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ANALYSIS OF A ALPHA-TYPE STIRLING ENGINE THROUGH ISOTHERMAL AND ADIABATIC MODELS

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Abstract. *Stirling engines are external combustion heat engines. This condition extends the list of power sources that can be used. Although this thermal engine precedes the internal combustion engines, it had its development suppressed due to technical problems of operation and influence of the petroleum industry. However, the research and development of Stirling engines has gained momentum in the last decade in view of a possible shortage of fossil fuels, as well as the huge amount of fuels that can provide thermal energy for Stirling engines such as biomass, solar, natural gas, oil companies, among others. The objective of this study is to develop the simulation for the Stirling type alpha motor comparing the Schmidt analysis based on the isothermal expansion and compression of an ideal gas and an adiabatic system. Parametric study is used to investigate the operational effects on engine performance. The effects on the expansion cylinder are analyzed as: dead volume, temperature and swept volume rate.*

Keywords: *Strling engine, Alpha-type, isothermal, adiabatic.*

1. INTRODUCTION

The use of fossil fuels as the main source for the generation of energy has caused several consequences to the planet, which are directly affecting the environment, an alternative to this problem would be the use of alternative sources for energy generation, which has been taking great the world (Borges *et al.*, 2017).

The main examples of renewable sources used for power generation can be mentioned such as wind power, hydropower, geothermal energy, biomass energy and also solar energy (Braz *et al.*, 2017). In the context of solar energy, the Stirling engine can be composed of parabolic dishes that concentrate solar radiation and transfer energy to a power system for the purpose of producing kinetic or electric energy (Hinrichs *et al.*, 2010).

The Stirling engine was the first engine able to perform useful work, created in 1816 by Robert Stirling, it is an external combustion heat engine powered by any sources of thermal energy. Its structure is composed of a piston-cylinder arrangement and inside the cylinder is confined the working fluid. Several gases have been tested as working fluid as helium, hydrogen, nitrogen or even a mixture between them (Sepulveda *et al.*, 2008).

The large amount of fuels that can be used as external energy source, natural gas, diesel fuel, biomass, diesel, solar energy, gasoline, among others is one of the strengths of this engine (Dias, 2016). In search of better engine yields, its creator proposed the use of a regenerative heat exchanger, which stores some of the heat that would be rejected by the engine and reheats the working fluid before entering the hot cylinder (Gingery, 1990).

Stirling engines have other advantages for their use, operate well with fuels containing low carbon / hydrogen ratio, low noise, low maintenance costs and can achieve 30% efficiency in thermal generation and 90% in cogeneration (da Rosa, 2015; Sepulveda *et al.*, 2008). On the other hand, some technical problems hamper operation and development such as: sealing for operating pressures greater than atmospheric, high response time due to limitations imposed by the heat transfer processes, and when compared to diesel engines they are expensive (Wilke and Lora, 1977).

Another important point of the Stirling engines is the diversity of applications, which can be in the automotive sector, electric microgeneration systems and even in military and medical operations due to their quiet and clean operation (Tlili *et al.*, 2008).

Seeking to know the potential of the Stirling engine in the automotive sector, a survey conducted by the Ford Motor Company and Philips compared the results with a spark-ignition engine (Kitzner, 1977). The study presented positive results, which can be observed the increase of power, good operation under conditions of high temperatures and pressure, fuel economy and low emissions of gases (Ernest and Shaltens, 1997). However, the disadvantages present in the engine, such as cylinder sealing, speed control and slow start make it difficult to implement the Stirling engine in this sector (Dias, 2016).

Besides that, there are applications for the engine from renewable sources such as biomass and solar. A solar concentrator coupled to the Stirling engine, concentrates the solar radiation in search of higher temperatures, thus increasing the exergetic availability for the thermal machine. This system is composed of solar dishes positioned at strategic points seeking the best use of radiation. The engine drives an alternator that generates output power (Hillig, 2016). The biomass, after an anaerobic fermentation process produces the biogas, in turn, this when it can be used to drive the Stirling engine (Dias, 2016). These systems are viable in terms of Distributed Generation (DG), which seeks the generation of electricity close to the consumer, especially in isolated areas (Sepulveda *et al.*, 2008).

Another source of fuel for the Stirling engine, which is used by National Aeronautics and Space Administration (NASA), is the plutonium-238 radioisotope. With this radioactive source was possible the generation of heat and with that the production of electric energy (Dias, 2016). From a generator coupled to the engine, the system was implemented to supply space probes that operate far from the sun (Chan *et al.*, 2008).

In spite of the great potential of applying these equipment in tropical zones, due to the high availability of solar radiation, as well as the diversity of species that can supply biomass, there are few research groups working on technological development and there is no company producing on a commercial scale this relatively simple construction equipment in national territory.

In this context, the proposal of the present work is the isothermal and adiabatic modeling of a Stirling alpha-type engine and the comparison of its results, analyzing geometric and operational parameters such as the effect of dead volume rate, temperature of expansion and swept volume rate.

2. METHODOLOGY

Modeling is a technique with low execution cost, considering the exemption of experimental procedures for each of the evaluated conditions. This technique seeks to represent the physical or chemical phenomena, through mathematical equations that model these phenomena. The degree of complexity of the models varies greatly, ranging from simplified models such as Zero dimensions and non-reactional models to three-dimensional and reactional models.

The applied modeling depends on several factors, among them, the technical need for accuracy, knowledge of equations that model the phenomena involved, implementation time and computational costs. However, there is no guarantee that more complex models will deliver more realistic results.

The confrontation of results generated by mathematical models and experimental studies are fundamental to any model, and such stage of model development is the model validation.

The modeling proposed in this work is primarily zero dimensions and not reactionary. Thus, the modeling of the Stirling alpha-type engine was performed using MatLab software. This modeling requires the use of thermodynamic concepts for the thermal analysis of the process as well as the process of heat transfer. Thus, the general principles of thermodynamics applied to the Stirling cycle are presented below.

