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MIXING LAYER STABILITY ANALYSIS WITH STRONG TEMPERATURE GRADIENTS

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Abstract.
The stability of compressible flow relevant to aeronautical applications are investigated regarding the effect of large temperature gradients. The temperature gradients when large are responsible for strong variations in gas properties such as density, viscosity and conductivity. These variations change the characteristics of the flow both in terms of its stability parameters such as growth rate and wave speed and in terms of the flow topology and vorticity distribution. Direct numerical simulations and linear stability theory are used as tools for the intended study. Results show the stability characteristics of a chosen reference flow configuration against which the large temperature gradient cases are compared. Vorticity, pressure distribution and normal velocity components are presented for a range of Reynolds numbers for the reference flow configuration. The results show the change in growth rate, wavenumbers and wave speeds for the given changes in temperature of the lower mixing layer stream.

Keywords: *Kelvin-Helmhotz instability, compressible mixing layer, Direct Numerical Simulation*

1. INTRODUCTION

The use of model problems for parametric investigation of different flow conditions related to aerospace engineering problems allows fast and reliable analysis. The compressible mixing layer serves as a model problem for the analysis of high speed air breathing propulsion problems such as reactant mixing in a combustion chamber or noise generation in exhaust nozzles. In both cases two parallel streams at different velocities may be composed of different chemical species or with large temperature differences. In such cases properties variation may result in significant differences in flow stability characteristics such as growth rates and flow topology. The present investigation addresses the question of how temperature gradients affects the development of Kelvin-Helmholtz instability. The study is part of an on going project on stability and acoustic of compressible flows relevant to propulsive systems.

The research on the stability of binary mixing layers at the Instituto de Aeronáutica e Espaço (IAE) started in 2006 with the Masters' thesis of Salemi (2006) (Salemi and Mendonca, 2008) and Quirino (2006). The first used hydrodynamic stability theory to study binary mixing layers in compressible flow where the base flow was given by the similarity solution of the boundary layer equations. The second one used direct numerical simulation methods to solve the compressible Naver-Stokes equations to study the effect of strong heat sources on the stability of the mixing layer.

After these initial studies other works where conducted on more complex mixing layers either double mixing layers or mixing layers modified by jets and wakes (Mendonça, 2010; Souza, 2011; Souza *et al.*, 2014; Mendonca, 2014; Soares *et al.*, 2014; Fernandes *et al.*, 2014; Freitas *et al.*, 2014; Manco, 2014; Manco and Mendonca, 2014). Considering a canonical base flow given by a combination hyperbolic tangent and hyperbolic secant profiles Mendonça (2010) and Soares *et al.* (2014) studied different aspects of the influence of jets and wakes on the stability of compressible and incompressible mixing layers. Using direct numerical simulations Manco (2014); Manco and Mendonca (2014) are studying mixing layer stability looking into the nonlinear effects and the topological development of the resulting flow due to the interaction of mixing layers, jets and wakes.

Since the binary mixing layer velocity, temperature and mass fraction profiles are very different from the canonical profiles given by hyperbolic tangent and secant profiles, Mendonca (2014); Fernandes *et al.* (2014); Freitas *et al.* (2014) solved the boundary parabolic layer equations for a compressible binary flow in order to arrive at the base flow profiles. Given the resulting base flow profiles the stability analysis were performed using the compressible form of the Rayleigh equation given by the Gropengiesser transformation of variables (Gropengiesser, 1970; Salemi, 2006).

The present investigation extend these previous work using some of the methodologies developed by the group for the study of more complex mixing layer configurations.

2. METHODOLOGY

The study is performed through numerical simulation of a compressible mixing layer flow. The governing equations are the Navier-Stokes equations for compressible, inert flow in two dimensions, without body forces, heat sources and radiation. The numerical methodology is based on high order, low dissipation and low dispersion schemes.

2.1 Governing Equations

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix}, \qquad E = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho uv - \tau_{xy} \\ \rho eu + q_x - u\tau_{xx} - v\tau_{xy} \end{bmatrix}, \qquad F = \begin{bmatrix} \rho v \\ \rho uv - \tau_{xy} \\ \rho v^2 + p - \tau_{yy} \\ \rho ev + q_y - u\tau_{xy} - v\tau_{yy} \end{bmatrix} \tag{1}$$

Thus, the set of equations becomes:

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0. \tag{2}$$

Where the viscous stresses are related to the strain tensor linearly

$$\tau_{ij} = -\frac{2}{3}\mu\frac{\partial u_k}{\partial x_k} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{3}$$

These equations are non-dimensionalized using the upper stream variables as reference conditions and the mixing layer vorticity thickness as the reference length, such that

$$Re = \frac{\rho_1 U_1 \delta_w}{\mu_1}, \qquad Pr = \frac{\mu_1 cp_1}{k_1}, \qquad Ma = \frac{U_1}{a_1}. \tag{4}$$

Where, ρ is the density, U_1 is the fast stream velocity, μ_1 is the fast stream viscosity coefficient, c_p is the specific heat at constant pressure, k is the conductivity and a is the speed of sound. Subscript 1 refers to the fast stream.

