DINAME 2019 - Proceedings of the XV International Symposium on Dynamic Problems of Mechanics,
M.A. Savi, T.G. Ritto and W.M. Bessa (Editors), ABCM, Buzios, RJ, Brazil, March 10th to 15th, 2019

Calibrated Wake Oscillator Model of Horizontal Flexible Structure Moving in Uniform Flow

Victoria Kurushina [1,2], Ekaterina Pavlovskaia [1], and Marian Wiercigroch [1]

[1] Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, Kings College, Aberdeen, Scotland, United Kingdom, AB24 3FX

[2] Department of Transport of Hydrocarbon Resources, Institute of Transport, Industrial University of Tyumen, 38 Volodarskogo Street, Tyumen, Russia, 625040

Abstract: In this work, vortex-induced vibrations (or VIV) of a horizontal flexible structure capable of moving in cross-flow and in-line directions are considered using wake oscillator approach. The ongoing study aims to investigate different types of nonlinear damping in fluid oscillators and their role in accuracy of VIV prediction. Van der Pol and Rayleigh damping types were investigated in this project. The Galerkin-type sinusoidal discretization of the flexible beam equations was applied and the obtained system of ordinary differential equation was solved numerically. The proposed models were calibrated with the published experimental data by Sanaati and Kato (2012) and the prediction results were compared. It was observed that the Van der Pol damping was the most suitable to predict the vibrations of the considered horizontal structure.

Keywords: vortex-induced vibration (VIV), flexible structure, uniform flow, wake oscillator, multi-mode approach

INTRODUCTION

Offshore production systems and civil structures are exposed to various loadings which may result in substantial vibrations. In some cases vibrations of slender structures originate from the interaction with the surrounding flow of water or air. These vortex-induced vibrations cause flexible risers and cables to oscillate around initial position due to variations of the active fluid forces and often lead to significant reduction of the operational life of the structure.

This study aims to improve the accuracy of VIV predictions which are obtained employing the wake oscillator method. This approach implies modelling of fluctuations of the drag and lift forces using self-excited limit cycle oscillators. Equations of fluid forces are coupled with the structural equations for each direction. This method has the potential to achieve both high prediction accuracy and suitable computational time.

In the current research, the authors propose a novel wake oscillator model of flexible structure VIV considering various combinations of Van der Pol and Rayleigh damping terms in fluid equations and then investigate the dynamic responses of the most accurate model variations. The previous studies were focused on the improvement of prediction accuracy of the rigid structure models (Postnikov et al., 2017, Kurushina and Pavlovskaia, 2018, Kurushina et al., 2018) and on the VIV of flexible structures oscillating in cross-flow direction only (Pavlovskaia et al., 2016). Cases of vertical and horizontal structures interacting with the uniform flow are considered first before investigating prediction accuracy for more complex structures and flows.

MATHEMATICAL MODEL

In this study a horizontal flexible beam with circular cross-section and pinned-pinned ends is considered when subjected to the uniformed flow with the constant velocity U along the length of structure. Tension T_0 is applied to one of the ends of the structure, as shown in Fig. 1, and oscillations in cross-flow and in-line directions are investigated.

The governing partial differential equations for the horizontal flexible beam with purely linear characteristics are simplified using the Galerkin-type sinusoidal discretization and multi-mode approximations are derived and studied. Here, following the approach employing nonlinear oscillator equations of the Van der Pol type, the fluctuating lift and drag coefficients are modeled using two wake oscillators $q(\zeta,\tau)$ and $w(\zeta,\tau)$ variables. The obtained equations in non-dimensional form are:

$$\ddot{X}_i + 2a\Omega_R\dot{X}_i + \omega_{ni}^2 X_i = \frac{a\Omega_R^2}{i\pi^2 St}(1-\cos(i\pi)) + \frac{b\Omega_R^2}{4\pi St}w_i - b\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N} w_n\dot{X}_m\Pi_{nmi} + \frac{c\Omega_R}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} q_n\dot{Y}_m\Pi_{nmi} + \\ +2\pi Sta\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{X}_n\dot{X}_m\Pi_{nmi} + \pi Sta\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{Y}_n\dot{Y}_m\Pi_{nmi}; \tag{1}$$