2.1 The ideal and real thermodynamic cycle

The thermodynamic cycles are classified as gas cycles or steam cycles, depending on the working fluid phase, where in steam cycles there is a phase in which the working fluid is in the vapor phase and passes to the liquid phase, while in the gas cycles remain in the gas phase throughout the process (Çengel and Cimbala, 2015).

Another classification of the thermodynamic cycles is that of closed and open cycles, the closed cycle is defined by which at the end of its course it returns to its initial state and repeats the cycle, already an open cycle is that at the end of each cycle the fluid of work is renewed (Çengel and Cimbala, 2015).

The understanding of thermodynamic cycles is of paramount importance, since most of the devices that generate power operate in cycles, but a real analysis of these cycles is complicated, and for the sake of simplification, it is necessary to study an ideal cycle. For the analysis of the ideal cycle are used simplification as not having friction in the cycle, thus, there is no loss of pressure in pipes and heat exchangers, expansion and compression occur almost quasi-static and the system is very well isolated and losses in heat exchangers are negligible (Çengel and Cimbala, 2015).

The Stirling engine features a closed and regenerative gas cycle, involves two isothermal processes and two isochoric regeneration processes, and the working fluid remains confined in a cylinder. In order to increase the efficiency of the engine the regenerator stores and transfers heat to the working fluid during the cycle (Çengel and Cimbala, 2015). The Stirling cycle of pressure versus volume and temperature versus entropy are showing in Fig. 1.

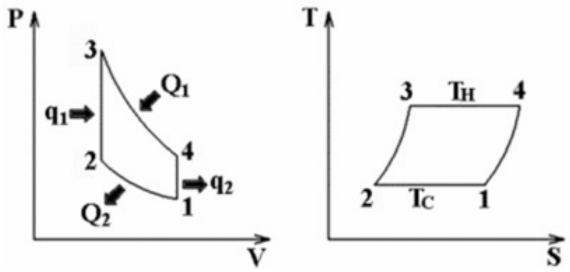

Figure 1. Diagrams of P-V and T-S of the ideal Stirling cycle (da Cruz, 2012)

Figure 1 represents the four stages of the Stirling cycle (Çengel and Cimbala, 2015):

1 → 2 - An isothermal compression process where there is heat rejection to the cold source and the work required for gas compression is the area below the curve between points 1 and 2.

2 → 3 - Process of isochoric regeneration, where the heat of the regenerator is given to the working fluid, in this step there is an increase of the internal energy of the system due to the increase in temperature.

3 → 4 - An isothermal expansion process, where the external source provides heat to the system, thereby increasing the gas volume and decreasing the pressure. It is at this stage that the heat received is converted into work on the piston, and can be observed in the area below the curve between points 3 and 4.

4 → 1 - Process of isochoric regeneration, where the heat received by the working fluid is transferred to the regenerator. The temperature of the system in this stage decreases and with this, there is a decrease of the internal energy of the system.

One of the characteristics of the Stirling engine is that the mass of the working fluid is fixed and from the exchange of heat between the hot and cold cylinders the expansion and contraction of the gas occurs. Thus, the change of temperature of the gas generates a difference of pressure, which generates a force that is delivered to the shaft from the power pistons (da Cruz, 2012).

Three factors hamper the ideal engine process such as sinusoidal piston motion, imperfect regeneration and dead volume. During regeneration, some of the heat is lost during cooling because freshly heated fluid comes into equilibrium with the cold source and the regenerator transposed to the hot source from that fluid, so there is less energy expenditure. Dead volume is the biggest cause of inefficiency because some of the gas remains in the engine chambers (Stine, 2001). These factors round the P-V diagram of the system as shown in Fig. 2.

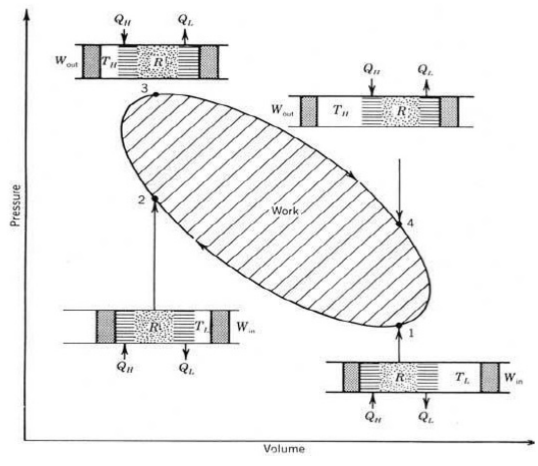

Figure 2. Diagram P-V real cycle Stirling engine (Stine, 2001)

For alpha-type engines, the total volume of the working fluid is determined by the relative displacements of its two pistons.

2.2 Stirling engine components and their classification according to the configuration of the pistons

Capable of converting thermal energy into mechanical energy, the Stirling alpha-type engine consists of two pistons, a hot cylinder and a cold cylinder connected in series. The working fluid is confined in the cylinders, which, through an external source of heat, undergoes consecutive cyclical processes of heating, expansion, cooling and compression. With the change in temperature of the working fluid, the gas moves between the cylinders and thus generates a pressure difference causing the reciprocating movement of the pistons, and that moment of force is delivered to the shaft (Dias, 2016).

Figure 3 shows the engine components in the alpha-type configuration.

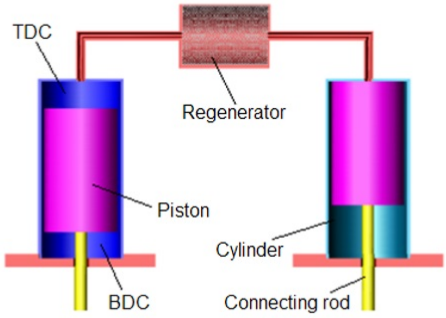

Figure 3. Schematic view of Stirling engine. Adapted from (Velázquez, 2007)

The pistons, which are connected by the crankshaft and connecting rods, rotate with a quarter of a revolution difference from the other (θ = 90 °). Each cylinder has a piston, where in the hot cylinder the gas expansion process will occur and in the cold cylinder its compression (Cachute, 2006). Each cylinder has a top dead center (TDC) and a bottom dead center (BDC), where these values mean that the piston is as close to or as far away from the head as possible (Brunetti, 2013).