2.2 Numerical Methodology

In order to properly capture disturbance evolutions associated with hydrodynamic stability problems, the numerical method must be of high order, have low dissipation and low dispersion properties. Therefore, for spatial and temporal discretization the following schemes are available in the code.

Spatial Schemes

- 4th order Central Finite Difference.
- 6th order Compact Finite Difference (Lele, 1992).
- 4th order Dispersion Relation Preserving Finite Difference (Tam and Webb, 1993).

Temporal Schemes

- 4th order, 4 steps Runge Kutta.
- 4th order, Low Dissipation and Low Dispersion Runge-Kutta (Hu *et al.*, 1996).
- 4th order, Low storage six stage Runge Kutta for non-linear operators (Berland *et al.*, 2006).

Non-reflecting boundary conditions are also applied in order to avoid noise generated at the boundaries from reaching the domain of interest. More details about the numerical schemes and the boundary condition treatment may be found in Manco (2014).

3. RESULTS AND DISCUSSIONS

3.1 Code Verification

The present viscous Navier-Stokes code was developed from an Euler code developed by Manco (2014). In order to verify the implemented numerical scheme comparisons are presented for a mixing layer where the upper stream has Mach number equal to $Ma = 0.8$ and the lower scheme has $Ma = 0.2$. Figure 1, taken from Manco and Mendonca (2018), show comparisons between amplification rate obtained with a inviscid linear stability analysis code and Euler simulations for a range of frequencies. The growth rate α_i is based on the evolution of the kinetic energy along the streamwise direction.

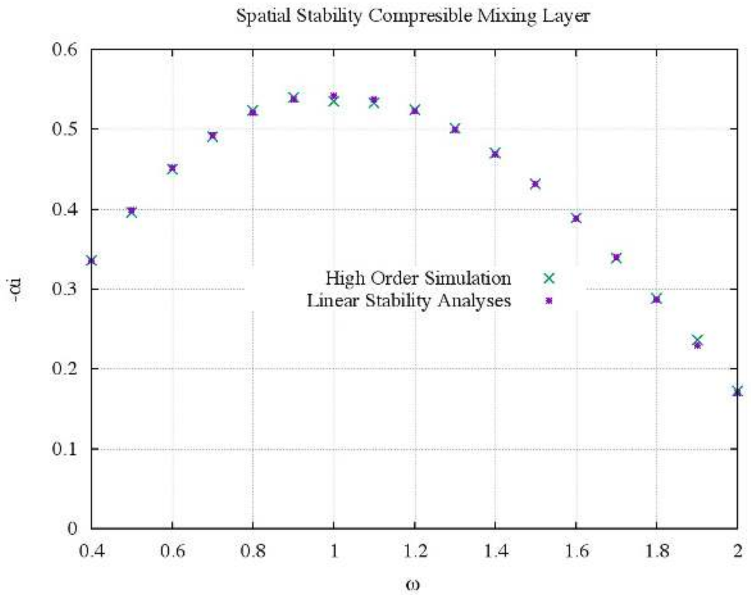


Figure 1. Growth rate versus frequency ω, comparison between linear stability analysis and Euler numerical simulation.

3.2 Reference Case

The effect of temperature gradients will be investigated comparing results with a low temperature gradient reference case. Results are presented in terms of flow topology, given by the vorticity, normal velocity and pressure distribution.

3.2.1 Base flow parameters

The base flow is given by a canonical laminar state defined by the distribution of density, non-dimensional streamwise velocity, and pressure. The flow is assumed parallel and the velocity distribution follows a hyperbolic tangent profile.

$$U(y) = \frac{1}{2}\left[(U_1 + U_2) + (U_1 - U_2)\tanh\left(\frac{2u}{\delta}\right)\right] \tag{5}$$

The fast stream has velocity $U_1 = 0.8$ and temperature $T_1 = 1$, while at the slow stream $U_2 = 0.2$ and $T_2 = 0.8$.

The laminar temperature distribution $T(y)$ is given by the Crocco-Busemann relation and the corresponding non-dimensional density is $1/T(y)$

3.2.2 Reference flow results

Figures 2 through 4 show the evolution of a mixing layer with a mild temperature gradient with $T_1 = 1$ and $T_2 = 0.8$. The vorticity distributions for the Euler solution and three Reynolds number viscous solutions are presented in Fig. 2. The Euler solution, although somewhat noisy show a stronger growth than the viscous solutions. One expected result from the viscous simulation is the attenuation of noise radiated from the boundaries. The lowest Reynolds number solution show a strong stabilization of the Kelvin-Helmholtz structures due to viscous effects. Figure 3 show the pressure distribution of this same test case out of which the same conclusions are drawn, but the solution is not as noise as the vorticity, which involve computations of gradients of the velocity field. Finally, the normal velocity field is presented in Fig. 4, where the instability attenuation due to viscous effects are again noticeable.

