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ESTIMATION OF THERMAL PROPERTIES USING THE SOBOL SEQUENCE AND MERSENNE TWISTER WITH THE TOPOGRAPHICAL GLOBAL OPTIMIZATION

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Abstract. *When one wishes to estimate parameters of a model from available experimental data, a traditional approach is the procedure of maximum likelihood, which results in an objective function to be minimized. A robust global optimization method must be employed to prevent stagnation in local minima. In this work, the technique known as Topographical Global Optimization (TGO) will be used as an optimization strategy. Fundamentally, TGO distributes random points in a search space and, through the topographic information of the objective function, selects considered topographical minimum points. These minima are then used as the initial solution for a local search method. The objective of the present work is to present investigations of how a random point generator can influence the final result of the method, to do so, the Wilcoxon Signed-Rank Test is performed. The Sobol Sequence was tested, a quasi-random sequence with low discrepancy, the pseudo-random Mersenne Twister, a generator that tends more to real randomness and the built-in routine RandomReal of Mathematica to represent a software package solution. The results obtained reinforce the efficiency of the TGO and, in addition, show the potential of the Sobol Sequence as initial point sampler.*

Keywords: *Topographical Global Optimization, Sobol Sequence, Mersenne Twister, Inverse Problem, Thermal Properties*

1. INTRODUCTION

The formulation and solution of an inverse heat conduction problem using thermography have many applications in science and engineering, such as the detection of tumors (Mital et al, 2007), the characterization of the heat flux in grinding processes (Brosse, 2008), the design of conjugated heat transfer in nanocomposite heat spreaders with multiple microchannels (Knupp et al, 2014), CPU and microelectronics cooling (Silvéro et al, 2016) and others.

When some parameters are unknown but experimental measurements of the temperature profile are available, it is possible to determine such unknowns by minimizing a residue function containing the squared difference of the calculated and measured temperatures (Silva Neto and Moura Neto, 2005). In other words, this inverse problem is modelled as an optimization problem and, to be effectively solved, it requires a method that can overcome local minimas. In this work, the technique known as Topographical Global Optimization, introduced by Törn and Viitanen (1992), used recently on the problem of a nuclear core reactor design (Sacco et al., 2014) and chemical equilibrium (Henderson et al, 2015). This optimization method extracts points considered as topographical optima of a set of randomly distributed points in the search space to use them as initial solution for a local optimization method.

The main objective of this work is to investigate the outcome of using three different random points generator on the first step of TGO: the pseudo-random Mersenne Twister (Matsumoto and Nishimura, 1998), the quasi-random Sobol Sequence (1967) and the built-in routine *RandomReal* of the *Wolfram Mathematica 11.0* software. The influence on the final quality of these generators will be measured by the number of evaluations of the objective function each one have to make until obtain a certain objective function value. Recent literature presents this comparison in several kinds of problems and methods, Sacco et. al (2014) have investigated the use of different population initialization schemes on the Differential Evolution Algorithm, Dalal et al. (2016) solved the inductance problem using Monte-Carlo and Quasi-Monte Carlo approximations, Zhang et al. (2016) compared both point generators – and others – by testing them on several compact genetic algorithms, just to cite a few. To perform this comparison the Wilcoxon Signed-Rank Test (Wilcoxon, 1945) will provide a feasible conclusion on whether there is a difference between the results or not.

The chosen method for the local search of TGO is the Nelder-Mead (Nelder and Mead, 1965) algorithm. Although it's a relatively old method, its characteristic of slightly overcoming local mininas is great for the investigation of this work. The TGO algorithm, of simple implementation, is not very well known by the scientific community. This work consists of the first investigations of the random points generators influence on the quality on the final results.

2. METHODOLOGY

Consider a system with a sandwich-like configuration, plate-resistance-plate, as presented in Fig. 1 (Knupp, 2012), with lateral dimensions of $L_x \times L_y$ mm and thickness of L_z. A thin electrical resistance is employed in the middle of the superior half of the sandwich system, transferring the generated heat to the surface of the plates.

