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A MODEL FOR UNSTEADY TURBULENT FRICTION IN PIPE FLOWS

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Abstract. *This work presents a one-dimensional model for frequency-dependent friction for turbulent flows in deformable pipes. This model is derived by employing a virtual structured mixture model for the fluid, which is formed by n constituents obeying the same equations of state. The identification of the constitutive constants of the friction model forms a crucial part of the work and are easily calculated in a thermo-mechanical consistent basis. A comparison with experimental data has shown good agreement what upholds this model to real-time applications of any type of Newtonian fluid transients in enclosed pipes.*

Keywords: *unsteady friction, fluid transients in pipes, turbulent flow.*

1. INTRODUCTION

Rapid closures of valves due to planned or unexpected events cause changes in the spatial and temporal fields of velocity and pressure in pipeline systems. The frequency-dependent friction has a remarkable role to describe all these alterations. Such perturbations are important to designing safe pipeline structures, leakage detection techniques, and water quality management. (Guidaoui, *et al.,* 2005).

Due to the importance of the unsteady friction to transient hydraulic analysis, different models of transient friction have been developed in the last century. The most elementary approach and yet most used model in commercial software packages is the quasi-steady onedimensional friction model. In this model, the Darcy-Weisbach factor is used to describe friction in transient pipe flows. However, several experiments have shown that this assumption was inaccurate since it predicted wrongly the velocity and pressure profiles in phase and magnitude. Such models overestimated the pressure peaks which in turn caused over-dimensioned projects of the pipes and therefore higher costs.

In the late sixties, Zilke (1968) theoretically derived an expression for the one-dimensional unsteady friction in laminar flow. This work basically put an end to this matter in laminar flows. On the contrary, the first reasonable and theoretically consistent attempt to model unsteady friction in turbulent flows was recorded by Vardy and Brown (1995).

Basically, there are two main groups of models for turbulent flows, the Convolution Integral Models, which follows the Zielke's approach, and the Instantaneous Acceleration Models. As the principal authors of the first group, the continuing works of Vardy and Brown (2003, 2004, 2007) and Zarzycki (Zarzycki, 1994; Zarzycki and Urbanowicz, 2012) can be highlighted. Both have attained an excellent agreement with experimental data. In the second group, it is assumed that the friction is dependent on the instantaneous acceleration of the fluid (Brunone, *et al.,*1991), and in contrast with the first group, the expression is basically derived from experimental and empirical data.

Two-dimensional models have also been developed (Pezzinga, 1999 and Araya and Chaudry, 1997) but their application is still limited, since, in industry, real-time calculations are extensively requested. Besides, it is hard to properly treat complex two-dimensional boundary conditions in pipe flows (Brunone, *et al.,* 2004).

Although these models have attained good results in some situations, the second group is far from being general, violating the restrictions imposed by the Second Law of Thermodynamics (Vitkovsky, *et al.,* 2006). On the other hand, the first group has quite complex procedures to find the material constants of the model.

This work presents a one-dimensional thermodynamically consistent unsteady friction model for turbulent flows derived from the context of the continuum mixture theory. This approach follows the strategy used to derive an unsteady laminar friction model in pipe flows (Cunha and Rachid, 2005).

2. METHODOLOGY

Usually, the length of pipes conveying fluids is far greater than its internal diameters, so the flow can be described as one-dimensional with almost no lack of generality as demonstrated with the good results in the former one-dimensional models. With that, a one-dimensional unsteady friction is derived based on the mixture theory governing equations (Drew and Passman, 1998). The mixture, in this context, means that all the n constituents coexist at every material point at any instant of time t. Each constituent has its own independent kinematics along the pipe centerline, whose the length is L.

The governing equations of mass for a constituent; momentum for a constituent and mixture as a whole are respectively (Drew and Passman, 1998):

$$\frac{\partial}{\partial t}\left(\rho_j A\right) + \frac{\partial}{\partial x}\left(\rho_j A v_j\right) = 0; \tag{1}$$

$$\rho_j \frac{D^j v_j}{Dt} = \frac{\partial t_j}{\partial x} + b_j - a_j - m_j ; \tag{2}$$

$$\sum_1^n m_j = 0. \tag{3}$$

When applied to each constituent, the Second Law of Thermodynamics (SLT) takes the following form:

$$-\rho_j \frac{d\Psi}{dt} + t_j^I \frac{\partial v_j}{\partial x} - \frac{t_j^R}{\rho_j}\frac{d\rho_j}{dt} + a_j v_j \geq 0; \tag{4}$$

In these equations, $v_j(x,t)$ and $\rho_j(x,t)$ are the velocity and density of each component; m_j is an internal interaction force per unit of cross-sectional area exerted by the other constituents on the *j-th* constituent; t_j^I and t_j^R are the irreversible and reversible surface forces per unit of the cross-section area; Ψ is the Helmholtz free energy per unit mass; b_j stands for body forces per unit of the cross-sectional area; and a_j is the reactive contact friction force per unit lateral area that acts on the pipewall-fluid interface, all along the *x*-direction.

2.1 Structured Virtual Model

The governing equations described above are quite general to an arbitrary mixture flowing inside a pipe. To narrow our continuum analysis to further application, a description of the fluid and pipe characteristics, as well as constitutive equations for the interactive forces among the constituents m_j and reactive force at the wall a_j must be derived.