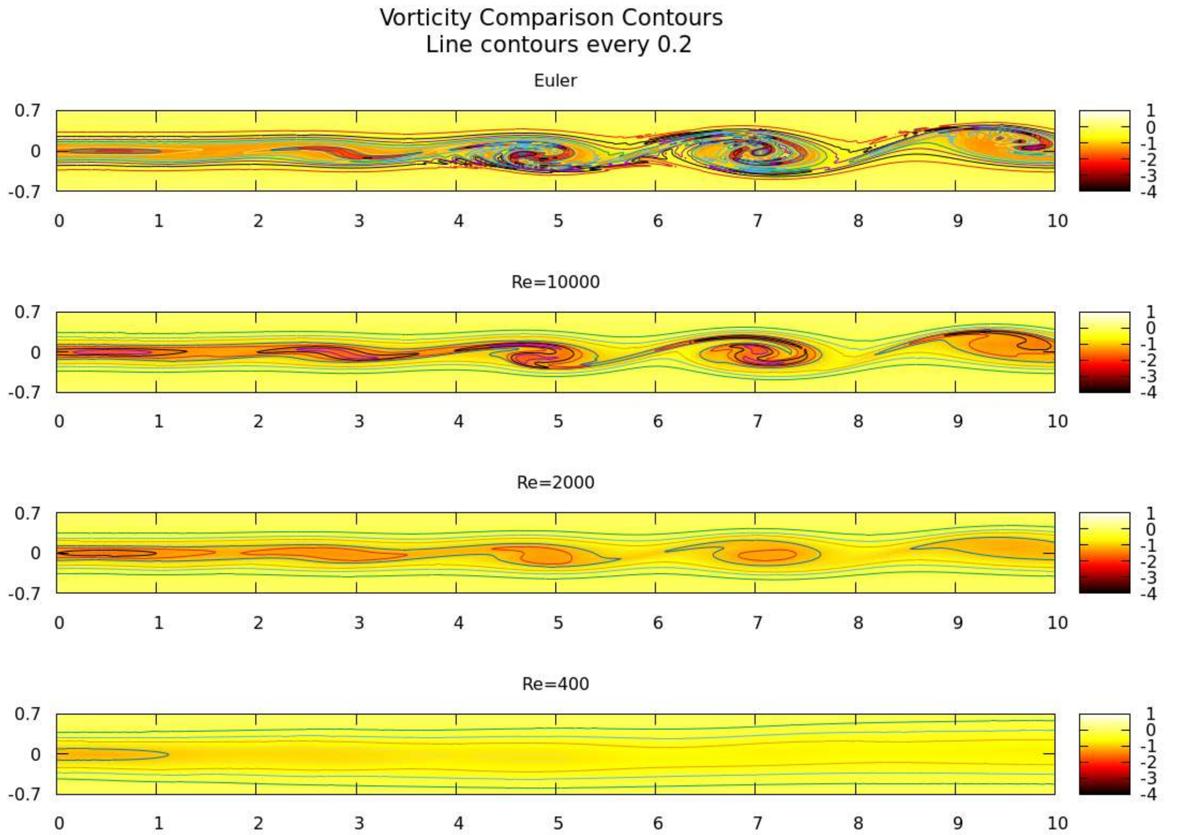


Figure 2. reference flow vorticity distribution

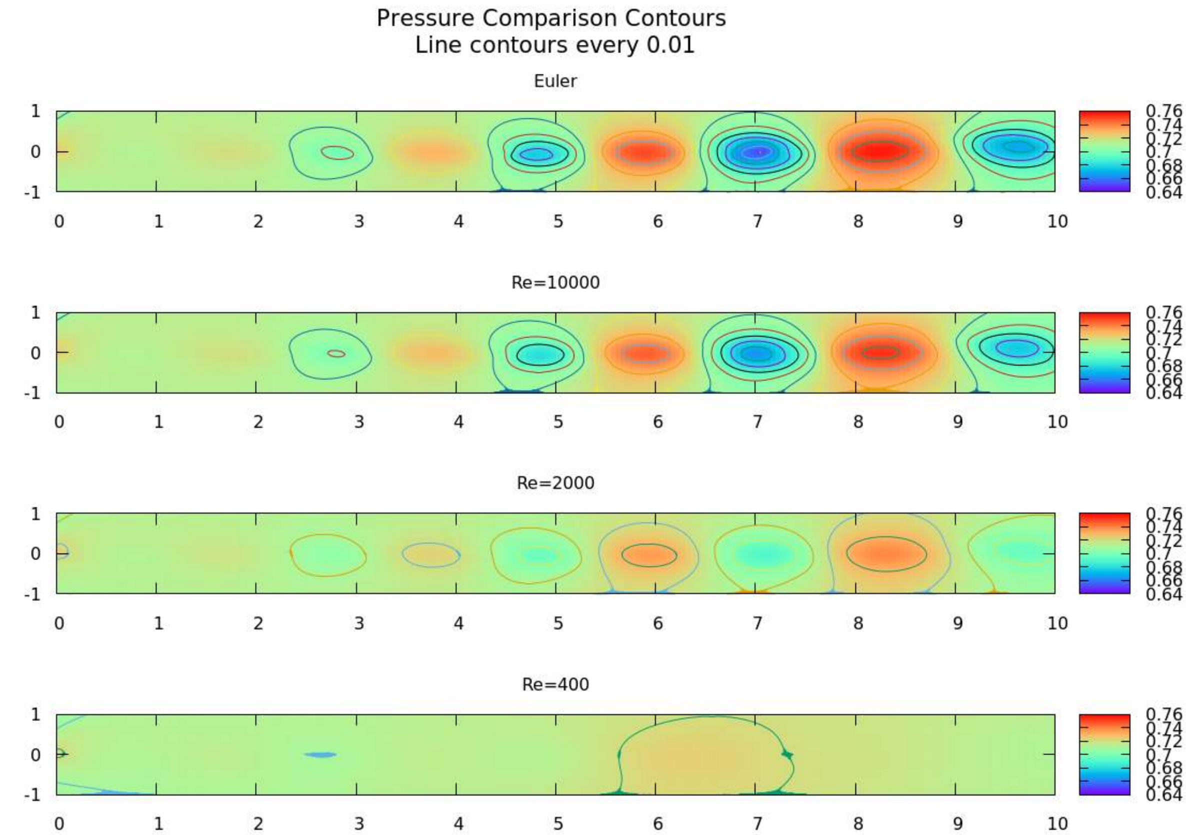