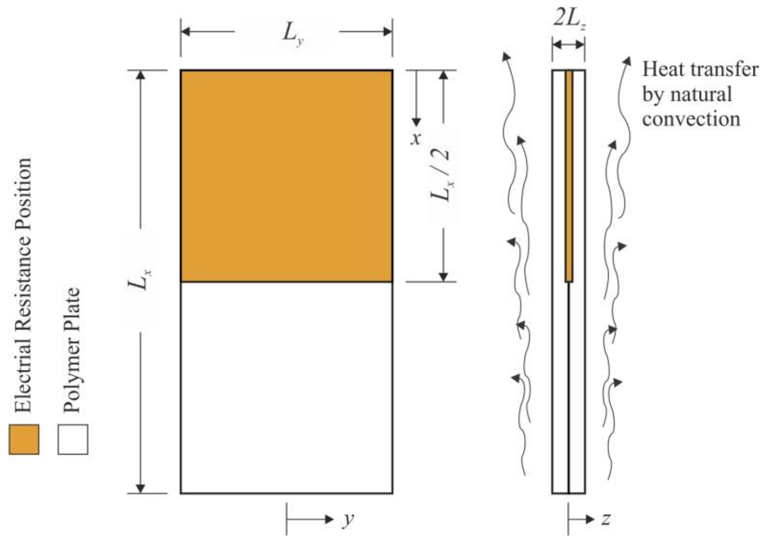


Figure 1. Schematic representation of the plate-resistance-plate system

This problem can be directly solved using the heat diffusion equation and, using the lumped analysis technique, it can be simplified to one spatial variable. When all the variables, parameters and conditions are known, the result obtained is the temperature as a function of space and time: this is the direct problem.

2.1 Formulation and solution of the Direct Problem

The hypothesis considered is that the power generated by the heater is equally divided between the plates, that is, the plane $z = 0$ defines a plane of symmetry, making it possible to analyze the problem in only one of the sandwich plates. In addition, the temperature gradient in the y direction will be neglected, this is reasonable since the heater is sufficiently uniform. We must also consider as constants – both spatially and temporally – all properties, thermal or not. Equqation (1) describes the thermal diffusion on the plate in general (Ozisik, 1993)

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho c_p \frac{\partial T}{\partial t} \quad , \tag{1a}$$

$$-k\left.\frac{\partial T}{\partial z}\right|_{z=0}=q_{aq}(x), \quad -k\left.\frac{\partial T}{\partial z}\right|_{z=L_z}=h_{ef}(x)\left[T(x,z,t)-T_\infty\right], \tag{1b,c}$$

$$k\left.\frac{\partial T}{\partial x}\right|_{x=0}=0\,, \quad k\left.\frac{\partial T}{\partial x}\right|_{x=L_x}=0\,, \tag{1d,e}$$

$$T(x,z,0)=T_\infty \tag{1f}$$

where T_∞ is the ambient temperature during the experiment, h_{ef} (x) is the convection coefficient in the boundary of the plate, q_{aq} (x) is the dissipated heat flux due to the electrical resistance, c_p is the specific heat and k is the thermal conductivity of the material.

In order to transform the problem described by Eq. (1) into a 1D problem, the use of the classical technique of lumped parameters will result in an approximation in which the temperature on the plate $z = L_z$ boundary is very close to the average temperature of the plate at that point x (Knupp et al. 2012). The use of this technique is extremely useful and sometimes even mandatory when a simplified formulation of transient heat conduction is needed (Su and Cotta, 2001). This approach restricts the use of the technique for problems with Biot number less than 0.1 (Regis et al., 2000). The highest number of Biot in the plate will occur at $x < L_x / 2$ and will be $Bi = 0.0975$. Therefore it is acceptable that the temperature at the edge is very close to the average temperature along the thickness of the plate.

Using this technique, the problem can be represented by only one spatial dimension, that is, the temperature distribution is now a function only of the x position and the time t and it is an average temperature, expressed as T_m. The direct problem then becomes the partial differential equation represented by Eq. (2)

$$k\frac{\partial^2 T_m(x,t)}{\partial x^2}-\frac{h_{ef}(x)\left[T_m(x,t)-T_\infty\right]}{L_z}+\frac{q''_{aq}(x)}{L_z}=\rho c_p\frac{\partial T_m(x,t)}{\partial t}, \tag{2a}$$

$$k\left.\frac{\partial T_m}{\partial x}\right|_{x=0}=0,\quad k\left.\frac{\partial T_m}{\partial x}\right|_{x=L_x}=0, \tag{2b}$$

$$T_m(x,0)=\mathrm{T}_\infty . \tag{2c}$$

The solution of the Direct Problem described by Eq. (2) is achieved by the numerical technique of Finite Differences with implicit formulation (Ozisik, 1994), in which the spatial term of the equation is evaluated at time level $m + 1$, where m represents the temporal iteration. This will entail a coupling of the equations for each spatial node i at time $m + 1$ and a system of algebraic equations must be solved as time goes on. In this work the solution of this system is achieved via the built-in routine *LinearSolve* of the software Wolfram Mathematica 11.0 (Wolfram, 2005).

