}{\partial \beta}, T_n(z), \frac{\partial T_n(z)}{\partial \beta}, w_n(z), \frac{\partial w_n(z)}{\partial \beta}, R_v, R_h, Q_v \right) = 0 \end{cases} \quad (14)$$

where  $\frac{\partial \omega}{\partial \alpha}$  and  $\frac{\partial \omega}{\partial \beta}$  are the streamwise and spanwise group velocities. An important point in this methodology is consider the derivatives of the eigenfunction in relation the wavenumbers as another eigenfunction. So the boundary condition for theses new eigenfunction are

$$\begin{cases} \frac{\partial w_n(z)}{\partial \alpha} = \frac{\partial T_n(z)}{\partial \alpha} = 0 \\ \frac{\partial w_n(z)}{\partial \beta} = \frac{\partial T_n(z)}{\partial \beta} = 0 \end{cases} \quad \text{at } z = \pm 1/2. \quad (15)$$

The critical values are achieved solving simultaneously the differential dispersion relation, Eqn. (12), and its derivatives, Eqn. (14), considering its boundary conditions Eqn. (13) and (15). The initial guesses for the shooting method were the results from Brevdo (2009).

### 3.2 Absolute analysis

The physical difference between the convective and the absolute instability is that the perturbation propagates just in downstream direction in the convective instability, while, in the absolute instability, it propagates in both direction, downstream and upstream. A necessary but not sufficient condition to the imminence of the absolute instability is the group velocity is equal to zero. To impose this condition, it's necessary derivate Eqn. (12) again, however, the imaginary part of the complex wavenumbers are not equal to zero.

$$\begin{cases} \frac{dD}{d\alpha} = D_\alpha \left( \alpha, \beta, \omega_R, T_n(z), \frac{\partial T_n(z)}{\partial \alpha}, w_n(z), \frac{\partial w_n(z)}{\partial \alpha}, R_v, R_h, Q_v \right) = 0 \\ \frac{dD}{d\beta} = D_\beta \left( \alpha, \beta, \omega_R, T_n(z), \frac{\partial T_n(z)}{\partial \beta}, w_n(z), \frac{\partial w_n(z)}{\partial \beta}, R_v, R_h, Q_v \right) = 0 \end{cases}, \quad \omega_i = 0 \quad (16)$$

Then, to find the absolute critical value, the Eq. (12) and (16) must be solve simultaneously, together with their boundary conditions, Eq. (13) and (15). This method guarantees a saddle point, however, to be a pinching point, the collision criterion developed by Brevdo (1991) must be satisfied. The initial guesses for the shooting method were the convective results from Brevdo (2009), since the convective instability occurs before the transition to absolute.

## 4. RESULTS AND DISCUSSION

In this section will be addressed the obtained results in both convective and absolute analysis, including the verification and the new results.

### 4.1 Convective Results

The following three tables show the critical values of control parameter  $R_v$ , real part of the wavenumber and the group velocities for the three-dimensional convective instability. The results present in the this tables were compared with those found in Brevdo (2009) with the same number of significant digits. The maximum relative errors for vertical Rayleigh number, wavenumber and group velocities are, respectively, 0.0099%, 0.23% and 1.5%. All critical modes are non-oscillatory, i.e.,  $\omega_R = 0$ .

By the table 1, it is possible to observe a stabilization effect with the increase of both Péclet and horizontal Rayleigh. All critical values of control parameter are smaller than the two-dimensional results of the transversal modes found in (Brevdo and Ruderman, 2009a), therefore, the nature of the destabilization is three-dimensional.

The spanwise wavenumber is an increasing function of both  $R_h$  and  $Q_v$  and the streamwise wavenumber is always equal to zero, as can be observed by the table 2. This implies that the convection rolls are longitudinal. By the table 3, it is possible to analyze the group velocities, and it is observed that when the product  $R_h \cdot Q_v$  is equal to zero, both group velocities is also zero and nature of the destabilization is absolute, unlike, it is different to zero and the transition is convective.

	Rh=0	10	20	30	40	50	60
Qv=0	39.478	42.008	49.549	61.957	78.966	100.12	124.47
1	40.875	43.403	50.940	63.336	80.316	101.41	125.65
2	45.078	47.603	55.127	67.486	84.380	105.29	129.18
3	52.068	54.588	62.088	74.381	91.128	111.74	135.05
4	61.666	64.171	71.616	83.786	100.28	120.39	142.53
5	73.415	75.879	83.189	95.078	111.02	129.93	148.98
6	86.619	88.994	96.008	107.30	122.08	138.82	155.56
7	100.58	102.80	109.33	119.67	132.92	147.73	162.89
8	114.83	116.86	122.77	132.05	143.85	157.12	171.00

