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AN ALGORITHM FOR SOLVING AN IMPLICIT SOLUTION FOR FULLY DEVELOPED FLOW IN A CHANNEL OF A GIESEKUS FLUID

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Abstract. *This work presents the development of a semi-analytical solution of the Giesekus constitutive equation that is compared to the numerical solution obtained with a Direct Numerical Simulation (DNS) code where the governing equations are written in a vorticity-velocity formulation. The numerical simulation is applied to a two-dimensional viscoelastic fluid flow in a straight channel using a high-order compact finite difference scheme for the spatial derivatives discretization. The semi-analytical solution is used to study the minimal distance necessary to achieve the fully developed solution in the non-Newtonian flow. The results show the Reynolds and Weissenberg number influence in this minimal distance.*

Keywords: *Semi-analytical solution, Giesekus fluid, Straight channel flow, Direct Numerical Simulation*

1. INTRODUCTION

Computational fluid dynamics is an area of significant industrial interest in the chemical, food and oil fields. Among the various problems that have arisen in this area is the treatment of non-Newtonian fluids flows, since viscoelastic fluids find application in many industrial processes and theirs study represents a great challenge. Therefore, a great deal of effort has been expended over the last twenty years or so in attempting to find robust and stable numerical methods for viscoelastic flow problems. However, this is not an easy work because the viscoelastic fluid constitutive equations are complex and are difficult to treat in computationally.

Analytical solutions are a valuable tool to understand the complexity of fluid dynamics. The Cauchy equation together with a rheological constitutive equation, allow the determination of the flow characteristics of non-Newtonian fluids. However, these are complex equations for which analytical solutions can only be obtained for basic flows in simple geometries. Adding boundary conditions to this system of equations increases the complexity to obtain analytical solutions (Ferrás *et al.*, 2012). The derivation of analytical solutions for the Giesekus fluid is well explained by Yoo and Choi (Yoo and Choi, 1989), Schleiniger and Weinacht (Schleiniger and Weinacht, 1991) and Raisi *et al.* (Raisi *et al.*, 2007).

The quest for analytical solutions of the most frequently used viscoelastic rheological models in relatively simple flows is, a matter of great importance but one which has been largely overlooked (Cruz *et al.*, 2005). In many applications, as in many numerical techniques, for example Direct Numerical Simulation (DNS) or Linear Stability Theory (LST), it is important to know the baseflow for simulations at high Reynolds numbers, and this baseflow solution is not always known analytically. Since this baseflow is generated numerically, as in the straight channel flow problem, the channel size must be too large to reach the fully developed solution, which is often impractical due to the high computational cost.

The Direct Numerical Simulation of transitional and turbulent incompressible flows is an area that is increasing with the advance in computational resources. The DNS technique solves directly the Navier-Stokes equations, usually employing high-order methods, such that all scales of the flow are simulated, from the largest and most energetic to the smallest.

The main restriction of this technique is related to the computational cost.

In this work, we are concerned with numerical simulation of two-dimensional viscoelastic fluid flow using the Giesekus constitutive equation. We deal specifically with the investigation of the baseflow in a straight channel flow. The analysis is carried out by means of Direct Numerical Simulation and compared with a semi-analytical solution. In the DNS formulation, the governing equations are written in a vorticity-velocity formulation and the linear system arising from the numerical solution of the Poisson equation is solved by a multigrid methods. The spatial derivatives are discretized by compact finite difference schemes. The time integration is carried out by a fourth-order Runge-Kutta method. In order to evaluate the baseflow in a Poiseuille flow, different values of dimensionless parameters are tested for Newtonian and non-Newtonian fluid flows, comparing the results obtained by DNS and the semi-analytical solution.

2. MATHEMATICAL FORMULATION

In this work, we consider a non-Newtonian, two-dimensional and incompressible fluid flow which it is assumed to be unsteady and without body forces. The governing equations are the continuity