2.2 Formulation the Inverse Problem

Suppose that some thermal properties of the material are unknown, but experimental measurements of the temperature profile $T_{\exp,j}$, j = 1,2, ..., N_d are available, where N_d is the number of available experimental data. The solution of the inverse problem, then, is to minimize the objective function given by the sum of the quadratic residuals between the experimentally measured temperatures and the values reached through the solution of the direct problem, that is, one must obtain the global optimum of the function

$$Q(\mathbf{u})=\sum_{j=1}^{N_d}\left[T_j(\mathbf{u})-T_{\exp,\,j}\right]^2, \tag{3}$$

where $\mathbf{u}$ is the vector of unknowns and, for the problem described they are the thermal properties k and c_p, so the vector $\mathbf{u}$ is $\mathbf{u} = \{k, c_p\}$. The experimental data used in this work will be simulated computationally through the solution of the direct problem, given by Eq. (2), and the addition of noise drawn from a normal distribution, with known variance σ^2, as described by Eq. (4).

$$T_{\exp,j}=T_j(\mathbf{u})+e_i,\qquad e_i\sim \mathrm{N}\,(0,\sigma^2) \tag{4}$$

2.3 Solution of the Inverse Problem via TGO

Introduced by Törn and Viitanen (1992), TGO uses topographic heuristics to determine minimums in a set of initial points. These minimum findings are used as the initial solution for a local optimization method (Sacco et al., 2014). It can be described in three steps:

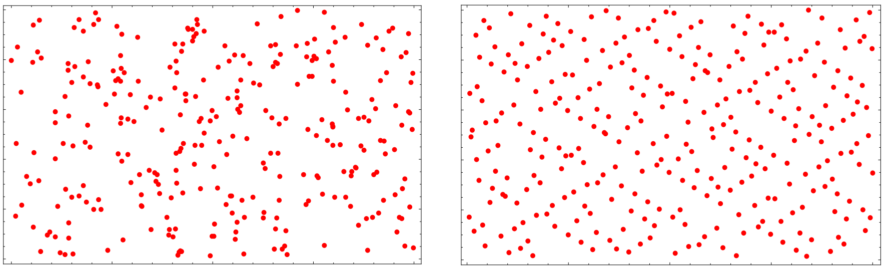
1. Sample the search space with N uniformly distributed random points.

2. Construct of the topography by analyzing the objective function value at each one of the N points with respect to their K-nearest neighbors. When a point P_u has neighbors which evaluate to higher objective function values, this point Pi is considered to be the topography minimum.
3. All the topographical minima from Step 2 are set as initial solutions for a local optimization method. The global minimum is the lowest function evaluation from all the local search executions.

The first step consists of selecting N random points distributed evenly in the search space where, for each point P, an index u, i.e. P_u, is associated with $u = 1, 2, ..., N$ (Henderson et al. al., 2015). A poor distribution of these points may negatively influence the final outcome of the method, since all areas of the search space should be equally covered (Törn and Viitanen, 1996).

In this work we use three random point generators, a pseudo-random known as Mersenne Twister, a quasi-random, also known as Sobol Sequence and the built-in routine “RandomReal”, which behaviors like a pseudo-random generator also. The numbers coming from a quasi-random generator are organized in such a way as to maximally avoid each other, tending to imitate a uniform distribution. On the other hand, a pseudo-random generator tries to mimic a random behavior (Maaranen et al., 2004).

Figure 2 presents two illustrative examples of 200 points generated in a two-dimensional space for the Mersenne Twister (a) and the Sobol Sequence (b). Observe that the Sobol Sequence generates a more uniform distribution, whereas the Mersenne Twister has a greater discrepancy.



(a) (b)

Figure 2. Example of 200 points in a two-dimensional space: (a) Mersenne Twister and (b) Sobol Sequence

For the third step of TGO, the choice for the local search method is the Nelder-Mead (Nelder and Mead, 1965). Despite being considered a local search method, NM has some characteristics that can overcome local optimums. In this work, all the parameters of the method, as well as the stopping criterion, will be fixed in order to ensure that the results are influenced, majorly, by the points selected in Step 2 of TGO. The Morozov’s Discrepancy Principle is used as stop criterion for the TGO algorithm. When the objective function value obtained is equal or small than M, then solution is considered reached, with $M = N_d\, \sigma^2$, where σ is the standard deviation of the experimental error in Eq. (4).