Table 1. Convective critical value of the parameter control  $R_v$

	Rh=0	10	20	30	40	50	60
Qv=0	(0, 3.14)	(0, 3.14)	(0, 3.15)	(0, 3.16)	(0, 3.22)	(0, 3.34)	(0, 3.67)
1	(0, 3.18)	(0, 3.18)	(0, 3.18)	(0, 3.20)	(0, 3.25)	(0, 3.38)	(0, 3.71)
2	(0, 3.29)	(0, 3.29)	(0, 3.30)	(0, 3.31)	(0, 3.37)	(0, 3.50)	(0, 3.83)
3	(0, 3.49)	(0, 3.49)	(0, 3.50)	(0, 3.52)	(0, 3.58)	(0, 3.73)	(0, 4.11)
4	(0, 3.79)	(0, 3.79)	(0, 3.81)	(0, 3.85)	(0, 3.95)	(0, 4.18)	(0, 5.11)
5	(0, 4.20)	(0, 4.21)	(0, 4.25)	(0, 4.35)	(0, 4.58)	(0, 5.22)	(0, 6.68)
6	(0, 4.73)	(0, 4.76)	(0, 4.85)	(0, 5.06)	(0, 5.52)	(0, 6.40)	(0, 7.46)
7	(0, 5.38)	(0, 5.43)	(0, 5.58)	(0, 5.89)	(0, 6.45)	(0, 7.24)	(0, 8.10)
8	(0, 6.09)	(0, 6.15)	(0, 6.34)	(0, 6.70)	(0, 7.25)	(0, 7.95)	(0, 8.70)

Table 2. Convective critical value of  $(\alpha_R, \beta_R)$

	Rh=0	10	20	30	40	50	60
Qv=0	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
1	(0, 0)	(0.0937, 0)	(0.179, 0)	(0.246, 0)	(0.281, 0)	(0.268, 0)	(0.198, 0)
2	(0, 0)	(0.218, 0)	(0.423, 0)	(0.597, 0)	(0.721, 0)	(0.772, 0)	(0.774, 0)
3	(0, 0)	(0.402, 0)	(0.793, 0)	(1.16, 0)	(1.50, 0)	(1.82, 0)	(2.40, 0)
4	(0, 0)	(0.666, 0)	(1.34, 0)	(2.06, 0)	(2.86, 0)	(4.03, 0)	(8.71, 0)
5	(0, 0)	(1.02, 0)	(2.10, 0)	(3.35, 0)	(5.08, 0)	(8.62, 0)	(15.7, 0)
6	(0, 0)	(1.43, 0)	(2.99, 0)	(4.94, 0)	(7.78, 0)	(12.2, 0)	(17.4, 0)
7	(0, 0)	(1.84, 0)	(3.88, 0)	(6.38, 0)	(9.69, 0)	(13.9, 0)	(18.4, 0)
8	(0, 0)	(2.21, 0)	(4.62, 0)	(7.46, 0)	(10.9, 0)	(14.9, 0)	(19.1, 0)

Table 3. Convective critical value of the group velocities  $(\partial\omega/\partial\alpha, \partial\omega/\partial\beta)$

## 4.2 Absolute Results

The convective results were used as initial guesses for the absolute analysis, especially, the cases where the group velocities are equal to zero. A study about the transversal, longitudinal and oblique modes were done. Although all convective critical modes are longitudinal, in the transition to absolute instability, there is a competition between the transversal and the longitudinal modes to be the critical.

	Rh=0	10	20	30	40	50	60
Qv=0	39.478	42.008	49.549	61.957	78.966	100.12	124.47
1	40.875	43.448	50.979	63.365	80.335	101.41	125.65
2	45.078	47.846	55.341	67.656	84.498	105.36	129.22
3	52.068	55.410	62.834	75.012	91.616	112.08	135.29
4	61.666	66.429	73.729	85.674	101.90	121.76	144.02
5	73.415	80.680	88.187	99.875	115.53	134.54	155.53
6	86.619	94.343	106.68	117.75	132.68	150.48	169.83
7	100.58	108.01	125.90	139.30	153.13	169.52	186.87
8	114.83	121.65	138.04	159.50	176.84	191.44	206.43

Table 4. Absolute critical value of the control parameter  $R_v$

The tables 4, 5, 6 and the 7 present, respectively, the critical values of the vertical Rayleigh number,  $R_v$ , the wave length,  $\alpha_R$  and  $\beta_R$ , the spatial growth,  $\alpha_i$  and  $\beta_i$ , and the frequency,  $\omega_R$ . The table 4 shows that a higher vertical temperature gradient is required to occurs the absolute instability when increase both vertical throughflow and horizontal temperature gradient. By the table 5, it is observed that for the range  $0 \leq R_h \leq 30$  and  $0 \leq Q_v \leq 8$ , the longitudinal and transversal rolls alternate between themselves to be the critical. Analyzing the table 6, the spatial growth in  $x$ -direction has a tendency to decrease for the longitudinal modes and increase for the transversal mode when the horizontal thermal gradient is increased. When the product  $R_h \cdot Q_v$  is equal the zero, there is no spatial growth in both direction.