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

and the Navier-Stokes equation

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) \right) = -\nabla p + \nabla^2 \mathbf{u} + \nabla \cdot \mathbf{T}, \tag{2}$$

where $\mathbf{u}$ denotes the velocity field, t is the time, ρ is the fluid density, p is the pressure and $\mathbf{T}$ is the extra-stress tensor which must obey an appropriate constitutive equation. In this paper, we worked with viscoelastic flows governed by the non-linear Giesekus constitutive equation (Giesekus, 1982), given by:

$$\mathbf{T} + \lambda \left(\overset{\nabla}{\mathbf{T}} + \frac{\alpha_G}{\eta_p} (\mathbf{T} \cdot \mathbf{T}) \right) = 2\eta_p \mathbf{D}, \tag{3}$$

where $\mathbf{D} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ is the rate of deformation tensor, λ is the relaxation-time of the fluid, α_G is the so-called mobility parameter, η_p is the polymer-contributed viscosity and $\overset{\nabla}{\mathbf{T}}$ is the upper-convected derivative of $\mathbf{T}$, defined by:

$$\overset{\nabla}{\mathbf{T}} = \frac{\partial \mathbf{T}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{T}) - \mathbf{T} \cdot (\nabla \mathbf{u})^T - (\nabla \mathbf{u}) \cdot \mathbf{T}. \tag{4}$$

Introducing the following non-dimensionalized scalings:

$$\mathbf{x}^* = \frac{x}{L}, \quad \mathbf{u}^* = \frac{\mathbf{u}}{U}, \quad t^* = \frac{U}{L}t, \quad p^* = \frac{p}{\rho U^2}, \quad \mathbf{T}^* = \frac{\mathbf{T}}{\rho U^2}, \tag{5}$$

where L and U denote length and velocity scales, respectively. These equations can then be written (omitting the symbol $*$ for convenience)

$$\nabla \cdot \mathbf{u} = 0, \tag{6}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \frac{\beta}{Re} \nabla^2 \mathbf{u} + \nabla \cdot \mathbf{T}, \tag{7}$$

$$\mathbf{T} + Wi\, \overset{\nabla}{\mathbf{T}} + \frac{\alpha_G\, Wi\, Re}{(1-\beta)} (\mathbf{T} \cdot \mathbf{T}) = 2\frac{(1-\beta)}{Re} \mathbf{D}, \tag{8}$$

where the dimensionless parameters $Re = (\rho U L)/(\eta)$ and $Wi = (\lambda U)/(L)$ denote the associated Reynolds and Weissenberg numbers, respectively. The amount of Newtonian solvent is controlled by the dimensionless solvent viscosity coefficient, $\beta = \eta_s/\eta_0$, where $\eta_0 = \eta_s + \eta_p$ denotes the total shear viscosity; η_s and η_p represent the Newtonian solvent and polymeric viscosities, respectively.

2.1 Direct Numerical Simulation

To simulate all scales of the flow, from the largest and most energetic to the smallest, it is employed a high order finite difference schemes, without adding closing equations.

In order to eliminate the pressure term in the Navier-Stokes equations, it is adopted the vorticity-velocity formulation. Thus, the vorticity in direction z, ω_z, is defined by:

$$\omega_z = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}. \tag{9}$$

Therefore, the equations (6)-(8) in the two-dimensional and non-dimensional form can be rewritten as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{10}$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -\frac{\partial \omega_z}{\partial x}, \tag{11}$$

$$\frac{\partial \omega_z}{\partial t} + \frac{\partial (u\omega_z)}{\partial x} + \frac{\partial (v\omega_z)}{\partial y} = \frac{\beta}{Re}\left(\frac{\partial^2 \omega_z}{\partial x^2} + \frac{\partial^2 \omega_z}{\partial y^2}\right) + \frac{\partial^2 T^{xx}}{\partial x \partial y} + \frac{\partial^2 T^{xy}}{\partial y^2} - \frac{\partial^2 T^{xy}}{\partial x^2} - \frac{\partial^2 T^{yy}}{\partial x \partial y}, \tag{12}$$

$$T^{xx} + Wi\Big(\frac{\partial T^{xx}}{\partial t} + \frac{\partial (uT^{xx})}{\partial x} + \frac{\partial (vT^{xx})}{\partial y} - 2T^{xx}\frac{\partial u}{\partial x} - 2T^{xy}\frac{\partial u}{\partial y}\Big) + \alpha_G \frac{WiRe}{(1-\beta)}(T^{xx^2} + T^{xy^2}) = = 2\frac{(1-\beta)}{Re}\frac{\partial u}{\partial x}, \tag{13}$$

$$T^{xy} + Wi\Big(\frac{\partial T^{xy}}{\partial t} + \frac{\partial (uT^{xy})}{\partial x} + \frac{\partial (vT^{xy})}{\partial y} - T^{xx}\frac{\partial v}{\partial x} - T^{yy}\frac{\partial u}{\partial y}\Big) + \alpha_G \frac{WiRe}{(1-\beta)}(T^{xx}T^{xy} + T^{xy}T^{yy}) = = \frac{(1-\beta)}{Re}\Big(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\Big), \tag{14}$$