Figure 3. reference flow pressure distribution

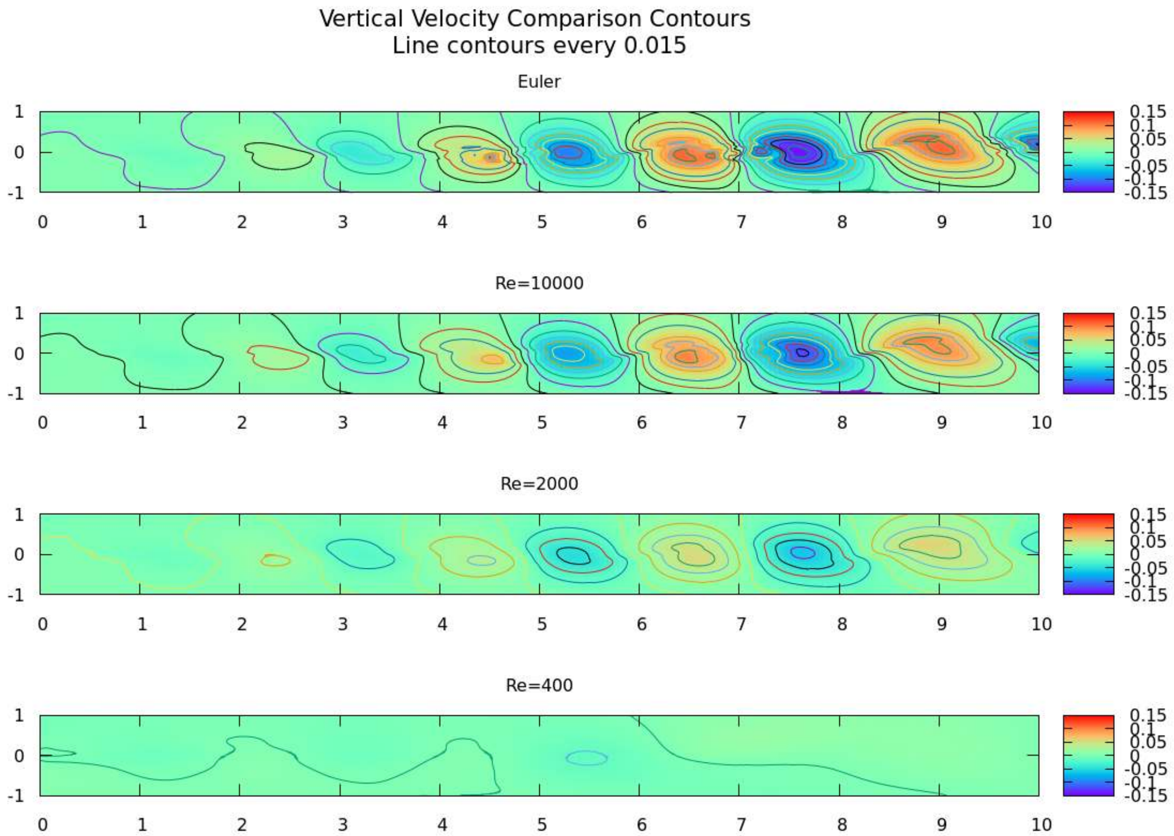
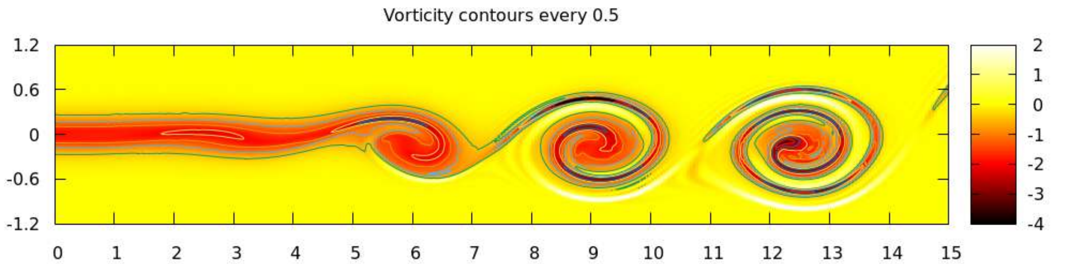