2.4 The Wilcoxon Signed-Rrank Test

The comparison of the points generators is based on the Wilcoxon signed-rank test (1945), which is a test for finding if there is a difference in two groups of results, observations or samples, i.e., it is a pairwise test that aims to detect significant differences between two samples (Derrac et. al, 2011).

The Wilcoxon signed-rank test is performed by taking the difference between observations and raking the absolute values obtained, from the smallest to the largest, where the smallest difference gets the rank 1, the second smallest the rank 2, and so on. Ties must receive the averaged rank. Let $T_{(+)}$ be the sum of all ranks obtained from positive differences, and $T_{(-)}$ the sum obtained from all the negative differences. The smaller number obtained from $T_{(+)}$ and $T_{(-)}$ is the test statistic T_s (McDonald, 2009). If T_s is less than or equal to the critical values of the distribution of Wilcoxon, then the null hypothesis is rejected, meaning that there is a difference between the two samples compared. A brief table of critical values for a two tailed test is displayed in Tab. (1), which is small excerpt from the one found in the work of McCornack (1965). Here α is the level of significance and n is the degrees of freedom.

The observation to compare is the number of function evaluations (NEF) necessary to reach the Morozov’s Discrepancy Principle. Since there are three random point generators to investigate, three Wilcoxon test must be performed, each one comparing two of the generators.

Table 1. Critical values for the Wilcoxon Signed-Rank Test considering three different levels of significance α. Adapted from McCornack (1965).

n	α = 5%	α = 2%	α = 1%
5	0	-	-
6	0	-	-
7	2	0	-
8	3	1	0
9	5	3	1
10	8	5	3
11	10	7	5
12	13	9	7
13	17	12	9
14	21	15	12

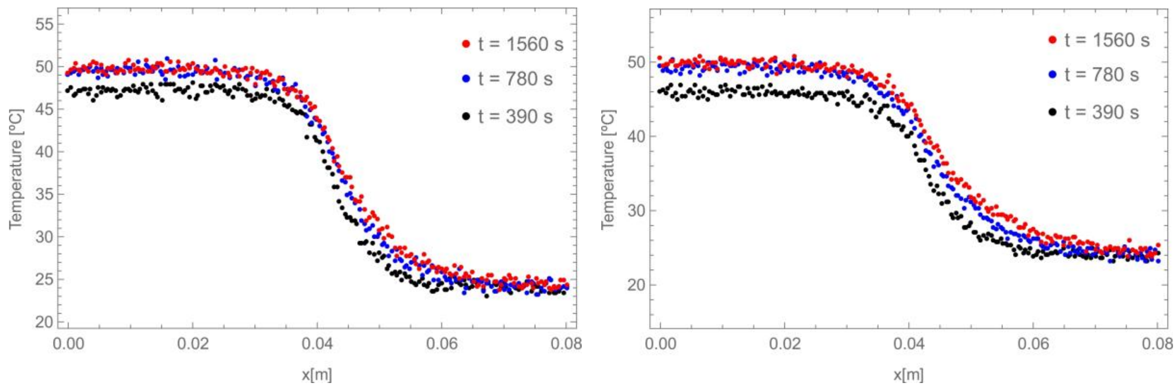
3. RESULTS AND DISCUSSION

The materials tested in the present work are the Nylon 6,12 ($Ny_{6,12}$) and the High Density Polyethilene (HDPE). The physical properties used for both materials, such as the specific mass, the thermal conductivity and the specific heat, are presented in Tab. (2), which were taken in the range of 0ºC to 49ºC (Lienhard, 2008).

Table 2. Summary of test materials with the exact values for the unknown parameters and respective search space.

	Nylon 6,12	**HDPE**	**Search Space**
ρ	1060 kg/m³	960 kg/m³	-
k	0.22 W/m K	0.33 W/m K	$0.1 < k < 10$ W/m K
c_p	1680 J/kg K	2260 J/kg K	$1000 < c_p < 5000$ J/kg K

Using σ = 0.5 in Eq. (4), the experimental data are acquired at three instants of time: t = 390 s, t = 780 s and t = 1560 s. For each of these time levels, 201 temperature data are acquired along the x-direction, resulting in N_d = 603 experimental data in total. Figure 3 presents the simulated experimental data for these three levels of time. These experimental data are temperature profiles on the exposed face of the plate at different times, such as those presented by Knupp et al. (2012), which were obtained via infrared thermography.



(a) (b)

Figure 3. Experimental data for three levels of time for (a) the $Ny_{6,12}$ and (b) the HPDE experiments, both using simulated noise of σ = 0.5 in Eq. (4).