$$\ddot{Y}_i + 2a\Omega_R\dot{Y}_i + \omega_{ni}^2 Y_i = -\frac{b}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} w_n\dot{Y}_m\Pi_{nmi} + \frac{c}{4\pi St}\Omega_R^2 q_i - c\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N} q_n\dot{X}_m\Pi_{nmi} + 2\pi Sta\sum_{n=1}^{N}\sum_{m=1}^{N}\dot{X}_n\dot{Y}_m\Pi_{nmi};$$

$$\ddot{w}_i + 2\varepsilon_x\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{w}_n w_m w_l \Psi_{nmli}) - 2\varepsilon_x\Omega_R\dot{w}_i + 4\Omega_R^2 w_i = A_x\ddot{X}_i;$$

$$\ddot{q}_i + \varepsilon_y\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{q}_n q_m q_l \Psi_{nmli}) - \varepsilon_y\Omega_R\dot{q}_i + \Omega_R^2 q_i = A_y\ddot{Y}_i,$$

where X_i, Y_i are in-line and cross-flow modal coefficients of the i-th mode respectively; w_i, q_i are wake coefficients, representing fluctuations of the fluid forces; ε_x, ε_y are dimensionless damping coefficients in the fluid equations; a, b, c are dimensionless coefficients depending on the initial drag, fluctuating drag and lift coefficients respectively; *St* is the Strouhal number (assumed 0.2 in this study); m_* is mass per unit length; L is structural length; Ω_R is dimensionless vortex shedding frequency; ω_{ni} is the natural frequency of the i-th mode; Π_{nmi}, Ψ_{nmli} are dimensionless constants depending on the number of considered modes; N is the number of considered modes. The final displacements are obtained from the time-dependent modal coefficients for a cross-section $\zeta = \frac{\bar{Z}}{L}$, where $\bar{Z}$ is the axial coordinate along the beam, as $X(\zeta, \tau) = \sum_{i=1}^{N} X_i\tilde{X}_i$ and $Y(\zeta, \tau) = \sum_{i=1}^{N} Y_i\tilde{Y}_i$, where $\tilde{X}_i$ and $\tilde{Y}_i$ are assumed as sinusoidal functions dependent on ζ.

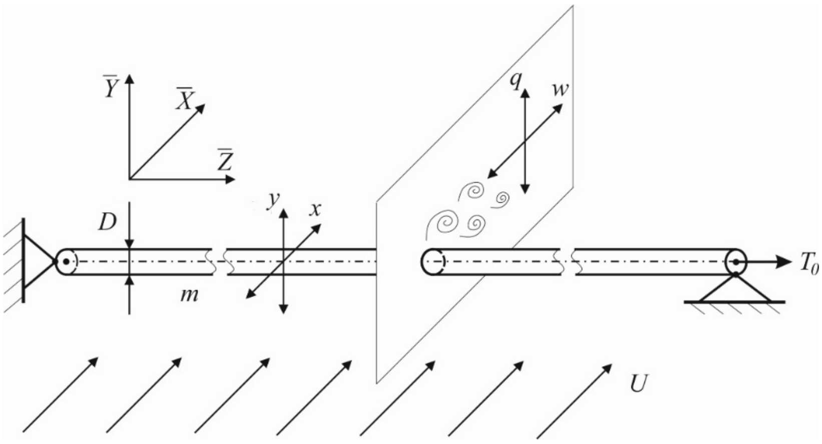

Figure 1 – Horizontal flexible structure in uniform flow

In this work the authors consider various nonlinear damping terms presented in Table 1 following previously proposed models by Kurushina and Pavlovskaia (2017) for the rigid structure VIV. Overall number of the model variations, considered in this study, is 8, including the original option with Van der Pol – Van der Pol damping types.