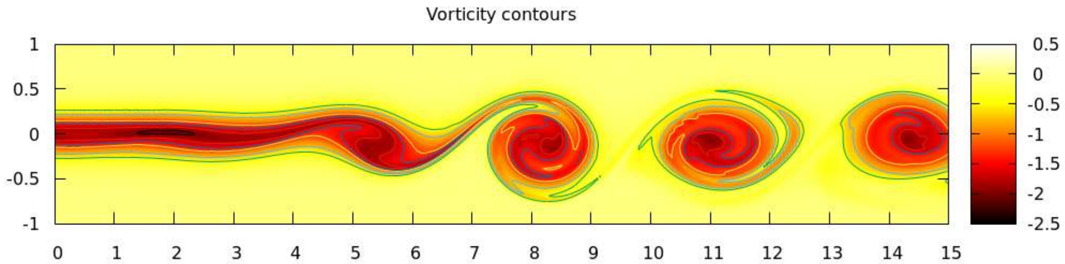
Figure 4. reference flow normal velocity component distribution

3.3 Strong Temperature Gradient Cases

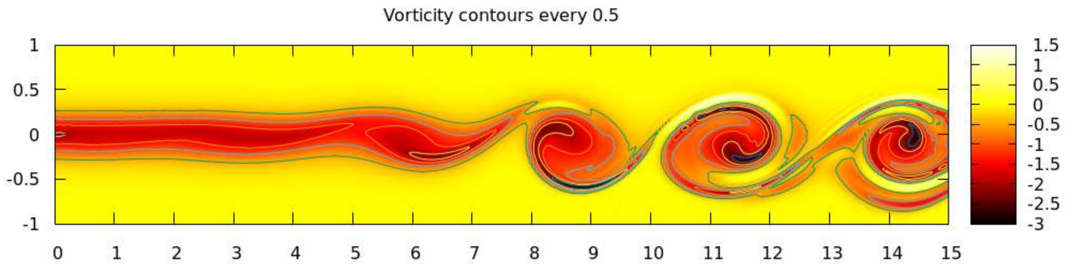
The effect of temperature gradients is presented in this section. The upper stream nondimensional temperature is kept $T_1 = 1$, while the lower stream temperature is fixed at $T_2 = 1$, $T_2 = 0.5$ and $T_2 = 2$. The frequency of each case was selected to correspond to the highest growth rate frequency obtained from a linear stability analysis. For $T_2 = 0.5$ the frequency is $\omega = 0.8$, for $T_2 = 1$, $\omega = 1$ and for $T_2 = 2$, $\omega = 1.2$. The results are shown in Figs. 5 through 7 in terms of vorticity distribution pressure and normal velocity component. According to Fig. 5, the topology of the flow structure is not significantly affected, but the disturbance growth is clearly stronger for the $T_2 = 0.5$ case, while for $T_2 = 2$ the disturbance is clearly the weakest, as seen in Fig. 6 and 7. This results are consistent with the linear stability analysis presented in the next section. The wavenumber change due to changes in T_2 is also consistent with the linear stability results, where one has to observe that each case correspond to a different frequency.



$T_1 = 1$ and $T_2 = 0.5$, $\omega = 0.8$.



$T_1 = 1$ and $T_2 = 1$, $\omega = 1$.