According to empirical correlations for natural convection and linearized radiation, the coefficient of heat transfer by convection have a magnitude of 17 W/m² K for the heated portion and 4 W/m² K for the non-heated portion of the plate, as described in Eq. (5a).The dissipated heat due to the electrical resistance only acts on the upper half of the set, so the term q_{aq} (x), referring to the heat flux yielded by the resistance in the z = 0 boundary, can be written as Eq. (5b)

$$h_{ef}(x) = \begin{cases} 17 \ \text{W/m}^2\text{K}, & x \le L_x/2 \\ 4 \ \text{W/m}^2\text{K}, & x > L_x/2 \end{cases}, \quad q_{aq}(x) = \begin{cases} 440 \ \text{W/m}^2\text{K}, & x \le L_x/2 \\ 0 \ \text{W/m}^2\text{K}, & x > L_x/2 \end{cases} \tag{5a,b}$$

where the magnitude of 440 W/m²K is calculated by the Joule's law of heat produced by an electrical current – for this experiment we simulated a real condition of a resistance heater with 39.9 Ω resistance, powered with a 7.5 V power supply. This calculation takes in consideration the resistance with dimensions of 40 × 40 mm, where the generated heat is equally distributed to the two plates of the sandwich.

To obtain all the results on the present work, the increment of spatial discretization is Δx = 0.004 m and the time increment is Δt = 5 s. With the implicit formulation such increments do not present any stability or convergence problems. Another constants of the experiment are: The ambient temperature T_∞ = 24 ºC; The dimensions L_x = 80 mm, L_y = 40 mm and L_z = 1.6 mm, of the plate.

All the results presented in the following sub-section 3.1 and 3.2 are calculated as the mean of 50 independent executions of the whole method, in order to reduce the randomness effect, and the term NFE refers to the of Number of Fitness Evaluations necessary to reach M = 150.75 ºC².

3.1 Results obtained for the Nylon 6,12

Table 3 presents the results obtained for the Nylon 6,12 as building material – see Tab. 1. Different configurations of N initial points and K-nearest neighbors are tested, going from N = 100 to N = 1600. The results show that in 8 out of 13 configurations, the Sobol Sequence achieved the smallest NFE average, especially in the ones with higher initial points N.

Table 3. Average of the number of fitness evaluations to reach the stop criterion of $Q \le M$ with the Nylon 6,12 material. Calculated with σ = 0.5 level of error in the experimental data for the Mersenne Twister [MT], Sobol Sequence [SS] and RandomReal [RR].

N	K	MT [ºC²]	SS [ºC²]	RR [ºC²]
100	K = 10	165.26	**154.20**	164.28
	K = 15	137.66	**136.44**	136.76
200	K = 20	**247.78**	258.10	258.98
	K = 25	242.52	240.28	**237.46**
400	K = 30	**464.26**	465.56	471.44
	K = 40	**441.32**	444.78	446.50
	K = 45	436.16	**434.60**	438.50
800	K = 60	846.40	833.32	**831.54**
	K = 65	840.50	**828.68**	834.36
	K = 75	831.54	**822.44**	832.26
1600	K = 90	1642.40	**1630.62**	1646.44
	K = 100	1633.80	**1620.48**	1638.94
	K = 110	1619.32	**1615.04**	1630.02
Avg		**734.532**	**729.580**	**735.960**

Table 4 shows the difference between the results obtained on Tab. 3, the column named as "Abs. Rank" gives the ranks, in ascending order, of the absolute values of those differences. As described in sub-section 2.4, the terms $T_{(-)}$ and $T_{(+)}$ are the sum of all ranks of negative and positive differences, respectively. If the smallest of those sums is less or equal to the critical values presented in Tab. 1, then there is a difference between the two results and one can conclude which one is better by the average values of NEF in Tab 3.

Using the number of data n = 13 and α = 1% – see Tab. 1 – the tests show that there is no difference between the Mersenne Twister and the RandomReal results. For the Sobol Sequence and the other two points generators, the smallest T obtained is $T_{(-)}$ = 15 (MT – SS) and $T_{(+)}$ = 9 (RR – SS). With α = 1% one can conclude that the Sobol Sequence generated better results against the RandomReal generator. But that claim against the Mersenne Twister can only be done with α = 5%.