Table 1 – Alternative wake oscillator equations

Equation type	In-line oscillator	Cross-flow oscillator
Modified Van der Pol	$\ddot{w}_i + 2\varepsilon_{x1}\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{w}_n w_m w_l\Psi_{nmli}) - 2\varepsilon_{x2}\Omega_R\dot{w}_i + +4\Omega_R^2 w_i = A_x\ddot{X}_i$	$\ddot{q}_i + \varepsilon_{y1}\Omega_R\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{q}_n q_m q_l\Psi_{nmli}) - \varepsilon_{y2}\Omega_R\dot{q}_i + +\Omega_R^2 q_i = A_y\ddot{Y}_i$
Rayleigh	$\ddot{w}_i + 2\frac{\varepsilon_x}{\Omega_R}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{w}_n\dot{w}_m\dot{w}_l\Psi_{nmli}) - 2\varepsilon_x\Omega_R\dot{w}_i + +4\Omega_R^2 w_i = A_x\ddot{X}_i$	$\ddot{q}_i + \frac{\varepsilon_y}{\Omega_R}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{q}_n\dot{q}_m\dot{q}_l\Psi_{nmli}) - \varepsilon_y\Omega_R\dot{q}_i + +\Omega_R^2 q_i = A_y\ddot{Y}_i$
Modified Rayleigh	$\ddot{w}_i + 2\frac{\varepsilon_{x1}}{\Omega_R}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{q}_n\dot{q}_m\dot{q}_l\Psi_{nmli}) - 2\varepsilon_{x2}\Omega_R\dot{q}_i + +4\Omega_R^2 w_i = A_x\ddot{X}_i$	$\ddot{q}_i + \frac{\varepsilon_{y1}}{\Omega_R}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=1}^{N}(\dot{q}_n\dot{q}_m\dot{q}_l\Psi_{nmli}) - \varepsilon_{y2}\Omega_R\dot{q}_i + +\Omega_R^2 q_i = A_y\ddot{Y}_i$

CALIBRATION RESULTS

Calibration of the empirical coefficients is conducted using the experimental data by Sanaati and Kato (2012). Accuracy of prediction is assessed using the system of priorities previously applied for the rigid structure models (Kurushina et al., 2018), where the main focus is on the highest displacement amplitude in the target experimental record.

The calibration procedure and selection of the options based on the formulated objective function resulted in 3 suitable versions with the appropriate coefficients, as detailed in Table 2. These model versions are listed in order from the smallest value of the obtained objective function to the highest one, and the accuracy of the fit in illustrated in Fig. 2. In the work by Sanaati and Kato (2012), all measurements are performed for the cross-section in the middle of the structure (or ζ = 0.50), and the displacement is recorded as the standard deviation. This is why, the statistics of displacement signal generated by the considered model versions is also standard deviation in this research, and the data displayed in figures are for the middle cross-section only.

Fig. 2b shows that Option 1 with Van der Pol – Van der Pol damping provides the most accurate prediction of the initial and super-upper branches of the target cross-flow displacement observed in the experiment (Sanaati and Kato, 2012). Fig. 2a illustrates the differences in the generated in-line responses, where Option 2 with Rayleigh – Rayleigh damping implies a high first peak of the in-line displacement amplitudes around $U_R = 3.45$.

The generated modal coefficients for all three model versions in comparison are given in Fig. 3 for $U_R = 3.45$. Option 2 is distinct from the two other model variations in the signal of in-line displacement of the highest amplitude, which corresponds to the mentioned first peak in the in-line direction, and also this signal is the least modulated among all the options. On the contrary, the signal of the cross-flow displacement of Option 2 is the only modulated among three options. The authors consider these differences to result from applying Rayleigh damping in the lift oscillator.

Table 2 – The most accurate model versions with the sets of calibrated coefficients

Option	Fluid nonlinearities	C_{L0}	C_{D0}	C_{D0}^{fl}	ε_x	ε_y	A_x	A_y	C_A	K
1	Van der Pol – Van der Pol	0.74	1.59	0.01	0.8154	1.0116	11.96	3.94	2.77	1.35
2	Rayleigh – Rayleigh	0.72	2.60	1.55	0.7041	0.0825	12.37	8.43	2.15	0.88
3	Modified Rayleigh – Modified Van der Pol	0.29	1.74	0.19	0.3498, 0.3028	0.0478, 0.0645	11.98	11.90	1.17	0.47

