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Dynamic analysis and control of drill strings using state-space method

Maryam Ghandich Tehrani [1], Thiago G Ritto [2]

[1]University of Southampton, Institute of Sound and Vibration Research, UK

[2] Universidade Federal do Rio de Janeiro, Department of Mechanical Engineering, Brazil

Abstract: In this paper, active vibration control of a drill string is considered. A discretized drill string is provided where only torsional motion is considered. The dynamic equations are presented in the form of state-space, taking into account the nonlinear behaviour due to the nonlinear bit-rock interaction. The eigenvalues of the linearized system and its the stability map are obtained for different values of imposed speed at the top and weight-on-bit. Active vibration control strategy in the form of eigenvalue assignment is considered to assign the poles of the linearized model to prescribed locations to reduce the amplitude of torsional oscillation. The response of the system with and without the controller is compared and the performance of the control system in terms of stick-slip motion presented very good results.

Keywords: drill string dynamics, nonlinear bit-rock interaction, stability, stick-slip mitigation, active control

INTRODUCTION

Figure 1 sketches the system under analysis. A torque is applied at the top of a vertical slender column (few kilometers of length), which rotates, and the bit, at the bottom, drills the rock. This work is an extension of a previous work (Ritto and Ghandchi-Tehrani, 2018), and it aims at controlling torsional vibrations in drill-strings using eigenvalue assignment.

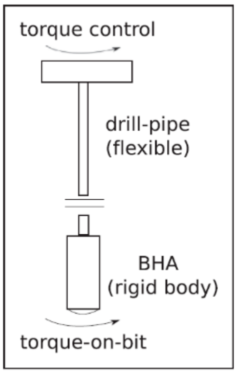


Figure1. Sketch of a drill string

DYNAMIC MODEL

Only torsional motion is considered, and a linear torsional bar model is discretized by means of the finite element method (Nogueira and Ritto, 2018). The FE model of the drill-string with n degrees of can be written in state-space form such that,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{1}$$

where, the linearized system matrix is

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{2n\times 2n} \tag{2}$$

The nonlinearity arises from the bit-rock interaction model, which is a function of the bit speed (Tucker and Wang, 1997). The derivative of this function dT_{bit} is added to the damping term of the n th degree of freedom. The bit-rock interaction is linearized around the nominal rotational speed Ω and is a function of the weight-on-bit, W_{bit}, rock properties, α_1, α_2 and friction coefficient, μ .

$$dT_{bit} = -W_{bit}\mu \frac{\alpha_1}{\alpha_2\Omega^2 + 1} - \tanh\left(\Omega^2\right) - \frac{2\alpha_1\alpha_2\Omega^2}{\left(\alpha_2\Omega^2 + 1\right)^2 + 1} \tag{3}$$

The control distribution is $\mathbf{B}$ and the control force is a state feedback control such as,

$$\mathbf{u} = -\mathbf{Kx} \tag{4}$$

The control force is obtained to stabilize the linearized system by assigning the eigenvalues of the closed-loop system to the prescribed location in the complex plane. The closed-loop system matrix $\mathbf{A} - \mathbf{BK}$ provides the eigenvalues after control.

NUMERICAL SIMULATIONS

In this first investigation a numerical simulation is carried out with only two finite elements to test if the proposed strategy of active control can stabilize the system. The data used in the simulations are the following: length of the drill pipe 4744.6 m, length of the bottom hole assembly 466.45 m, outer/inner radius 0.07/0.056 and 0.08/0.036 m, density 7800 kg/m^3, shear modulus 85.3 GPa, α_1, α_2 = 2 and 1, and μ =0.04.

The real and imaginary parts of the two eigenvalues are plotted as a function of weight-on-bit and rotational speed. The rotational speed, Ω , varies from 100 to 200rpm and W_{bit} varies from -50kN to -150kN. The imaginary part of the first two eigenvalues does not change much as can be seen in Fig., 2 and it is about 0.85 for the first and 15.6 for the second eigenvalue. However, the real part, which determines the stability of the linearized system, can vary significantly.