$$T^{yy} + Wi\Big(\frac{\partial T^{yy}}{\partial t} + \frac{\partial (uT^{yy})}{\partial x} + \frac{\partial (vT^{yy})}{\partial y} - 2T^{xy}\frac{\partial v}{\partial x} - 2T^{yy}\frac{\partial v}{\partial y}\Big) + \alpha_G \frac{WiRe}{(1-\beta)}(T^{xy^2} + T^{yy^2}) = = 2\frac{(1-\beta)}{Re}\frac{\partial v}{\partial y}. \tag{15}$$

For the boundaries conditions of the computational domain, Fig. 1, the following conditions were used:

- At the inflow:

$$\begin{aligned} &u = U(y), \quad v = 0, \\ &T^{xx} = T^{xx}(y), \quad T^{xy} = T^{xy}(y) \quad \text{and} \quad T^{yy} = T^{yy}(y). \end{aligned} \tag{16}$$

- On wall boundaries: no-slip condition and impermeability ($u = 0, v = 0$) are employed.
- At the outflow: the second derivative of variables in respect to x are set to zero.

3. ANALYTICAL SOLUTION FOR STEADY POISEUILLE FLOWS OF A GIESEKUS FLUID

In this section will be presented the steps to solve analytically the non-linear system of equations which represents the steady state of the isothermal, incompressible flows of a Giesekus fluid with a Newtonian solvent in a channel based on (Schleiniger and Weinacht, 1991), who presented them using a different dimensionless form of the system. Schleiniger and Weinacht (1991) solved and discussed mathematically the solutions for steady Poiseuille flows in two-dimension Cartesian coordinates considering Giesekus fluids with and without Newtonian solvent; and they also commented about Giesekus fluids without Newtonian solvent for the axi-symmetric case. In that paper, the solution is not obtained explicitly, i.e. the derivative of the component of the velocity is given by an implicit equation, moreover some procedures for the calculation of the solution may be not checked out directly by the reader, hence in this section, it will be provided a detailed explanation to achieve the analytical solution and a numerical algorithm for solving the implicit equation which is named herein by “semi-analytical solution”.

Consider the system of equations representing the isothermal, incompressible flows of a Giesekus fluid with a Newtonian solvent in two-dimensional Cartesian coordinates (x, y) and dimensionless form under fully developed conditions, such as: steady state flows $\left(\frac{\partial(\cdot)}{\partial t} = 0\right)$, no variation of the velocity and tensor along the streamwise direction $\left(\frac{\partial(\cdot)}{\partial x} = 0, \mathbf{u} = \mathbf{u}(y), \mathbf{T} = \mathbf{T}(y)\right)$ and null cross component of the velocity ($\mathbf{u} = (u(y), 0)$) and a constant streamwise gradient of pressure $\left(\frac{\partial p(x,y)}{\partial x} = p_x < 0\right)$. According to these assumptions, it is being considered a horizontal channel where the fluid flows along the x-direction, hence it will be taken the following system into account,

$$p_x = \frac{\beta}{Re}u'' + T^{xy\prime} \tag{17}$$

$$\frac{\partial p(x,y)}{\partial y} = T^{yy\prime} \tag{18}$$

$$T^{xx} - 2WiT^{xy}u' + \frac{\alpha ReWi}{(1-\beta)}(T^{xx2} + T^{xy2}) = 0 \tag{19}$$

$$T^{xy} - WiT^{yy}u' + \frac{\alpha ReWi}{(1-\beta)}T^{xy}(T^{xx} + T^{yy}) = \frac{(1-\beta)}{Re}u' \tag{20}$$

$$T^{yy} + \frac{\alpha ReWi}{(1-\beta)}(T^{xy2} + T^{yy2}) = 0 \tag{21}$$

where $(\cdot)' = \frac{d(\cdot)}{dy}$ and $(\cdot)'' = \frac{d^2(\cdot)}{dy^2}$ for simplicity of the notation. In addition, it will be considered $-1 \leq y \leq 1$ and, thus, $T^{xx}(0) = T^{xy}(0) = T^{yy}(0) = u'(0) = 0$ at the centerline of the channel.