Table 4. Wilcoxon test for the Nylon 6,12 for all the three random points generators

N	*K*	MT – SS [ºC²]		MT – RR [ºC²]		SS – RR [ºC²]	
		Difference	Abs. Rank	Difference	Abs. Rank	Difference	Rank
100	*K* = 10	11.06	9 (+)	0.98	3 (+)	– 10.08	10 (–)
	K = 15	1.22	1 (+)	0.90	2 (+)	– 0.32	1 (–)
200	*K* = 20	– 10.32	8 (–)	– 11.20	12 (–)	– 0.88	2 (–)
	K = 25	2.24	4 (+)	5.06	6 (+)	2.82	5 (+)
400	*K* = 30	– 1.30	2 (–)	– 7.18	10 (–)	– 5.88	8 (–)
	K = 40	– 3.46	5 (–)	– 5.18	8 (–)	– 1.72	3 (–)
	K = 45	1.56	3 (+)	– 2.34	4 (–)	– 3.90	6 (–)
800	*K* = 60	13.08	12 (+)	14.86	13 (+)	1.78	4 (+)
	K = 65	11.82	11 (+)	6.14	9 (+)	– 5.68	7 (–)
	K = 75	9.10	7 (+)	– 0.72	1 (–)	– 9.82	9 (–)
1600	*K* = 90	11.78	10 (+)	– 4.04	5 (–)	– 15.82	12 (–)
	K = 100	13.32	13 (+)	– 5.14	7 (–)	– 18.46	13 (–)
	K = 110	4.28	6 (+)	– 10.70	11 (–)	– 14.98	11 (–)
		$T_{(-)} = 15$ $T_{(+)} = 76$		$T_{(-)} = 58$ $T_{(+)} = 33$		$T_{(-)} = 82$ $T_{(+)} = 9$	

3.2 Results obtained for the High Density Polyethylene

Table 5 presents the results obtained for the HDPE as building material – see Tab. 1. These results somewhat similar to the ones obtained for the Nylon 6,12, but here the mean of the objective function values are greater. This can be explained by the random nature of the experimental noise added, which yields different value ranges for the objective function.

Table 5. Average of the number of fitness evaluations to reach the stop criterion of $Q \leq M$ with the HDPE material. Calculated with σ = 0.5 level of error in the experimental data for the Mersenne Twister [MT], Sobol Sequence [SS] and RandomReal [RR].

N	*K*	MT [ºC²]	SS [ºC²]	RR [ºC²]
100	*K* = 10	188.36	**183.22**	193.16
	K = 15	153.82	**152.28**	153.20
200	*K* = 20	**282.66**	284.86	283.02
	K = 25	262.06	**257.36**	263.92
400	*K* = 30	509.84	**500.96**	506.08
	K = 40	470.34	**465.78**	470.02
	K = 45	**454.84**	457.00	457.70
800	*K* = 60	888.12	**863.16**	882.46
	K = 65	874.50	**860.42**	871.74
	K = 75	852.90	**841.34**	854.78
1600	*K* = 90	1696.46	**1687.60**	1697.16
	K = 100	1678.34	**1658.56**	1675.50
	K = 110	1663.44	**1643.92**	1661.76
Avg		**767.360**	**758.189**	**766.962**

Observe that for this material, the Sobol Sequence achieved the smallest count of function evaluations for 12 out of 13 different configurations. This outstanding performance is even better than the one obtained for the Nylon 6,12. The Wilcoxon test at this point will certainly support that claim. The differences and ranks for the comparisons tests are displayed in Tab. 6.

The comparisons here are similar to the ones obtained for the Nylon 6,12. Observe that the Sobol Sequence yielded T–values of 3 and 5 for both comparisons, which can reject the null hypothesis with $\alpha = 1\%$ (this level of certainty was not achieved for the Nylon 6,12 experiment – see Tab. 4). Again, the comparisons between the Mersenne Twister and RandomReal led to accept the null hypothesis which states that there is no difference between the two of them.