A regenerator connects the cylinders, which can be stainless steel wool, rearranged tubes or matrix with plates. The regenerator, used to increase the efficiency of the engine, should store part of the thermal energy and then return it to the working fluid and, therefore, must have high thermal capacity and low thermal conductivity (Dias, 2016).

The Stirling engine has two heat exchangers, the function of the heat exchanger being the supply of heat to the working fluid from the external source during expansion. During compression, the cooling exchanger should dissipate the heat, and cooling may be free or forced generally using air, by means of fins, or running water (Lloyd, 2009).

The pistons have a linear movement and, when connected to the connecting rod, this device is able to convert the force received by the movement of the pistons into torque, and in this way, the crankshaft that is connected to the connecting rods makes the rotation of the shaft. The steering wheel assists in the rotation of the crankshaft, its function is to store kinetic energy increasing its angular velocity (Dias, 2016).

In the configuration of the Stirling type beta engine the displacement and power pistons are mounted in the same cylinder and a same line of action, in this way, they are more compact and can reach higher compression ratios (Velázquez, 2007). In Figure 4 below, the configurations of the beta and gamma engines are present.

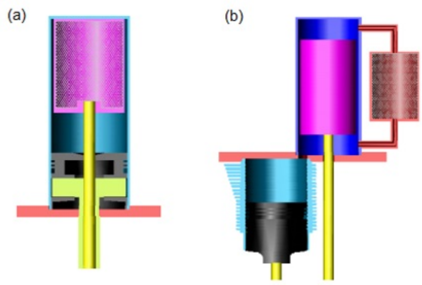

Figure 4. Engine configurations (a) beta and (b) gamma. Adapted from (Velázquez, 2007)

The engine type gamma has two cylinders, but the compression and displacement piston are well defined (Velázquez, 2007).

2.3 Isothermal model of the alpha-type Stirling engine with Schmidt analysis

The objective of isotherm analysis is to obtain the conducted work as a result of the pressure changes and temperature of the operating gas using the heat transfer to the inside of the engine.

The actual thermodynamic cycles do not have such well-delineated steps as shown in the ideal cycle diagrams, but Schmidt analysis represents reality well, although it is a very idealized analysis of the Stirling engine. The analysis makes it possible to obtain the net work of the cycle and the power of the engine, considering that the regeneration is perfect and brings the sinusoidal variation of the volume of the cylinders (Thombare and Verma, 2008).

Schmidt analysis takes into account other parameters such as no internal pressure differences, no heat loss in heat exchangers, expansion and compression processes are isothermal, during the cycle the dead volume of expansion maintains the gas temperature at expansion T_E, and the dead volume of compression maintains the temperature of the gas in compression, T_C. From the average of the gas temperature in the expansion and in the compression the temperature of the gas in the regenerator is estimated (Hirata, 1997).

The main variables of the Stirling alpha-type engine are showing in Fig. 5.

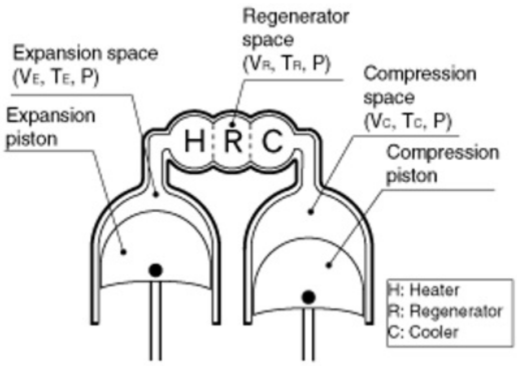

Figure 5. Alpha-type Stirling engine (Hirata, 1997)

The expansion and compression volumes must first be determined. The instantaneous expansion volume V_E, Eq. (1), is obtained from the crank angle, x=0° to x=360°, totaling one cycle. This is a function of the volume swept by the expansion piston, V_{SE}, and the dead volume of expansion, V_{DE}.

$$V_E = \frac{V_{SE}}{2}(1 - cos(x)) + V_{DE} \tag{1}$$

The instantaneous compression volume V_C is obtained from the Eq. (2). This is a function of the volume swept by the compression piston, V_{SC}, and the dead volume of compression, V_{DC}.

$$V_C = \frac{V_{SC}}{2}[1 - cos(x - dx)] + V_{DC} \tag{2}$$

Thus, by stipulating a volume for the regenerator, V_R, it is possible to obtain the total instantaneous volume, given in Eq. (3) below.

$$V = V_E + V_C + V_R \tag{3}$$

From these three equations the sinusoidal variations of the volumes are obtained, thus rearranging the ideal gas equation, Eq. (4), it is possible to estimate the mass of the working fluid.

$$m = \frac{PV_E}{RT_E} + \frac{PV_R}{RT_R} + \frac{PV_C}{RT_C} \tag{4}$$

In the equation above, the engine pressure is P, V each volume and the gas constant, R. The temperature of the gas in the expansion space is T_E, the temperature of the gas in the compression space is T_C, and these temperatures can be stipulated for the temperature that is desired to operate the cylinder, since T_R is the arithmetic mean between them.

$$T_R = \frac{T_E + T_C}{2} \tag{5}$$

The relative travel volume v, the relative temperature t, and the relative dead volumes of expansion, compression and regenerator are respectively obtained from the following equations.

$$v = \frac{V_{SC}}{V_{SE}} \tag{6}$$

$$t = \frac{T_C}{T_E} \tag{7}$$

$$X_{DE} = \frac{V_{DE}}{V_{SE}} \tag{8}$$

$$X_{DC} = \frac{V_{DC}}{V_{SE}} \tag{9}$$

$$X_R = \frac{V_R}{V_{SE}} \tag{10}$$

Substituting Eq. (5) into Eq. (4):

$$m = \frac{P}{RT_C}(V_E t + \frac{2tV_R}{(1+t)} + V_C) \quad (11)$$

Substituting Eq. (1) and Eq. (2) and simplifying.