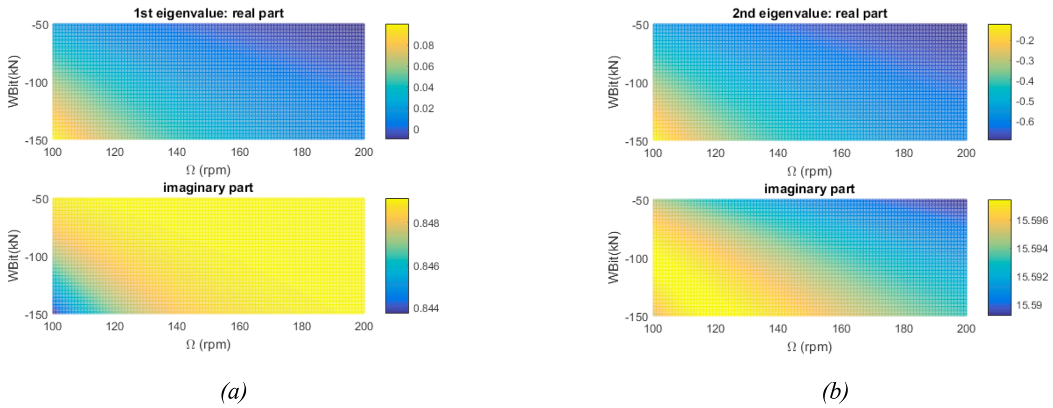
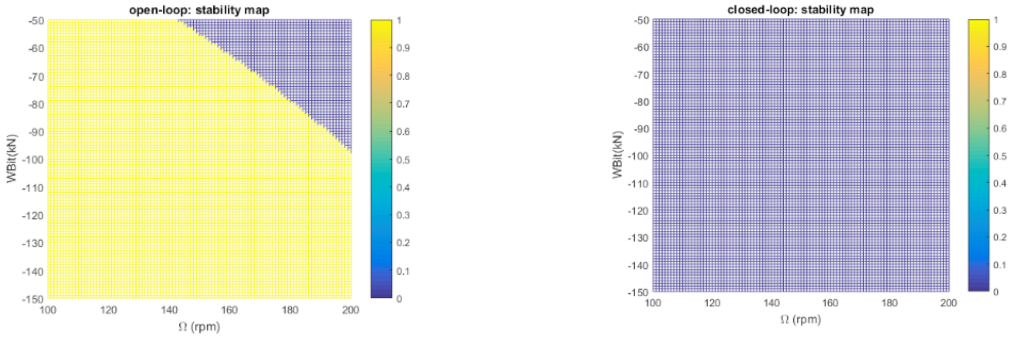


Figure 2. Eigenvalues of the linearized open-loop system (a) first eigenvalue, (b) second eigenvalue

The stability map is provided for the system before applying the control strategy as shown in Figure 3(a). For rotational speeds below 150rpm the system is unstable, independently of the values of W_{bit}. The yellow region, which is attributed the value one shows the instability due to torsional phenomena (stick-slip oscillations in the severe case), while the blue region, with values zero, is the stable area. Most the analyzed range are dominated by the unstable region (yellow color). Therefore, the aim of active control is to increase the stability region by assigning the eigenvalues to the prescribed locations. In this example, we assign the two eigenvalues to $-0.1 \pm 1i$ and $-0.3 \pm 15i$ using $\mathbf{B}^T = [1 \quad 0 \quad 1 \quad 0]$. The strategy works and closed-loop system becomes stable, as shown in Figure 3(b).



(a) *(b)*

Figure 3. Stability map for the (a) open-loop, (b) closed-loop linearised system

The control gains $\mathbf{K} = [K_1 \quad K_2 \quad K_3 \quad K_4]$ are obtained at every rotational speed and weight-on-bit, and they are shown in Fig. 4.

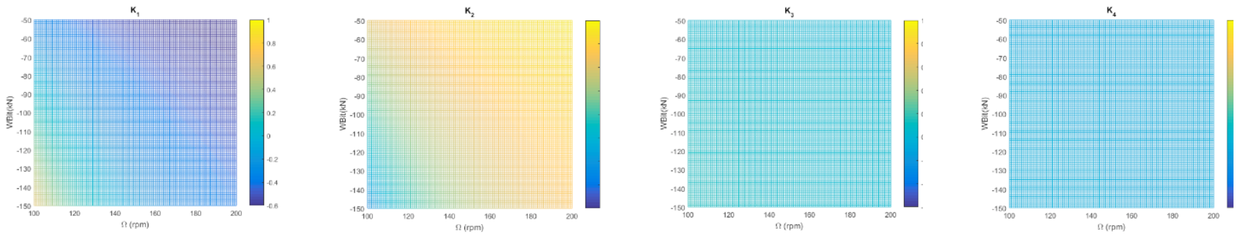


Figure 4. The feedback gain $\mathbf{K} = [K_1 \quad K_2 \quad K_3 \quad K_4]$

CONCLUSION

This paper presented the use of eigenvalue assignment for torsional vibration mitigation in drill strings. For the proposed state feedback strategy, the stability region of the linearized system is increased. The method should then be applied to systems with higher degrees of freedom. Time domain simulation of the controller will be considered for future work.

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