We point out that the system of equations (17)-(21) is analogous to the system (2.1)-(2.5) solved by Schleiniger and Weinacht (1991) unless the dimensionless parameters.

Rewriting Eq. (21) in a equivalent form, we get

$$\left(T^{yy} + \frac{(1-\beta)}{2\alpha ReWi}\right)^2 + T^{xy2} = \frac{(1-\beta)^2}{4\alpha^2 Re^2 Wi^2}\ , \tag{22}$$

which provides two expressions to T^{yy} as a function of T^{xy},

$$T^{yy} = \frac{-(1-\beta) \pm \sqrt{(1-\beta)^2 - 4\alpha^2 Re^2 Wi^2 T^{xy2}}}{2\alpha ReWi}\ . \tag{23}$$

As the tensor is null along the centreline of the channel, the best choice in Eq. (23) is the plus sign:

$$T^{yy} = \frac{-(1-\beta) + \sqrt{(1-\beta)^2 - 4\alpha^2 Re^2 Wi^2 T^{xy2}}}{2\alpha ReWi}\ . \tag{24}$$

Adding Eq. (19) and Eq. (21), we have

$$(T^{xx} + T^{yy}) + \frac{\alpha ReWi}{(1-\beta)}[(T^{xx} + T^{yy})^2 - 2T^{xx}T^{yy} + 2T^{xy2}] - 2WiT^{xy}u' = 0\ , \tag{25}$$

or equivalently,

$$\frac{\alpha ReWi}{(1-\beta)}(T^{xx} + T^{yy})^2 + (T^{xx} + T^{yy}) - \frac{2\alpha ReWi}{(1-\beta)}T^{xx}T^{yy} + \frac{2\alpha ReWi}{(1-\beta)}T^{xy2} = 2WiT^{xy}u'\ . \tag{26}$$

Now, expressing $(T^{xx} + T^{yy})$ from Eq. (20), we get,

$$(T^{xx} + T^{yy}) = \frac{\frac{(1-\beta)}{Re}u' + WiT^{yy}u' - T^{xy}}{\frac{\alpha ReWi}{(1-\beta)}T^{xy}}\ . \tag{27}$$

Moreover, Eq. (27) implies

$$T^{xx} = \frac{(1-\beta)[(1-\beta) + ReWiT^{yy}]u'}{\alpha Re^2 WiT^{xy}} - \frac{[(1-\beta) + \alpha ReWiT^{yy}]}{\alpha ReWi}\ . \tag{28}$$

Equations (27) and (28) are well established for all $y \neq 0$. By taking T^{xx}, T^{yy}, T^{xy} and u' are well known at $y = 0$ into account, that restriction will not be a task in this paper.

Substituting Eq. (27) and Eq. (28) into Eq. (26) and after several algebraic manipulations use Eq. (21) to simplify the calculations in

$$u' = \frac{\left[\frac{(1-\beta)}{Re} + bWiT^{yy}\right]T^{xy}}{\left[\frac{(1-\beta)}{Re} + WiT^{yy}\right]^2}\ , \quad \text{where} \quad b = 2\alpha - 1\ . \tag{29}$$

Equation (29) shows that u' depends on T^{yy} and T^{xy}. As Eq. (24) displays the component T^{yy} depending on the component T^{xy}, then let substitute Eq. (24) into Eq. (29), and after some algebraic manipulations again, we get a solution for u',

$$u' = \frac{2\alpha ReT^{xy}\left[(1-\beta) + b\sqrt{(1-\beta)^2 - 4\alpha^2 Re^2 Wi^2 T^{xy2}}\right]}{\left[b(1-\beta) + \sqrt{(1-\beta)^2 - 4\alpha^2 Re^2 Wi^2 T^{xy2}}\right]^2}\ . \tag{30}$$

It is worth some comments about the sign of the radical presents in Eq. (30). The solution of Gieskus model needs to satisfy all its equations, in particular Eq. (21), which was rewriting as Eq. (22). Equation (22) means that

$$\left(T^{yy}+\frac{(1-\beta)}{2\alpha ReWi}\right)^2 \leq \frac{(1-\beta)^2}{4\alpha^2 Re^2 Wi^2}\ , \tag{31}$$

and also

$$T^{xy2} \leq \frac{(1-\beta)^2}{4\alpha^2 Re^2 Wi^2}\ . \tag{32}$$

The restriction given by Eq. (32) leads to $(1-\beta)^2 - 4\alpha^2 Re^2 Wi^2 T^{xy2} \geq 0$, i.e., the radical presents in Eq. (30) is always non-negative since Eq. (21) is considered. Therefore, this restriction must be respected in this paper.