$$m = \frac{PV_{SE}}{2RT_C}[S - Bcos(x-a)] \quad (12)$$

$$S = t + 2tX_{DE} + \frac{4tX_R}{1+t} + v + 2X_{DC} \quad (13)$$

$$a = tan^{-1}(\frac{vsin(dx)}{t + cos(dx)}) \quad (14)$$

$$B = \sqrt{t^2 + 2tvcos(dx) + v^2} \quad (15)$$

Substituting equations (13), (14) and (15) into Eq. (11), instantaneous engine pressure is:

$$P = \frac{2mRT_C}{V_{SE}[S - Bcos(x-a)]} \quad (16)$$

When cos(x-a)=-1, in the Eq. (18), the engine pressure P becomes the minimum pressure P_{min} the next equation is introduced.

$$P_{min} = \frac{2mRT_C}{V_{SE}[S+B]} \quad (17)$$

Similarly, when cos(x-a)=1, the engine pressure P becomes the maximum pressure P_{max} is described in Eq. (18).

$$P_{max} = \frac{2mRT_C}{V_{SE}[S-B]} \quad (18)$$

The mean pressure P_{mean} can be calculated as follows:

$$P_{mean} = \frac{1}{2\pi}\oint Pdx = \frac{2mRT_C}{V_{SE}\sqrt{S^2 - B^2}} \quad (19)$$

Equation 22 defines c.

$$c = \frac{B}{S} \quad (20)$$

As a result, the engine pressure P, based the mean engine pressure is calculated in Eq. (21).

$$P = \frac{P_{mean}\sqrt{1-c}}{1 - c.cos(x-a)} \quad (21)$$

The expansion work W_e is given by Eq. (22) below.

$$W_E = \oint PdV_E = \frac{P_{mean}V_{SE}\pi c.sen(a)}{1 + \sqrt{1-c^2}} \quad (22)$$

The compression work W_c can be obtained from Eq. (23).

$$W_C = \oint PdV_C = \frac{P_{mean}V_{SE}\pi t.c.sen(a)}{1 + \sqrt{1-c^2}} \quad (23)$$

The net work W_{liq} performed by the cycle is expressed by Eq. (24).

$$W_{liq} = W_C + W_E \quad (24)$$

The p-V diagram of Alpha-type Stirling engine can be made with above equations.

2.4 Adiabatic model of the alpha-type Stirling engine

For this analysis will be taken as reference the work of Urieli and Berchowitz (1984) that proposed an adiabatic model assumed that the cooler and heater had infinite heat transfer and the isothermal condition are not true, because due to high engine speed it is impossible for this process to occur. The process isothermal is substituted for an adiabatic model, there is no heat exchange between the work fluid and expansion and compression chambers. In the analysis of the adiabatic model, using the energy equations and the equation of state of the ideal gas, are obtained heat transfer, work done and engine efficiency (Ziabasharhagh and Mahmoodi, 2012).

Finkelstein's adiabatic analysis is used and, as in the Schmidt analysis, it uses sine-wave expressions to describe the volume variations within the compression and expansion cylinders. However, all the simplifications adopted in the Schmidt model are retained but their result is more realistic (Walker, 1980).

The schematic model of the engine considered for the development of the analysis is presented in Figure 6, where the compartments are connected in the following order Compression (c), Cooling (k), Regeneration (r), Heating (h) and Expansion (e).

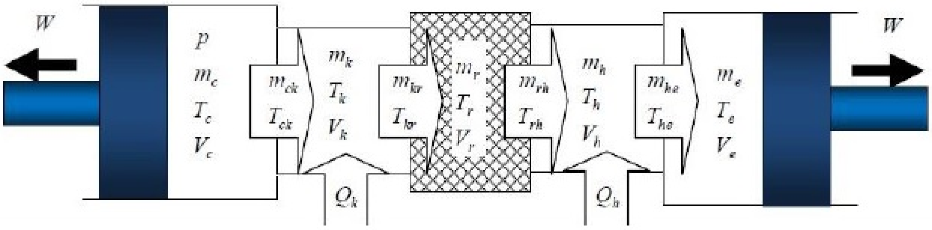

Figure 6. Model of the chambers of a Stirling engine (Ziabasharhagh and Mahmoodi, 2012)

It is important to note that the adiabatic process occurs in the compression and expansion cylinders so the working fluid temperatures vary during the processes and with the direction of the fluid. To calculate the temperature of the regenerator a simplification is adopted where it is convenient to use the effective average temperature Eq. (25), but this is not true, since a linear variation occurs in relation to the characteristic length of the regenerator.

$$T = \frac{T_h - T_k}{L_r} x + T_k \tag{25}$$

From the equation of state for ideal gases the following expression is obtained for the total mass of fluid inside the regenerator:

$$m_r = \int_0^{V_r} \rho dV_r = \int_0^{V_r} (\frac{p}{RT}) dV_r \tag{26}$$

The section area of the regenerator should be considered constant, so the volume differential and the total volume can be written as shown in Eq. (27) and Eq. (28).

$$dV_r = A_r dx \tag{27}$$

$$V_r = A_r L_r \tag{28}$$

Substituting Equations 25, 27 and 28 in Equation 26, we have that:

$$m_r = \frac{pV_r}{R} \int_0^{V_r} \frac{1}{(T_h - T_k)x + T_k L_r} dx = \frac{pV_r}{R(T_h - T_k)} ln(\frac{T_h}{T_k}) \tag{29}$$

In addition, it is possible to determine the mass of the fluid in the regenerator using the effective mean temperature as shown below.

$$m_r = \frac{pV_r}{RT_r} \tag{30}$$

Thus, effective mean regenerator temperature can be defined by equating Eqs. (29) and (30) and isolating T_r.

$$T_r = \frac{T_h - T_k}{ln(\frac{T_h}{T_k})} \tag{31}$$

The total mass of the engine is constant, since it is a closed system. Thus, one can write:

$$M = m_c + m_k + m_r + m_h + m_e \tag{32}$$

In the same way that the isothermal model considers that all the regions are in the same pressure, so the ideal gas equation can be used according to the Eq. (33).

$$M = \frac{p}{R}(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e}) \tag{33}$$

The pressure can be isolated, so an equation for the mean pressure of the system is obtained according to Eq. (34).

$$p = \frac{MR}{(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e})} \tag{34}$$

Differentiating equation (32):

$$0 = dm_c + dm_k + dm_r + dm_h + dm_e \tag{35}$$

The differential equations of mass of the heat exchangers are obtained from the equation of state for ideal gases in the differential form, where $dT = 0$ and $dV = 0$.

$$dm = \frac{dp}{R}\frac{V}{T} \tag{36}$$

The equations for the mass differentials of the compression and expansion regions of the engine are determined considering the control volume of Fig. 7.