	Rh=0	10	20	30	40	50	60
Qv=0	(0, 3.14)	(0, 3.14)	(0, 3.15)	(0, 3.16)	(0, 3.22)	(0, 3.34)	(0, 3.67)
1	(0, 3.18)	(0, 3.20)	(0, 3.18)	(0, 3.19)	(0, 3.24)	(0, 3.37)	(0, 3.70)
2	(0, 3.29)	(0, 3.41)	(0, 3.28)	(0, 3.28)	(0, 3.33)	(0, 3.46)	(0, 3.79)
3	(0, 3.49)	(0, 3.84)	(0, 3.47)	(0, 3.42)	(0, 3.46)	(0, 3.60)	(0, 3.93)
4	(0, 3.79)	(0, 4.57)	(0, 3.76)	(0, 3.62)	(0, 3.64)	(0, 3.78)	(0, 4.12)
5	(0, 4.20)	(4.26, 0)	(0, 4.19)	(0, 3.90)	(0, 3.88)	(0, 4.00)	(0, 4.35)
6	(0, 4.73)	(4.98, 0)	(0, 4.79)	(0, 4.28)	(0, 4.17)	(0, 4.28)	(0, 4.64)
7	(0, 5.38)	(5.75, 0)	(6.60, 0)	(0, 4.76)	(0, 4.55)	(0, 4.63)	(0, 4.99)
8	(0, 6.09)	(6.50, 0)	(7.31, 0)	(8.19, 0)	(0, 5.03)	(0, 5.06)	(0, 5.43)

Table 5. Absolute critical value of the  $(\alpha_R, \beta_R)$

	Rh=0	10	20	30	40	50	60
Qv=0	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
1	(0, 0)	(0.473, 0)	(0.213, 0)	(0.118, 0)	(0.0657, 0)	(0.0327, 0)	(0.0121, 0)
2	(0, 0)	(1.07, 0)	(0.487, 0)	(0.276, 0)	(0.161, 0)	(0.0889, 0)	(0.0432, 0)
3	(0, 0)	(1.88, 0)	(0.870, 0)	(0.507, 0)	(0.311, 0)	(0.188, 0)	(0.110, 0)
4	(0, 0)	(2.97, 0)	(1.39, 0)	(0.830, 0)	(0.531, 0)	(0.345, 0)	(0.226, 0)
5	(0, 0)	(0.958, 0)	(2.06, 0)	(1.25, 0)	(0.830, 0)	(0.567, 0)	(0.399, 0)
6	(0, 0)	(1.23, 0)	(2.87, 0)	(1.78, 0)	(1.21, 0)	(0.859, 0)	(0.637, 0)
7	(0, 0)	(1.37, 0)	(2.59, 0)	(2.41, 0)	(1.68, 0)	(1.23, 0)	(0.948, 0)
8	(0, 0)	(1.45, 0)	(2.71, 0)	(3.88, 0)	(2.23, 0)	(1.68, 0)	(1.34, 0)

Table 6. Absolute critical value of the  $(-\alpha_i, -\beta_i)$

Due the competition, the frequency become different to zero when the transversal mode is the critical. It is interesting to observe that when critical mode is the transversal, its critical values are exactly those found in two-dimensional absolute analysis done by Brevdo and Ruderman (2009b). It is already expected, because for this case, both  $\beta_R$  and  $\beta_i$  in the tables 5 and 6 are equal to zero.

	Rh=0	10	20	30	40	50	60
Qv=0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	5.06	0	0	0	0	0
6	0	8.15	0	0	0	0	0
7	0	11.7	32.3	0	0	0	0
8	0	15.3	39.1	73.5	0	0	0

Table 7. Absolute critical value of the frequency  $\omega_R$

The shooting method using the zero group velocity condition guarantee that the found results are just saddle point, but not necessary the critical state, i.e., *pinching point*. To show that the found results are truly pinching point, just two cases were chosen to verify the collision of the branches. The figures 2 and 3 show this verification for these two cases. This criterium established by (Brevdo, 1991) say that the collision of the branches must cross the imaginary plane of at least one wavenumber plane,  $\alpha$  or  $\beta$ .