Integrating Eq. (17) with respect y and using that $T^{xy} = u' = 0$ at the centreline $y = 0$, we get

$$T^{xy} = \frac{-\beta}{Re}u' + p_x y\ , \quad -1 \leq y \leq 1, \quad p_x \text{ is a negative constant.} \tag{33}$$

Substituting Eq. (33) into Eq. (30) we obtain an implicit expression for u',

$$u' = \frac{2\alpha Re\left(\frac{-\beta}{Re}u' + p_x y\right)\left[(1-\beta) + b\sqrt{(1-\beta)^2 - 4\alpha^2 Re^2 Wi^2\left(\frac{-\beta}{Re}u' + p_x y\right)^2}\right]}{\left[b(1-\beta) + \sqrt{(1-\beta)^2 - 4\alpha^2 Re^2 Wi^2\left(\frac{-\beta}{Re}u' + p_x y\right)^2}\right]^2}\ . \tag{34}$$

In order, to obtain the analytical solution it is necessary to follow the next steps:
1. Solve Eq. (34) to obtain u' for a p_x given properly.
2. Solve Eq. (33) to obtain $T^{xy}(y)$;
3. Solve Eq. (24) to obtain $T^{yy}(y)$;
4. Solve Eq. (28) to obtain $T^{xx}(y)$.

Again, we point out that Eqs. (34), (33), (24) and (28) are similar to Eqs. (5.5), (5.2), (5.3) and (5.4) from Schleiniger and Weinacht (1991), respectively, unless the dimensionless parameters.

Since the sequence 2., 3. and 4. is followed, it is possible to see that all the components of the tensor can be obtained explicitly, just using some algebraic calculations. However, in the step 1. is not easy to solve u' analytically. Therefore, in the next section it will be discussed the assumptions and the numerical strategies adopted to choose p_x properly, to calculate u' and to calcule the component of the velocity u for complementing this present section and also for providing an applicability of the Mathematical work of Schleiniger and Weinacht (1991) to the Engineering area.

3.1 A semi-analytical solution

In this section, the biggest task will be to deal with the step 1. appointed in the section 3.

The authors decided to calculate a p_x that promotes a desired average velocity U, as example $U = 2/3\ m/s$. By taking the definition of average velocity, integrating by parts and using the no-slip condition at the wall $u(1) = 0$, we have

$$U = \int_0^1 u(u)\ dy = -\int_0^1 y\frac{du(y)}{dy}\ dy \quad \Longrightarrow \quad \int_0^1 yu'\ dy = -U\ . \tag{35}$$

In order to adjust p_x, it is set-up two phases: I. and II. described as follow.

I. Find out the range $[a, b]$ where p_x belongs applying the Intermediate Value Theorem:

I.1 Enter an initial guess $p_x^{(n)}$ into Eq. (34);

I.2 Give the initial guesses $u'^{(n-1)}$ and $u'^{(n-2)}$ to obtain $u'^{(n)}$ by applying Secant's Method in I.1;

I.3 Verify if the $u'^{(n)}$ satisfies Eq. (35) by using 1/3 Simpson's Method;

I.3.1 If $\int_0^1 yu'^{(n)}\ dy = -U$, then the $p_x^{(n)} = p_x$ and $u'^{(n)} = u'$ is the required. Thus, step 1. is done.

I.3.2 If $\int_0^1 yu'^{(n)}\ dy < -U$, then keep the value of $p_x^{(n)} := a$, increment it to get a new $p_x^{(n)}$ and repeat I.1-I.3. untill $\int_0^1 yu'^{(n)}\ dy > -U$. Thus, keep the newest incremented value $p_x^{(n)}$ as b. The existence of a and b are assured by Intermediate Value Theorem.

II. As $p_x \in [a,b]$, apply Bissection's Method on the interval $[a,b]$ to reach it. In each iteration of the Bissection's Method, repeat I.1-I.3 until a tolerance prescribed by the user for 1/3 Simpson's Method to solve $\int_0^1 yu'^{(n)}\, dy$ is reached. After then, the p_x and u' for step 1. is obtained numerically.
III. Calculate numerically, again, the component of the velocity $u(y)$ by Euler's Method as an initial value problem assuming $u(y_0) = 0$, thus $u(y_j) = u(y_{j-1}) + \frac{du(y_{j-1})}{dy}$, $j = 1, \ldots, M$, where $y_j = \frac{j}{M}$, M a positive integer.