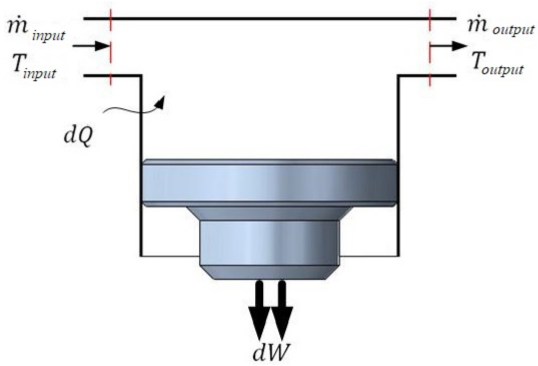

Figure 7. Volume of control for adiabatic analysis (Sant'anna and Yano, 2012)

Thus, the energy balance sums up to Eq. (37) considering as negligible the variations of kinetic energy and gravitational potential energy.

$$dU = dQ - dW + \dot{m}_{input}h_{input} - \dot{m}_{output}h_{output} \tag{37}$$

Considering the specific heats of the working fluid constant, Eq. (37) is rewritten as follows:

$$c_v d(mT) = dQ - dW + \dot{m}_{input}c_p T_{input} - \dot{m}_{output}c_p T_{output} \tag{38}$$

Thus, it is possible to use the state equation for ideal gases, where Eq. (38) is rewritten as follows.

$$\frac{c_v}{R}(pdV + Vdp) = dQ - dW + \dot{m}_{input}c_p T_{input} - \dot{m}_{output}c_p T_{output} \tag{39}$$

Equation (39) can be applied to the compression cylinder, knowing that from the mass balance for this space $dm_c = -\dot{m_{ck}}$ and replacing dW with pdV, it is rewritten according to Eq. (40).

$$\frac{c_v}{R}(pdV_c + V_c dp) = -pdV_c + dm_c c_p T_{ck} \tag{40}$$

The mass differential of the compression cylinder is isolated and the ratio of specific heats c_p/c_v is symbolized by γ.

$$dm_c = \frac{\frac{R}{C_p}pdV_c + \frac{c_v}{c_p}(pdV_c + V_c dp)}{RT_{ck}} = \frac{pdV_c + \frac{V_c dp}{\gamma}}{RT_{ck}} \tag{41}$$

Likewise, the mass differential of the expansion chamber is described according to Eq. (42).

$$dm_e = \frac{pdV_e + \frac{V_e dp}{\gamma}}{RT_{he}} \tag{42}$$

By rewriting equation (35) from the equations presented, the equation for the pressure differential is obtained.

$$dp = \frac{-\gamma p(\frac{dV_c}{T_{ck}} + \frac{dV_e}{T_{he}})}{[\frac{V_c}{T_{ck}} + \gamma(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}) + \frac{V_e}{T_{he}}]} \tag{43}$$

The energy balance is applied in order to obtain equations for the heat transferred in the heat exchangers. Thus, the equations for the heat exchanged in the cooling, regenerating and heating chamber are shown, respectively, below.

$$dQ_k = \frac{V_k c_v dp}{R} - c_p(\dot{m}_{ck}T_{ck} - \dot{m}_{kr}T_{kr}) \tag{44}$$

$$dQ_r = \frac{V_r c_v dp}{R} - c_p(\dot{m}_{kr}T_{kr} - \dot{m}_{rh}T_{rh}) \tag{45}$$

$$dQ_h = \frac{V_h c_v dp}{R} - c_p(\dot{m}_{rh}T_{rh} - \dot{m}_{he}T_{he}) \tag{46}$$

The temperature differentials for the engine compression and expansion spaces, respectively, are presented below from the state equation for ideal gases in differential form.

$$dT_c = T_c(\frac{dp}{p} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c}) \tag{47}$$

$$dT_e = T_e(\frac{dp}{p} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e}) \tag{48}$$

The equation (49) shows the working differential exerted by the engine.

$$dW = dW_c + dW_e = p(dV_c + dV_e) \tag{49}$$

It is further defined that the temperatures at the borders of the regenerator are equal to the temperatures of the heat exchangers, thus:

$$T_{kr} = T_k \tag{50}$$

$$T_{rh} = T_h \tag{51}$$

Other parameters that must be defined are the temperatures at the borders of the compression and expansion regions depend on the direction of the mass flow in them, so they are called conditionals.

$$\begin{cases} T_{ck} = T_c, \ \forall \, \dot{m}_{ck} \geq 0 \\ T_{ck} = T_k, \ \forall \, \dot{m}_{ck} < 0 \end{cases} \tag{52}$$

$$\begin{cases} T_{he} = T_h, \ \forall \, \dot{m}_{he} \geq 0 \\ T_{he} = T_e, \ \forall \, \dot{m}_{he} < 0 \end{cases} \tag{53}$$

All these equations must be solved simultaneously, so there is the difficulty of implementation of this model. However, the adiabatic model represents much better reality than isothermal.

3. RESULTS AND DISCUSSION

Currently, several researches have been developed in order to develop more efficient Stirling engines. In this context, in this section will be presented parameters to develop the small Stirling type alpha type engine. The modeling of the engine was performed from the Schmidt's Theorem for the isothermal model and the adiabatic analysis of Finkelstein for the adiabatic model, from an appropriate software. To begin the modeling of the engine it was necessary to adopt constructive and operations parameters described in Tab. 1.