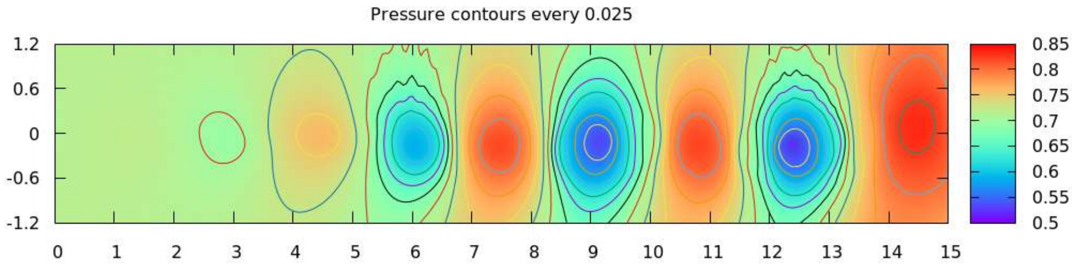


$T_1 = 1$ and $T_2 = 2$, $\omega = 1.2$.

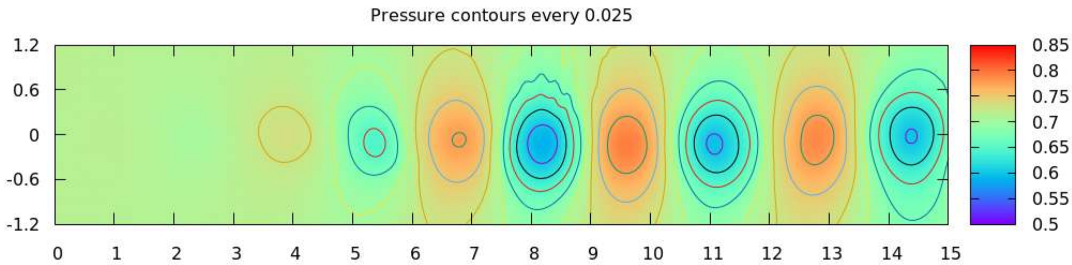
Figure 5. Vorticity Distribution.

3.4 Linear Stability Theory Analysis

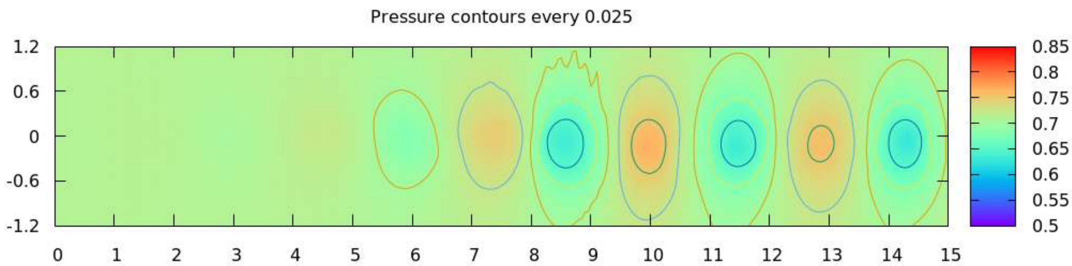
Figure 8 show comparisons of growth rate, phase speed and wavenumber for lower stream temperature T_2 from 0.1 to 10. The results show that reducing the lower stream temperature the range of unstable frequencies is reduced. The fastest growing mode for $T_2 = 0.5$ is a little larger than that for $T_2 = 1$ isothermal case, but as the slow stream temperature is decreased the flow becomes more stable, both in terms of the highest amplification rate and in terms of the range of unstable frequencies. When the slow stream temperature is higher than the fast stream, the mixing layer becomes more stable, but the range of unstable frequencies increase with increasing T_2. The phase speed increases continuously as the lower stream temperature is increased while the wavenumber reduces monotonically. The wavenumber observed in



$T_1 = 1$ and $T_2 = 0.5$, $\omega = 0.8$.



$T_1 = 1$ and $T_2 = 1$, $\omega = 1$.



$T_1 = 1$ and $T_2 = 2$, $\omega = 1.2$.

Figure 6. Pressure distribution.

the DNS solutions are compatible with this results, considering that each case was run at a different frequency, the one corresponding to the highest amplification rate. As T_2 is increased, the frequency considered is reduced, resulting in a lower wavenumber for the lower T_2 and a higher wavenumber for the higher T_2. Nevertheless the dispersion characteristic, with a relatively constant phase speed with frequency, does not change with T_2, where the results show that the phase speed levels off for higher frequencies.

4. CONCLUSIONS

Direct numerical simulation of mixing layers with large temperature gradients have been performed using a high order, low dissipation and low dispersion numerical method. The results show small changes in the flow topology in terms of the vorticity distribution. The results are consistent with linear stability theory results that show the change in amplification rate, wavenumber and wave speed with respect to the isothermal problem. This investigation will be extended to consider two different chemical species on the upper and lower streams as a preliminary study prior to the study of mixing layer with chemical reaction.