4. NUMERICAL METHOD

The system of equations (10)–(15) is solved numerically in the domain as shown in Fig. 1. The calculations are done on an orthogonal uniform grid, parallel to the wall. The fluid enters the computational domain at $x = x_0$ and exits at the outflow boundary $x = x_{max}$.

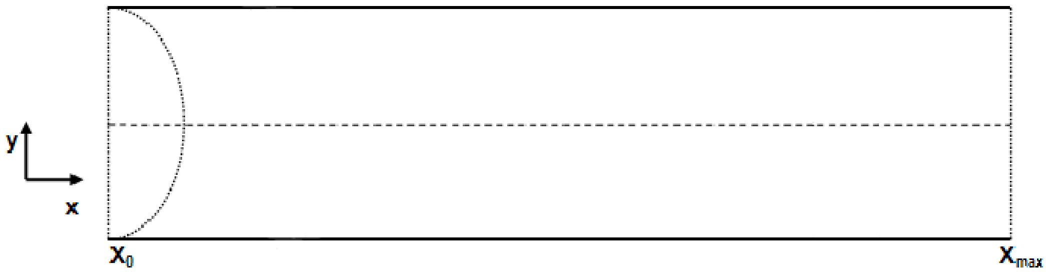


Figure 1. Computational domain.

Thus, the computational domain must be large enough for the flow to reach its fully developed state, where there is no more influence of the Newtonian flow profile imposed at the inflow, prevailing only the characteristics of the Giesekus model.

It is known that, unlike Newtonian flows, there is no hydrodynamic development relationship that guarantees the satisfactory domain size to be adopted that is related to non-Newtonian flow parameters such as Re, Wi, etc. Next section show the results obtained by numerical simulation and compare the values obtained with the semi-analytical solution.

5. NUMERICAL RESULTS

In order to study the influence and the minimal distance necessary to achieve the fully developed solution, different numerical simulations were performed by varying the parameters α and β from Giesekus model and the Reynolds and Weissenberg numbers.

The simulations were carried out using the same mesh with $imax = 6825$ and $jmax = 249$ points in x- and y-direction, respectively. Moreover, it were adopted $dx = \pi/8$ and $dy = 2/(jmax-1)$ as distance between two consecutive points in the x-direction and y-direction, respectively.

The metric adopted to measure the difference between the numerical and semi-analytical solution is:

$$E = \sum_{j=1}^{jmax} |u_n(y_j) - u_{an}(y_j)|^2, \tag{36}$$

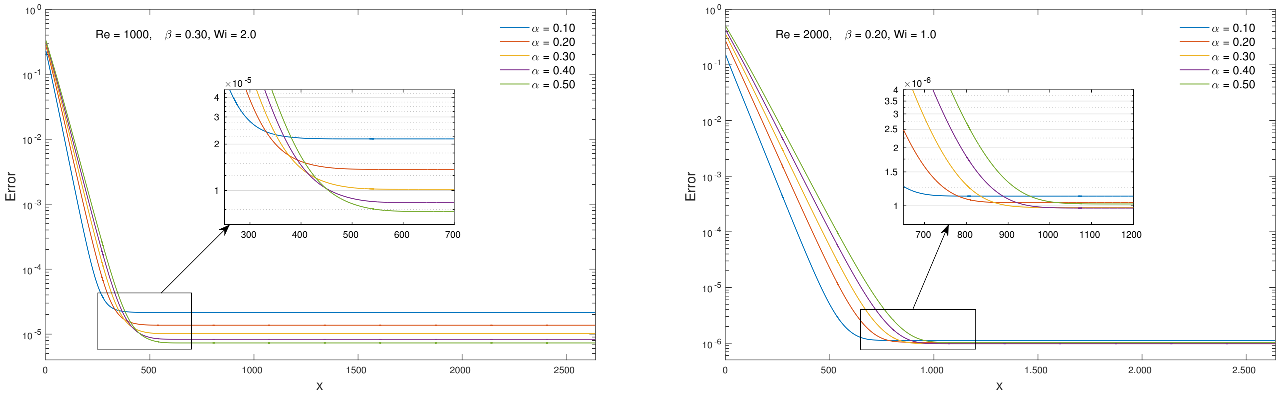
where u_n represents the numerical velocity obtained at the point y_j and u_{an} the respective semi-analytical solution.