From the engine specifications listed in Tab. 1 it was possible to obtain the volume variations according to the angle of rotation using Eq. (1), (2) and (3), according shows Fig. 8.

Table 1. Specific values used in the analysis.

Mechanical configuration	α
Expansion piston diameter (mm)	37.1
Expansion swept volume (cm^3)	40.192
Expansion dead volume (cm^3)	25.6
Compression piston diameter (mm)	37.1
Compression swept volume (cm^3)	40.192
Compression dead volume (cm^3)	25.6
Regenerator volume (cm^3)	25.6
Heating temperature (K)	673
Cooling temperature (K)	303
Phase angle (degree)	90
Working fluid	Air
Angular rotation	2000

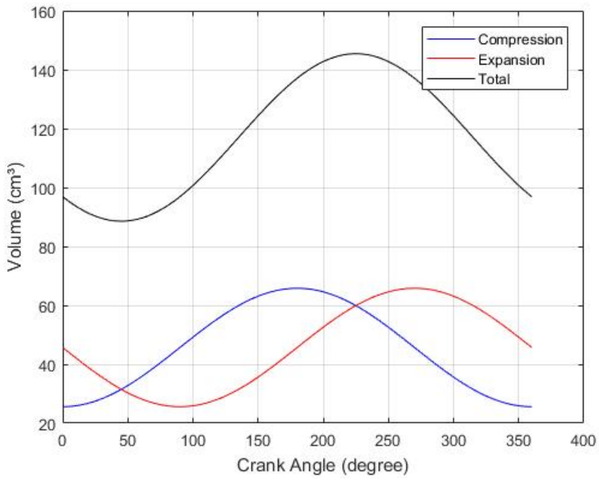

Figure 8. Volume change in the cylinders during the cycle

Figure 8 shows only one engine cycle, the black curve being the instantaneous sum of the expansion volumes, compression and regeneration. It is observed that the masses contained in the working spaces are variable, however, the total mass of the working fluid throughout the cylinder space is fixed because of mass conservation. The maximum instantaneous engine volume is 145.41 cm^3 and is obtained when the turning angle is 225°. For engines in which the compression and expansion volumes differ, the angle of rotation at which the maximum volume is located will be different from 225°.

Figure 9 below shows the p-V diagram obtained in the isothermal and adiabatic analyzes. It is remarkable the difference between the models, where the diagram of the adiabatic analysis presented an expected result that represents the reality better than the isothermal.

It is observed that the thermodynamic cycle p-V of the engine has an elliptical shape different from the ideal cycle, that is due to the sinusoidal variations of the volumes of the analyzes used. The work performed by the engine in the isothermal model was calculated from Eq. (24) is 0.76 J, and for the adiabatic analysis, the engine performed a work corresponding to 0.57 J calculated by Eq. (49). These values are perceptible by analyzing Fig. (9), where the area occupied by the p-V diagram of the adiabatic model is reduced.

Other factors that can be observed are the compression and expansion works. In the isothermal model, the compression work is 0.62 J and the expansion work is 1.38 J, where these values are higher than in the adiabatic analysis, which reached 0.56 J and 1.13 J, respectively. Comparing the compression jobs, these factors show a 10% difference between the analyzes. This means that the adiabatic model presents more realistic values and even if the implementation of its programming is more complex it is a more advantageous model to be used.

Thus, it is possible to determine the power in each model, where, for the isothermal model is 25.29 W and in the isothermal model 19.08 W, with a reduction of approximately 25%. Even with the high complexity of applying the adiabatic model, it is remarkable the importance of its use so that results closer to reality are obtained. Note that there is internal pressure variation, but it is the same across the engine. In addition, it is obtained that the minimum pressure is 66.5 kPa, the average is 101.3 kPa and the maximum is 154.04 kPa.

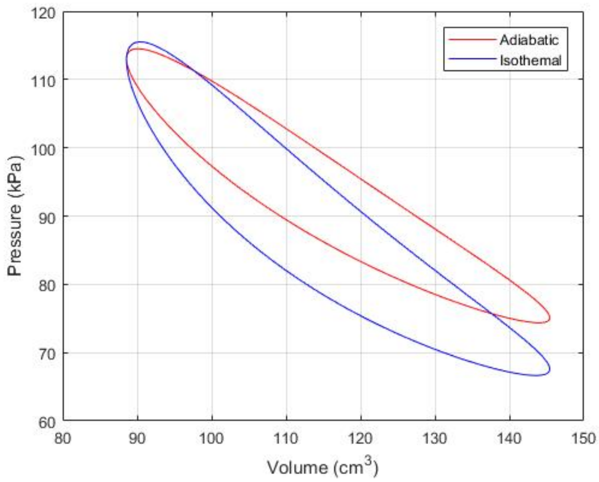

Figure 9. p-V diagram for the isothermal and adiabatic analyzes

As the engine in the isothermal analysis is very idealistic its efficiency is 54.98%, being equal to that of Carnot, however, in the adiabatic model is 50.66%, due to the hypotheses adopted in the model. The models presented consider perfect processes in which there is no heat loss from fluid to medium and machine different from hot and cold sources. The hypotheses that there are no internal pressure differences, there is no loss of heat, the expansion and compression processes are isothermal, during the cycle the dead volume of expansion maintains the temperature of the gas in the expansion and the dead volume of compression maintains the temperature of the gas in compression.

In Table 2 below, the data obtained during the analyzes are presented in order to facilitate the comparison between the results.

Table 2. Results obtained in the isothermal and adiabatic analyzes.

	Isothermal	Adiabatic
Compression work (J)	0.62	0.56
Expansion work (J)	1.38	1.13
Net work (J)	0.76	0.57
Power (W)	25.29	19.08
Thermal Efficiency (%)	54.98	50.66

A parametric study is important for behavioral evaluation of the engine in various operating situations that may be imposed. For the analysis of the variation of the temperature of expansion, one must adopt corresponding temperatures with other works, because the applied model is not able to identify the tempera that can compromise the integrality of the structure. The temperature variation was analyzed between 450 K and 950 K with a 5 K step, as shown in Fig. 10.