5. ACKNOWLEDGEMENTS

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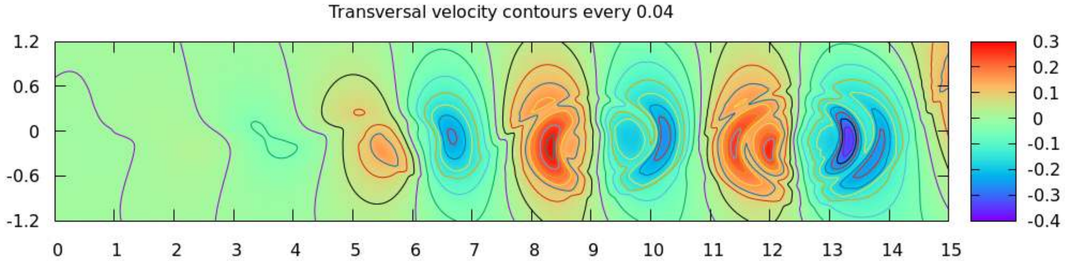
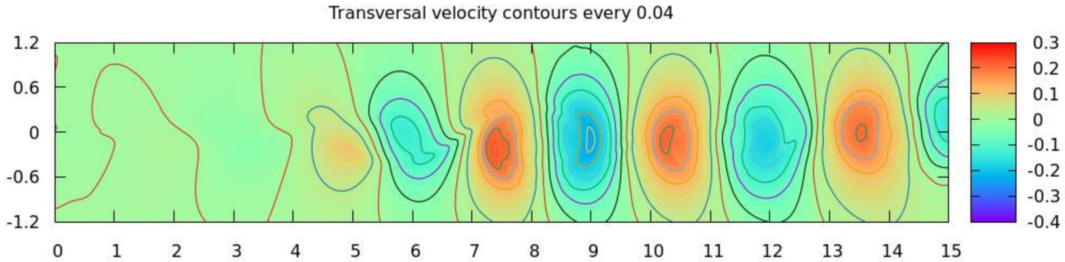
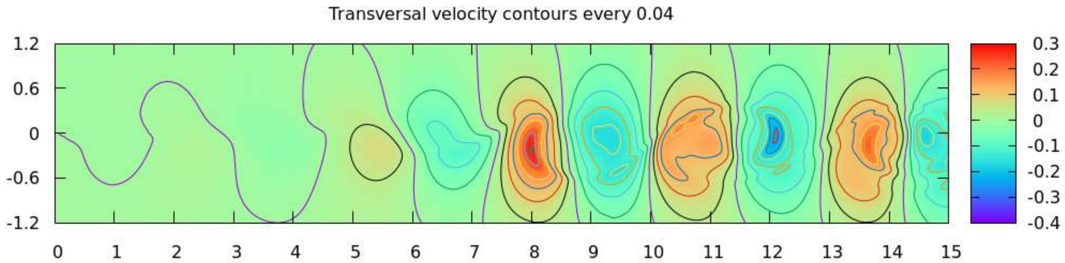


$T_1 = 1$ and $T_2 = 0.5, \omega = 0.8.$



$T_1 = 1$ and $T_2 = 1, \omega = 1.$



$T_1 = 1$ and $T_2 = 2, \omega = 1.2.$

Figure 7. Normal velocity component.

6. REFERENCES

Berland, J., Bogey, C. and Bailly, C., 2006. “Low-dissipation and low-dispersion fourth-order runge–kutta algorithm”. *Computers & Fluids*, Vol. 35, No. 10, pp. 1459–1463.

Fernandes, L.M., Freitas, R.B. and Mendonça, M.T., 2014. “Stability of o2/h2 binary mixing layers: Effect of temperature gradients, compressibility and three-dimensionality”. In *IX Escola de Primavera de Transição e Turbulência*. São Leopoldo, RS, pp. 1–10.

Freitas, R.B., Fernandes, L.M. and Mendonca, M.T., 2014. “Three-dimensional disturbances on binary mixing layers modified by jets and wakes”. In *IUTAM Symp. on Laminar-Turbulent Transition*. Rio de Janeiro, Brazil. Accepted for presentation.

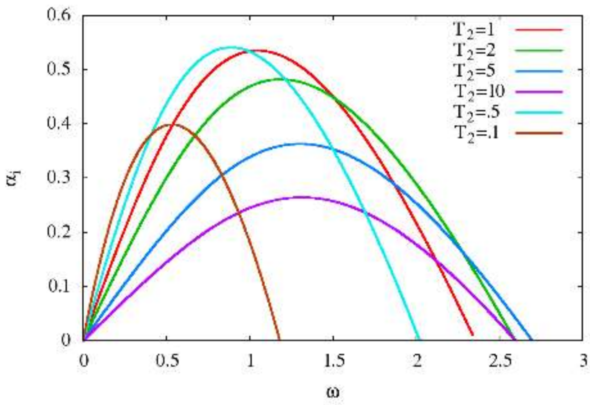
Gropengiesser, H., 1970. “Study on the stability of boundary layers and compressible fluids”. Technical Report Report TT F-12786, NASA.

Hu, F.Q., Hussaini, M.Y. and Manthey, J.L., 1996. “Low-dissipation and low-dispersion runge–kutta schemes for computational acoustics”. *Journal of Computational Physics*, Vol. 124, No. 1, pp. 177–191.