Figures 2-5 show the results obtained. In all the cases, it can be seen that the difference, calculated using Eq. (36), decays in the streamwise direction to a level where it remains constant. This position, known as inlet length, is taken as the minimal distance to achieve a fully developed solution. For each case studied, two combinations of the remaining parameters were made with the objective to show that the influence of the parameter analyzed remains.

The first analysis shows the influence of α parameter. Two set of parameter were adopted $Re = 1000, 2000$, $\beta = 0.2, 0.3$ and $Wi = 1.0, 2.0$ with $\alpha = 0.1, 0.2, \cdots, 0.5$. Figure 2 shows that the parameter α increases the minimum distance required to obtain the fully developed solution. The opposite occurs for beta parameter, as can be seen in Fig. 3. For this parameter the inlet length increases as beta decreases. This decrease in distance caused by the parameter β can be explained by the fact that the increase of the parameter β causes the flow to become closer to a Newtonian fluid, that corresponds to the inflow imposed condition. For the beta parameter results two set of parameters were adopted with $Re = 1000, 4000$, $\alpha = 0.25, 0.6$, $Wi = 1.0, 3.0$, and $\beta = 0.1, 0.3, 0.5, 0.7, 0.9$.

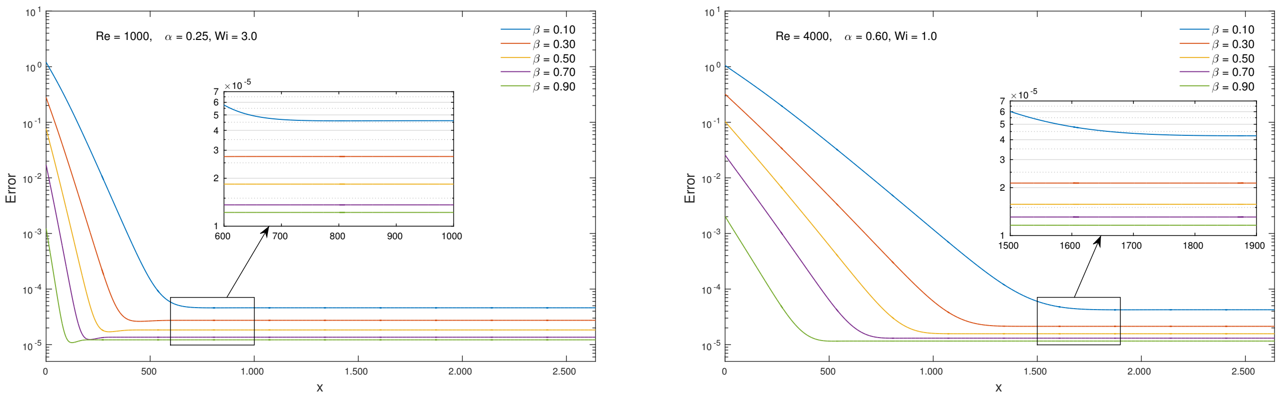
In the analysis of the influence of Reynolds number over the inlet length, shown in Fig. 4, the simulations was made using $\alpha = 0.25, 0.5$, $\beta = 0.25$, $Wi = 1.0$ with Re from 1000 to 7000 in steps of 1000. It can be observed that Re increase causes the flow to take longer to arrive at the fully developed solution so that for the simulations with $Re = 7000$, the size of the domain used was not sufficient to achieve this solution, as can be seen by the fact of the error do not become constant in the last points of the domain. This behavior can be explained by the fact of the increase in Reynolds number make the viscosity effect increasingly negligible. In this case, a larger domain is necessary to achieve the fully developed solution.

Figure 5 shows the results obtained from the Weissenberg number study, where it can be seen that its increase makes a decrease in the minimal distance to achieve the fully developed solution. In these simulations it were used as parameters $Re = 1000, 2000$, $\alpha = 0.1, 0.25$, $\beta = 0.25, 0.3$ with $Wi = 1.0, 3.0, 5.0, 7.0, 9.0, 10.0$.



(a) (b)

Figure 2. Comparison of the minimal distance to achieve the fully developed solution for different values of α.



(a) (b)

Figure 3. Comparison of the minimal distance to achieve the fully developed solution for different values of β.