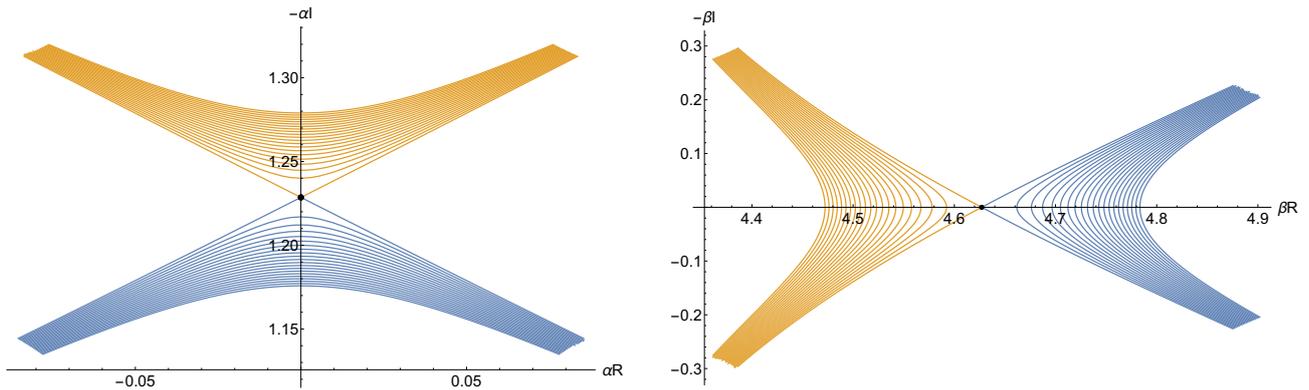


Figure 2. Collision check for the case  $R_h = 50$  e  $Q_v = 7$

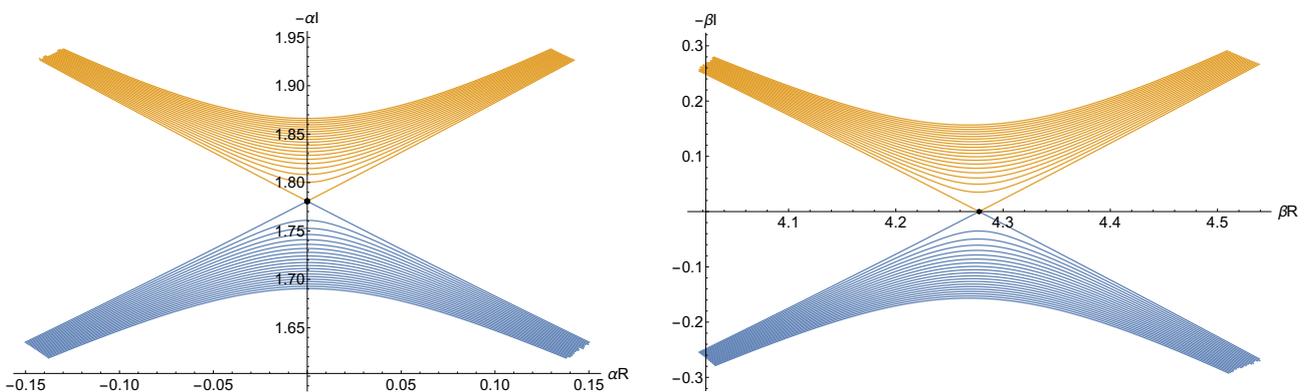


Figure 3. Collision check for the case  $R_h = 30$  e  $Q_v = 6$

## 5. CONCLUSION

In this paper, the three-dimensional convective and absolute instabilities in a saturated porous medium with inclined thermal gradient and horizontal and vertical throughflow were studied. The first analysis revealed that the destabilization the stabilization effect when increase the horizontal component of the temperature gradient and vertical throughflow represented, respectively, by  $R_h$  and  $Q_v$ , and all disturbances are non-oscillatory. In the second analysis, although there is a competition of transversal and longitudinal modes, the critical value of vertical Rayleigh number is an increasing function of both  $R_h$  and  $Q_v$  and there is not a spatial growth in  $y$ -direction for all cases analyzed. The transition to absolute instability has a three-dimensional nature, and when the transversal modes are critical, the disturbance is oscillatory, and when, the longitudinal modes are the critical, the disturbance is non-oscillatory. By the table of group velocities in convective analysis, the nature of the destabilization is absolute when the product  $R_h \cdot Q_v$  is equal to zero, otherwise, it is convective, exactly what was done in the absolute analysis in (Brevdo, 2009).

The computation used in both analysis proved to be a good method to found critical values for the imminence of convective and, especially, absolute instability. All convective results compare very well with the literature's results. Using the zero group velocity condition with shooting method is a great methodology to absolute instability, since the conventional way to analyze it is almost impossible. The two case chosen to verify the collision check are truly critical values assuring a considerable reliability.

## 6. ACKNOWLEDGEMENTS

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