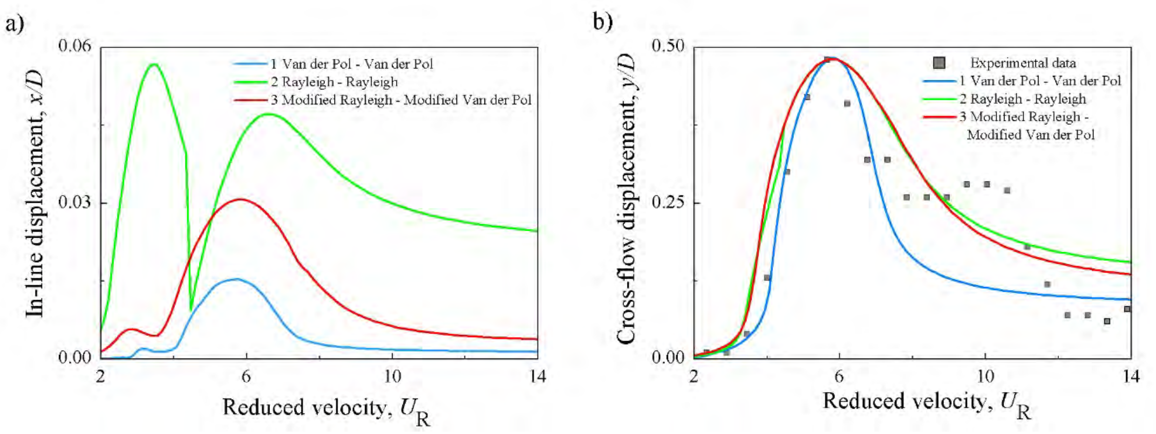

Figure 2 – Calibration results: a) obtained in-line displacement amplitudes; b) cross-flow displacement amplitudes calibrated with the experimental data by Sanaati and Kato (2012).

CONCLUSIONS

This research is focused on the development of a wake oscillator model with an appropriate damping type in the fluid equations in order to accurately predict VIV of a flexible structure. The model has been modified with 3 alternative damping terms in the equations for the lift and drag coefficients, which resulted in 8 alternative model versions to compare.

As a result of this study, the model option with Van der Pol – Van der Pol damping types, as shown in Table 2 and Fig. 2, with the appropriate set of coefficients is able to provide the most accurate prediction. The model version with Rayleigh – Rayleigh damping generates a distinct in-line displacement signal below $U_R = 5.0$, which is the result of applying Rayleigh damping in the lift oscillator equation.

The future research using wake oscillator method should focus on VIV predictions of various configurations of flexible structures, different boundary conditions and flow velocity profiles.

ACKNOWLEDGMENTS

V.K. would like to acknowledge the support of the Industrial University of Tyumen, Tyumen, Russia, and the State Program "Global Education", Russia. This work has been performed using the Maxwell High Performance Computing

Cluster funded by the University of Aberdeen. Authors also would like to express their gratitude to Dr. Andrew Starkey for advices regarding optimization procedures and to Naveed Khan for technical advices on accelerating computations.

REFERENCES

Kurushina, V., Pavlovskaia, E., 2018, "Fluid nonlinearities effect on wake oscillator model performance", MATEC Web of Conferences, Vol. 148 (04002), pp. 1-6.

Kurushina, V., Pavlovskaia, E., 2017, "Wake oscillator equations in modelling vortex-induced vibrations at low mass ratios", OCEANS 2017-Aberdeen, IEEE, pp. 1-6.

Kurushina, V., Pavlovskaia, E., Postnikov, A., Wiercigroch, M., 2018, "Calibration and comparison of VIV wake oscillator models for low mass ratio structures", International Journal of Mechanical Sciences, Vol. 142, pp. 547-560.

Pavlovskaia, E., Keber, M., Postnikov, A., Reddington, K., Wiercigroch, M., 2016, "Multi-modes approach to modelling of vortex-induced vibrations", International Journal of Non-Linear Mechanics, Vol. 80, pp.40-51.

Postnikov, A., Pavlovskaia, E., Wiercigroch, M., 2017, "2DOF CFD calibrated wake oscillator to investigate vortex-induced vibrations", International Journal of Mechanical Sciences, Vol. 127, pp.176-190.

Sanaati, B., Kato, N., 2012, "A study on the effects of axial stiffness and pre-tension on VIV dynamics of a flexible cylinder in uniform cross-flow", Applied Ocean Research, Vol. 37, pp. 198-210.