Table 6. Wilcoxon test for the HDPE for all the three random points generators

N	*K*	MT – SS [ºC²]		MT – RR [ºC²]		SS – RR [ºC²]	
		Difference	Abs. Rank	Difference	Abs. Rank	Difference	Rank
100	*K* = 10	5.14	6 (+)	– 4.80	12 (–)	– 9.94	8 (–)
	K = 15	1.54	1 (+)	0.62	3 (+)	– 0.92	2 (–)
200	*K* = 20	– 2.20	3 (–)	– 0.36	2 (–)	1.84	3 (+)
	K = 25	4.70	5 (+)	– 1.86	6 (–)	– 6.56	6 (–)
400	*K* = 30	8.88	8 (+)	3.76	11 (+)	– 5.12	5 (–)
	K = 40	4.56	4 (+)	0.32	1 (+)	– 4.24	4 (–)
	K = 45	– 2.16	2 (–)	– 2.86	10 (–)	– 0.70	1 (–)
800	*K* = 60	24.96	13 (+)	5.66	13 (+)	– 19.30	13 (–)
	K = 65	14.08	10 (+)	2.76	8 (+)	– 11.32	9 (–)
	K = 75	11.56	9 (+)	– 1.88	7 (–)	– 13.44	10 (–)
1600	*K* = 90	8.86	7 (+)	– 0.70	4 (–)	– 9.56	7 (–)
	K = 100	19.78	12 (+)	2.84	9 (+)	– 16.94	11 (–)
	K = 110	19.52	11 (+)	1.68	5 (+)	– 17.84	12 (–)
		$T_{(-)} = 5$ $T_{(+)} = 86$		$T_{(-)} = 41$ $T_{(+)} = 50$		$T_{(-)} = 88$ $T_{(+)} = 3$	

The results obtained for the estimation of the thermal properties k and c_p is presented in Tab. 7. We present only results using the Sobol Sequence as starting scheme for the TGO algorithm and configurations that yielded the lowest NFE for each range o N points. They are the ones with greater K-nearest neighbors, this can be explained by the fact that with more neighbors to analyze, more quality feasible is the points considered topographical optima.

Table 7. Thermal properties estimation for the Nylon 6,12 and HDPE using the Sobol Sequence as random point generator for TGO

	Nylon 6,12		HDPE	
	k [W / m K]	c_p [J / kg K]	k [W / m K]	c_p [J / kg K]
Exact Value	**0.220**	**1680**	**0.330**	**2260**
N = 100, *K* = 15	0.226	1670.37	0.340	2214.28
N = 200, *K* = 25	0.230	1676.22	0.341	2212.68
N = 400, *K* = 45	0.232	1676.59	0.342	2225.73
N = 800, *K* = 75	0.229	1673.85	0.342	2220.18
N = 1600, *K* = 110	0.232	1696.01	0.342	2225.92
Average	**0.229**	**1678.61**	**0.341**	**2219.76**

Observe that even with a stopping criterion such as the Morozov's Discrepancy Principle, the estimations are within a good confidence boundary. With this approach, the estimations do not get necessarily better as the number of initial points N increases, but the effort made by the local optimization method certainly reduces.

4. CONCLUSIONS

The problem addressed in this work deals with transient heat conduction. To solve the inverse problem of estimating thermal properties, an optimization method known as Topographical Global Optimization was used. This technique has as a first step the generation of random points in a search space. The focus of this work was to make the first investigation as to which random point generator can positively influence the final results of the method. We tested the pseudo-random Mersenne Twister, the also pseudo-random RandomReal (built-in routine of Wolfram Mathematica 11.0), and the quasi-random Sobol Sequence, which renders a low discrepancy arrange of points. The results were compared using the Wilcoxon Test, which supported the idea that the Sobol Sequence is the best point generator to use on this kind of inverse analysis. The Mersenne Twister and the RandomReal generators did not presented any difference of results when compared with the Wilcoxon Signed-Rank Test.

Much more investigations must continue on the subject. The work presented here points at the direction which should be the challenge for improving the TGO method. As an optimization technique, TGO deserves more attention due to its already proven effectiveness.

5. REFERENCES

Brosse, A., Naisson, P., Hamdi, H., Bergheau, J. M., 2008, "Temperature measurement and heat flux characterization in grinding using thermography", Journal of materials processing technology, 201(1-3), pp. 590-595.

Dalal, I.L., Stefan, D., Harwayne-Gidansky, J., 2008, "Low discrepancy sequences for Monte Carlo simulations on reconfigurable platforms." Application-Specific Systems, Architectures and Processors, ASAP 2008. International Conference. IEEE.

Henderson, N., Rêgo, M. de S., Sacco, W.F. and Rodrigues Jr., R.A., 2015, "A new look at the topographical global optimization method and its application to the phase stability analysis of mixtures", Chemical Engineering Science, Vol. 127, 151–174.

J. Derrac, J.S., García, D., Molina, F.H, 2011, "A practical tutorial on the use of nonparametric statistical test as a methodology for comparing evolutionary and swarm intelligence algorithms", Swarm and Evolutionary Computation 1, n 1 pp. 3–18.