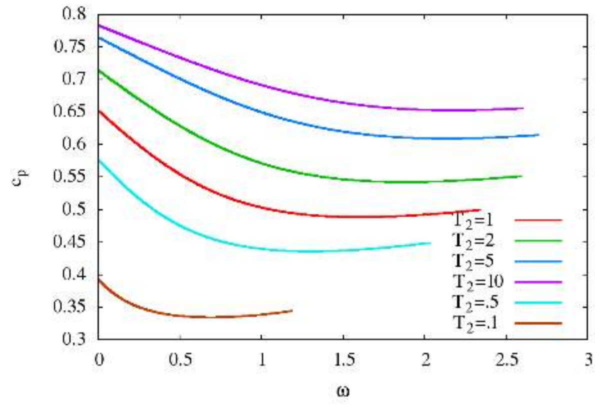
Lele, S.K., 1992. “Compact finite difference schemes with spectral-like resolution”. *Journal of computational physics*, Vol. 103, No. 1, pp. 16–42.

Manco, J.A.A., 2014. *Non-reflecting boundary conditions for high order numerical simulation of compressible Kelvin-Helmholtz instability*. Master’s degree in Spatial Engineering and Technology, Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos. INPE-17465-TDI/2256.

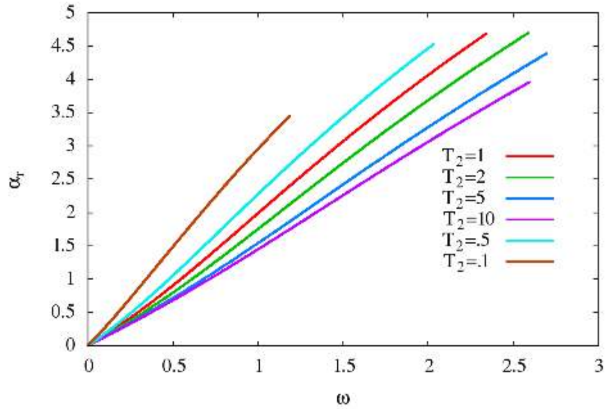
Manco, J.A.A. and Mendonca, M.T., 2018. “Comparative study of different non-reflecting boundary conditions for compressible flows”. *Computer and Fluids, Submitted for Publication*, Vol. 0, p. 0.



Amplification rate α_i.



Phase speed c_p.



Wave number α_r.

Figure 8. Linear Stability Theory results. Effect of Temperature gradient. $T_1 = 1$ through $T_2 = 0.1$ to 10.

Manco, J.A.A. and Mendonca, M.T., 2014. "Direct numerical simulation of binary free shear layer hydrodynamic stability". In *IUTAM Symp. on Laminar-Turbulent Transition*. Rio de Janeiro, Brazil. Accepted for presentation.

Mendonça, M.T., 2010. "Cooled gas turbine blade trailing edge flow analysis". In *13th Brazilian Congress of Engineering and Thermal Sciences - ENCIT 2010*. Uberlândia - MG, Brazil, pp. 1–9.

Mendonca, M.T., 2014. "Linear stability analysis of binary compressible mixing layers modified by a jet or a wake deficit". In *52nd AIAA Aerospace Sciences Meeting and Exhibit*. National Harbor, Maryland, pp. 1–13.

Quirino, S.F., 2006. *Simulação numérica direta de camada cisalhante compressível com fonte de calor*. Dissertação (mestrado em engenharia e tecnologia espaciais), Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos. (INPE-14661-TDI/1217.

Salemi, L. and Mendonca, M.T., 2008. "Spatial and temporal linear stability analysis of binary compressible shear layer". In *AIAA 38th Fluid Dynamics Conference*. Seattle, USA, AIAA paper 2008-3841, pp. 1–23.

Salemi, L.C., 2006. *Análise de Estabilidade Linear de Camada de Mistura Compressuvel Binária*. Dissertação (mestrado em engenharia e tecnologia espaciais), Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos.

Soares, M.S., Filho, F.F. and Mendonça, M.T., 2014. "Effect of thickness ratio on the stability of mixing layers with a wake component". In *IX Escola de Primavera de Transição e Turbulência*. São Leopoldo, RS, pp. 1–7.

Souza, O.M.R., Alves, L.S.B. and Mendonca, M.T., 2014. "Double mixing layer linear stability analysis". In *IX Escola de Primavera de Transição e Turbulência*. São Leopoldo, RS, pp. 1–8.

Souza, O.M.R., 2011. *Análise de Instabilidades Lineares em Camadas de Mistura Planas Duplas e Compressıeis*. Dissertação, Instituto Militar de Engenharia - IME, Rio de Janeiro.

Tam, C.K.W. and Webb, J.C., 1993. "Dispersion-relation-preserving schemes for computational acoustics". *Journal of Computational Physics*, Vol. 107, No. 184, pp. 262–281.

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