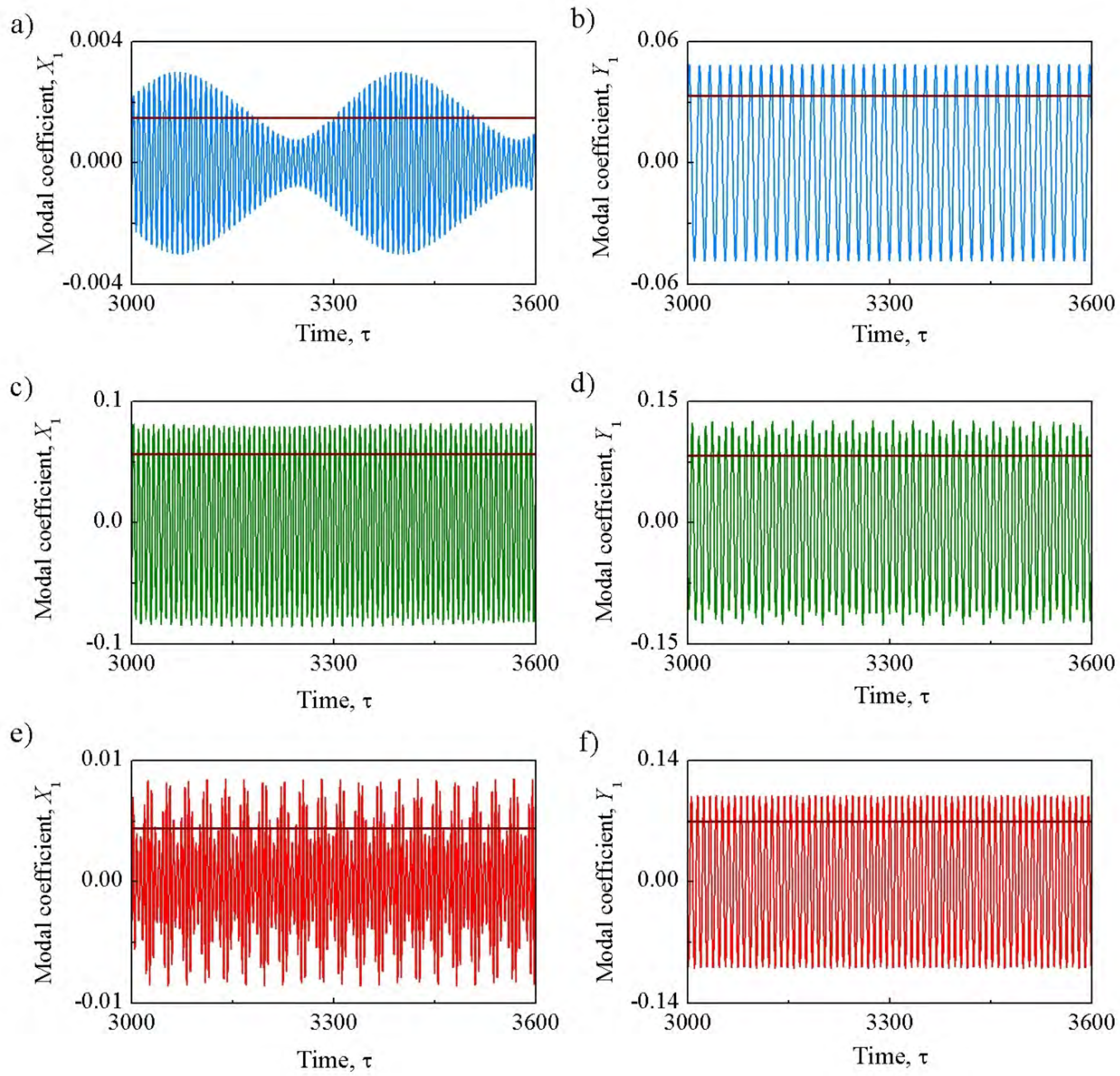

Figure 3 – Time histories of the modal coefficients generated by the model variations displayed in Fig. 2: a) in-line modal coefficient generated by Option 1; b) cross-flow modal coefficient generated by Option 1; c) in-line modal coefficient generated by Option 2; d) cross-flow modal coefficient generated by Option 2; e) in-line modal coefficient generated by Option 3; f) cross-flow modal coefficient generated by Option 3. All Options are given in the colour in accordance with Fig. 2. Brown lines show the values of standard deviations.