Knupp, D. C., Naveira-Cotta, C. P., Cotta, R. M., 2014, "Theoretical–experimental analysis of conjugated heat transfer in nanocomposite heat spreaders with multiple microchannels", International Journal of Heat and Mass Transfer, 74, pp. 306-318.

Knupp, D.C., Naveira-Cotta, C. P., Ayres, J. V., Cotta, R. M., Orlande, H. R., 2012, "Theoretical–experimental analysis of heat transfer in nonhomogeneous solids via improved lumped formulation, integral transforms and infrared thermography". International Journal of Thermal Sciences, 62, pp. 71-84.

Lienhard IV, J.H. and Lienhard V, J.H., 2008, *A heat transfer textbook*, 3rd Ed., Phlogiston, Cambridge, Massachusets, USA.

McCornack, R. L., 1965, "Extended tables of the Wilcoxon matched pair signed rank statistic." Journal of the American Statistical Association 60.311, pp. 864-871.

Mital, M., E.P. Scott, 2007, "Thermal detection of embedded tumors using infrared imaging", 366 J. Biomech. Eng. 129 (1), pp. 33–39.

Maaranen, H., Miettinen, K., Mäkelä, M.M., 2004, "Quasi-Random Initial Population for Genetic Algorithms", Comput. Math. Appl., Vol. 47, pp. 1885-1895.

Matsumoto, M., Nishimura, T., 1998. "Mersenne twister: a 623-dimensionally equidistributed uniform pseudo-random number generator", ACM Trans. Model. Comput. Simul. Vol. 8, pp. 3-20.

McDonald, J. H., 2009, "Handbook of biological statistics". Vol. 2. Baltimore, MD: sparky house publishing,.

Nelder, J.A. e Mead, R., 1965, "A simplex method for function minimization". Computer Journal 7, 308–313

Ozisik, M. N., 1993, "Heat conduction", 2nd ed., John Wiley & Sons, New York, NY, USA.

Ozisik, M. N., 1994, "Finite Differences methods in heat transfer", CRC Press, Boca Raton, FL, USA.

Regis, C.R., Cotta, R.M. e Su, J., (2000), "Iproved lumped analysis of transient heat conduction in a nuclear fuel rod", International Communications in Heat and Mass Transfer, Vol. 27, n.3, pp. 357-366.

Sacco, W.F., Henderson, N., Coelho, A. C. R., 2014, "Topographical global optimization applied to nuclear reactor core design: Some preliminary results", Annals of Nuclear Energy, Vol. 65, pp. 166-173.

Sacco, W.F., Rios-Coelho, A.C., Henderson, N, 2014, "Testing population initialisation schemes for differential evolution applied to a nuclear reactor core design." International Journal of Nuclear Energy Science and Technology 8.3, pp. 192-212.

Silva Neto, A.J. and Moura Neto, F.D., 2005, "Inverse problems–fundamental concepts and applications", EdUERJ, Rio de Janeiro.

Silvério, V., Cardoso, S., Moreira, A. L.N., 2016, "Evaluation of a new designed microchannel heat sink for CPU cooling based on IR–thermography synchronized with high-speed flow visualization." In: 18th International Symposium on the Application of Laser and Imaging Techniques to Fluid Mechanics, Lisbon, Portugal.

Sobol, I.M., 1967. "The distribution of points in a cube and the approximate evaluation of Integrals". USSR Comput. Math. Phys. 7, pp. 86-112.

Su, J. and Cotta, R.M., 2001, "Improved lumped parameter formulation for simplified LWR thermohydraulic analysis", Annals of Nuclear Energy, Vol. 28, pp. 1019-1031.

Törn, A. and Viitanen, S., 1992, "Topographical global optimization", In: C.A. Floudas and P.M. Pardalos (Eds.), Recent Advances in Global Optimization, Princeton University Press, pp. 384-398.

Törn, A. and Viitanen, S., 1996, "Iterative Topographical Global Optimization", Nonconvex Optimization and Its Applications, Vol. 7, pp. 353-363.

Wilcoxon, Frank. "Individual comparisons by ranking methods." Biometrics bulletin 1.6 (1945): 80-83.

Wolfram, S., "The Mathematica Book", Wolfram Media, Cambridge, 2005.

Zhang, Hong-Guang, Bi-Hua Tang, and Kai-Ming Liu., 2016, "The Influence Analysis of Pseudorandom Number Generators and Low Discrepancy Sequences for the Family of Compact Genetic Algorithms: Search Behavior Research from Outside Causes to Internal Causes." *Electronics, Communications and Networks V*. Springer, Singapore, pp. 385-396.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.

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