The results obtained satisfy the laws of thermodynamics, where it is possible to observe the increase of the output power and the efficiency of the engine due to the temperature gradient, in that the bigger this gradient the greater the work done by the engine, the greater its power.

For the proposed model the efficiency is of 68 W for the isothermal model and 65 W for the adiabatic with expansion temperature of 950 K. The temperature of 950 K is found in several works where solar radiation is used as a hot source of heat. In this system, the Stirling engine can be composed of parabolic dishes that concentrate the solar radiation and transfer energy to an energy system with the objective of producing kinetic energy, by the movement of the pistons and electric energy with the coupling of a generator.

In theory, the reduction of the efficiency of the thermal machines with the increase of the dead volume, is well-known fact, occurs that for the engines Stirling this volume can be raised to ensure that the area of transference of heat is sufficient to attend the process. In the study of the dead volume of expansion was evident that when null the value of power is maximum because the engine takes advantage of the whole cylinder to generate work and with the increase of the volume the power decreases. Figure 11 below shows the results obtained.

The maximum power obtained is approximately 18 W for the adiabatic model, while for the isothermal it is 31 W with a difference of approximately 40%, which again demonstrates the idealization of the isothermal model. However, it is not possible to build an engine where its dead volume is null as this makes it impossible to transfer heat to the working fluid.

Although the model applied in this study does not indicate the maximum volume of expansion, it is sensitive to compression and expansion volume. Thus, following data found in the literature, the ratio between compression and

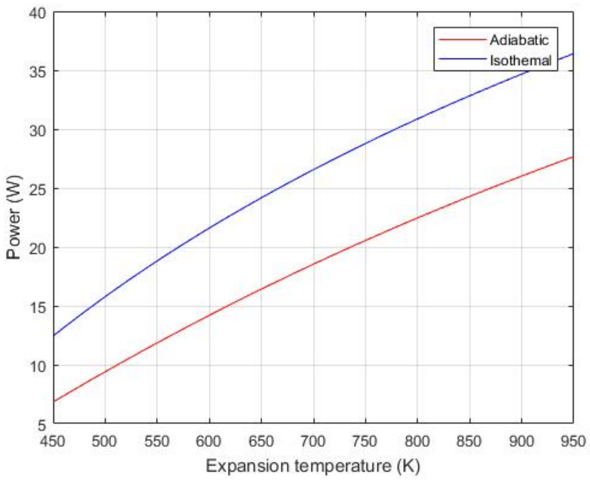

Figure 10. Comparison between models by varying the temperature of the expansion cylinder

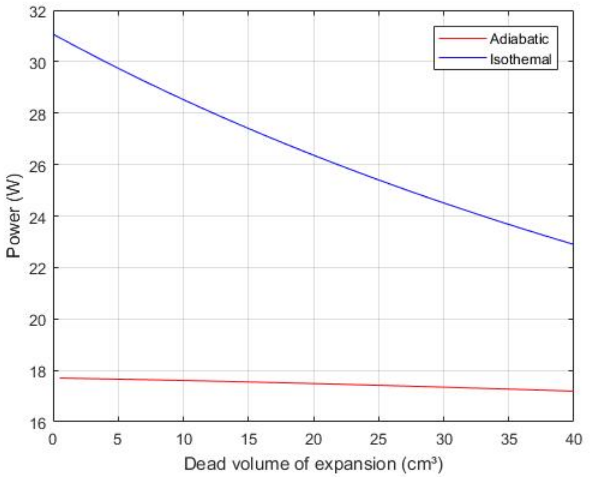

Figure 11. Power obtained from the variation of the dead volume of expansion

expansion volumes was evaluated in the range of 1.6 to 1.9. Data for this analysis are shown in Fig. 12.

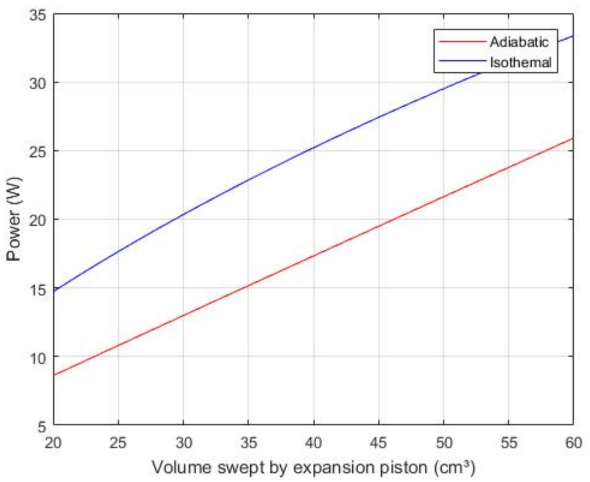

Figure 12. Power obtained from the variation of volume swept by expansion piston

The power gain is evident according to the increase in the volume swept by the expansion piston. This is due to the fact that the engine takes advantage of a larger part of the cylinder to generate work. In this study, the maximum power was obtained for a volume of 60 cm^3 with power of 33 W for the isothermal model and 26 W for the adiabatic model. This volume used represents a 50% increase in the initial engine volume.

All these parametric studies were performed in order to analyze the behavior of the engine in various operating conditions, being of great use in cases of performance losses due to thawing generated by continuous use, ensuring a prediction of what may occur.

4. CONCLUSIONS

Stirling engines have great flexibility in their operation and possible shortages of fossil fuels, studies on this engine are gaining prominence, focusing on solar energy and biomass as renewable energy sources. The development of this work had as study line the engine power considering it isothermal and its parametric study.

The objective of this study is to develop the simulation for the Stirling type alpha type motor comparing the Schmidt analysis based on the isothermal expansion and compression of an ideal gas and an adiabatic system proposed by Finkelstein.