6. CONCLUSIONS

The present work shows the development of a semi-analytical solution to the Giesekus constitutive equation of a non-Newtonian fluid flow in a straight channel. The results were used to study a relation between the non-Newtonian flow and the inlet length necessary to achieve the fully developed numerical solution. As results, the influence of the parameters α and β of the Giesekus model and the Reynolds and Weissenberg numbers on the minimal distance were analyzed.

The results show that the parameter α and the Reynolds and Weissenberg numbers increase the distance required to achieve the fully developed solution, while the parameter β decreases this distance, what can be explained by the fact that β closer to 1 causes the flow to become closer to a Newtonian fluid flow, which corresponds to the inflow imposed condition.

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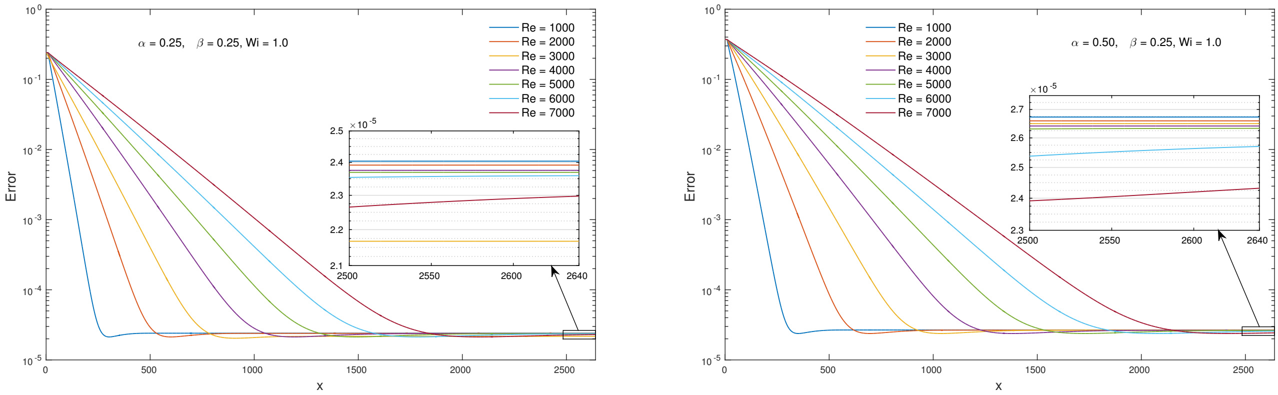
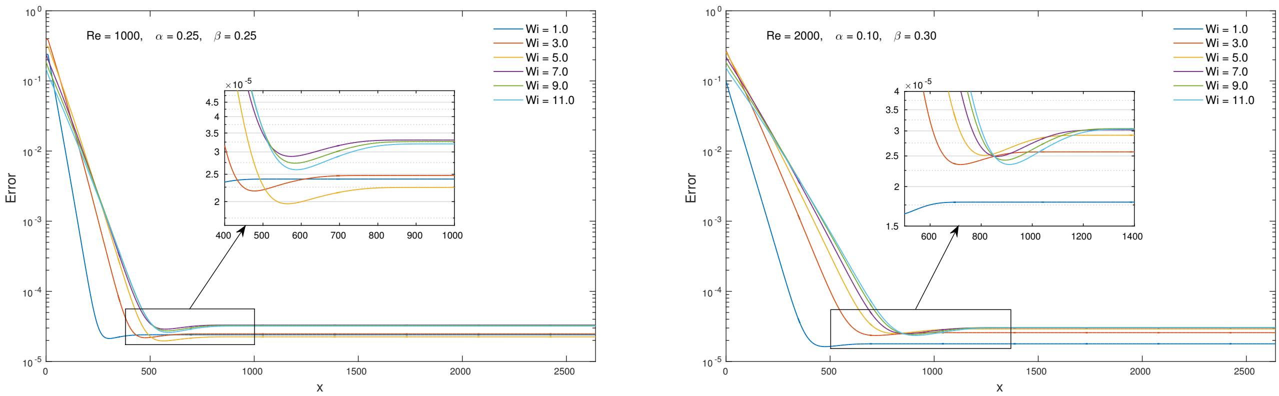


(a) (b)

Figure 4. Comparison of the minimal distance to achieve the developed solution for different values of Re.



(a) (b)

Figure 5. Comparison of the minimal distance to achieve the fully developed solution for different values of Wi.

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