The proposal to analyze the differences between two models is due to the great idealization of the isothermal model, with this, the implementation of the adiabatic model was important to demonstrate how much power and efficiency are reduced. However, both models still have values that discrepant from reality because engines that can be found in the market reach efficiencies around 30%.

The parametric study is used to investigate the effect of operation and geometric parameters on engine performance. Effects on the expansion cylinder are analyzed as: temperature, dead volume and volume swept by piston.

It is remarkable the power and efficiency reduction in the adiabatic model relative to the isothermal around 25%, and 4%, respectively. Comparing the compression jobs, these factors show a 10% difference between the analysis. With this, it is evident the need to implement this model even with the increase of its complexity, since the isothermal model is idealistic.

After the parametric evaluation between the models, it was noticed that the temperature variation of the hot source is governed by the Laws of Thermodynamics and, therefore, for all the increase of temperature there will be an increase of power and the model does not present a condition so that they do not compromise the integrity of the engine. For operating values that are found in literature the engine reaches 68 W.

Although the model does not indicate the maximum volume that can be traversed by the expansion piston, the results obtained show its influence, the analysis presented a 30% increase for the isothermal model and 36% for the adiabatic model. The study of variation of the dead volume of expansion showed that the best operating condition is when its value is zero, but this makes it impossible to transfer heat to the working fluid.

Even when analyzing a small motor and using a still quite idealized model such as the isothermal and the adiabatic model that present variations of expansion volumes and sinusoidal compression, the results obtained were satisfactory and proved useful for the execution of preliminary dimensions as well as evaluation of the main operational parameters.

5. REFERENCES

Borges, A.C.P., Alves, M.S. and Torres, E.A., 2017. "Renewable energy: a contextualization of the biomass as power". *REDE-Revista Eletrônica do PRODEMA*, Vol. 10, pp. 23–36.

Braz, C.A., Rodrigues, R.L. and Siqueira, H.V., 2017. "Geração de energia elétrica por meio de fontes de energias renováveis: uma revisão sistemática da literatura sobre energia eólica". *Gestão Industria*, Vol. 13, pp. 228–242.

Brunetti, F., 2013. *Motores de combustão interna*. Edgard Blücher, Brazil, 1st edition.

Cachute, L., 2006. *Análise Teórica e apresentação de Metodologia de Projeto de motor Stirling para uso em sistema de Resfriamento Evaporativo*. Master's thesis, Federal University of Taubaté, Taubaté.

Çengel, Y.A. and Cimbala, J.M., 2015. *Mecânica dos fluidos - Fundamentos e aplicações*. AMGH, Brazil, 3rd edition.

Chan, J., Hoye, T., Schulze, E. and Wiser, J., 2008. "Advanced stirling radioisotope generator: design overview and future plans". *AIP Conference Proceedings*, Vol. 969, p. 458.

da Cruz, V.G., 2012. *Desenvolvimento experimental de um motor stirling tipo gama*. Master's thesis, Federal University of Paraiba, João Pessoa.

da Rosa, A.V., 2015. *Processos de energias renováveis*. Elsevier, Brazil, 3rd edition.

Dias, L.V.R., 2016. *Modelagem e análise experimental de um protótipo didático de motor stirling*. Master's thesis, Federal University of Goias, Goiânia.

Ernest, W. and Shaltens, R., 1997. *Automotive Stirling Engine Development Project*. National Aeronautics And Space Administration - NASA, Cleveland, 1st edition.

Gingery, D.J., 1990. *Build a Two Cylinder Stirling Cycle Engine*. LLC, 1st edition.

Hillig, T., 2016. "A hybrid solution with concentrated solar power (csp) and fuel for baseload mining operations". *Ripasso Energy THEnergy study*.

Hinrichs, R.A., Kleinbach, M. and Reis, L.B., 2010. *Energia e Meio Ambiente*. Cengage Learning, Brazil, 4th edition.

Hirata, K., 1997. "Schmidt theory for stirling engines". *National Maritime Research Institute*.

Kitzner, E., 1977. *Automotive Stirling Engine Development Program*. National Aeronautics And Space Administration - NASA, Cleveland.

Lloyd, C., 2009. *A Low Temperature Differential Stirling Engine for Power Generation*. Master's thesis, University of Canterbury, Christchurch.

Sant'anna, B. and Yano, D., 2012. "Projero e simulação de um motor stirling". Monography, Federal Technological University of Paraná (UTFPR), Curitiba, Brazil.

Sepulveda, S., Wilkinson, J., Tiburcio, B. and Herrera, S., 2008. *Série desenvolvimento rural sustentável*. Brasília, Brazil, 1st edition.

Stine, W., 2001. "Power from the sun". 17 Apr. 2018 <http://www.powerfromthesun.net/book.html>.

Thombare, D.G. and Verma, S.K., 2008. "Technological development in the stirling cycle engines". *Renewable and Sustainable Energy Reviews*, Vol. 12, pp. 1–38.

Tlili, I., Timoumi, Y. and Nasrallah, S.B., 2008. "Analysis and design consideration of mean temperature differential stirling engine for solar appication". *Renewable Energy*, Vol. 33, pp. 1911–1921.

Urieli, I. and Berchowitz, D.M., 1984. *Stirling cycle engine analysis*. ISBN 0-85274-435-8 Intl Public Service, 1st edition.

Velázquez, D.O., 2007. *Estudio Teórico del Regenerador para un Motor Stirling*. Master's thesis, National Polytechnic Institute, Mexico D. F.

Walker, G., 1980. *Stirling Engines*. Nova Iorque: Oxford University Press, 1st edition.

Wilke, H. and Lora, E., 1977. "Desenvolvimento de um módulo combustor biomassa - motor stirling - aplicado a sistemas de geração isolada e baseados em gerador de indução". In *5th Encontro de Energia no Meio Rural*. Campinas, Brazil.

Ziabasharhagh, M. and Mahmoodi, M., 2012. "Numerical solution of beta-type stirling engine by optimizing heat regenerator for increasing output power and efficiency". *J Basic Appl Sci Res*, Vol. 22, pp. 1395–1406.