Along the usual description of these four matters, the mixture has also a form of structured concentric cylindrical shells, each of them representing one constituent, which occupies a volumetric fraction β_j. Assuming that R^0 set the intern radius of non-deformed pipe and the thickness of each constituent has the same value of R^0/n , a sketch of the virtual structure mixture takes the form illustrated in Figure 1.

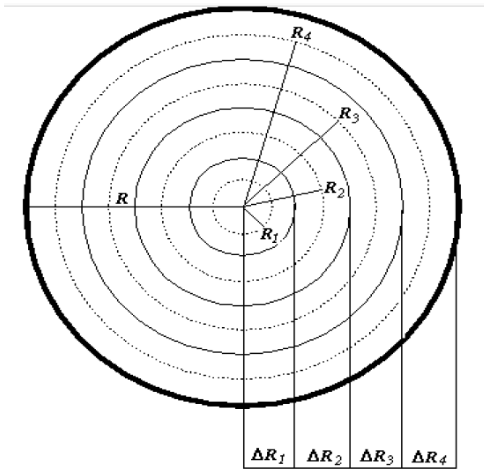

Figure 1. Virtual structured mixed model.

The momentum interaction force between adjacent constituents is scaled by the difference of the velocity of these constituents, having as unknowns the momentum material constants $C_{j,\,j+1}$ and $C_{j,\,j-1}$. As a result, this interaction is described as

$$m_j = C_{j,\,j-1}(v_j - v_{j-1}) + C_{j,\,j+1}(v_j - v_{j+1}) \tag{5}$$

, principle of action-and-reaction it comes out that $C_{j,\,j+1}=C_{j+1,j}$. Analyzing the nature of the model, it is easy to realize that $C_{0,\,1}\equiv 0,\ C_{n,\,n+1}\equiv 0$. In a similar fashion, the reactive friction force can be defined as

$$a_j=\begin{cases} 0, & for\ j=1,\dots,n-1 \\ Cv_n, & for\ j=n \end{cases} \tag{6}$$

in which C stands for the reactive material constant.

Disregarding the gravitational effects and with the assumption of low Mach number flows of a fluid slightly compressible inside an elastic pipe subjected to small deformation, it is obtained a hyperbolic system of equations of motion for the proposed virtual structure mixture.

$$\frac{1}{a^2}\frac{\partial p}{\partial t}+\frac{\partial v}{\partial t}=0 \tag{7}$$

$$\rho_0\frac{\partial v}{\partial t}+\frac{\partial p}{\partial x}+\frac{4}{D^0}\sum_{j=1}^{n} a_j=0 \tag{8}$$

$$\beta_j\rho_j\frac{\partial v_j}{\partial t}+\beta_j\frac{\partial p}{\partial x}+m_j+\frac{4}{D^0}a_j=0\ ,\ \text{for}\ j=2,\ \dots\ n \tag{9}$$

in which a is the wave velocity in the mixture:

$$a=\sqrt{\frac{\frac{K}{\rho_0}}{1+2K\psi/Ee}} \tag{10}$$

where E is the material Young modulus's; e is the pipe wall thickness; ψ represents a term that is dependent on the type of anchorage of the pipe (Guidaoui, 2005); ρ_0 represents the density of the fluid at the reference atmospheric condition; D^0 stands for the internal diameter of the pipe in the non-deformable configuration; and K is the isentropic bulk modulus of the fluid. Furthermore, for a complete flow model description, the second law of thermodynamics for isothermal conditions can be written for the mixture as a whole as (Drew and Passman, 1998),

$$\sum_{j=1}^{n} m_j v_j\ \geq\ 0, \tag{11}$$

and for each constituent as,

$$\frac{4}{D^0}a_j v_j\geq 0. \tag{12}$$

2.2 Determination of the Material Constants

The material constants are analytically determined by imposing that the motion equations in the steady state must resemble a prescribed turbulent velocity profile. This profile was developed in radial coordinates adopting the two- region turbulent viscosity definition of Ohmi and Usui (1976). In this turbulent model, the viscosity in the annular region varies linearly from the kinematic viscosity of the wall ν_w to a constant kinematic viscosity of the turbulent core ν_c. Vardy and Brown (2007) also used this description of the flow to perform sound predictions of the water-hammer effects. The velocity profile for the core region is

$$v(r)=\frac{dp}{dx}\frac{\left(r^2\text{-}R_M^2\right)}{4\rho_0\nu_c}+\frac{1}{2\rho^0}\frac{dp}{dx}\left\{-\frac{(0.2R^0)^2}{\nu_w\text{-}\nu_c}-\frac{(\text{-}4\nu_w+5\nu_c)(0.2R^0)^2}{(\nu_w\text{-}\nu_c)^2}ln[\sigma_{cw}]\right\} \tag{13}$$

, whereas for the annular region, the velocity profile can be written as

$$v(r) = \frac{1}{2\rho_0}\frac{dp}{dx}\left\{\frac{(r\text{-}R^0)(0.2R^0)}{(v_w\text{-}v_c)} + \left[\frac{(-4v_w+5v_c)(0.2R^0)^2}{(v_w\text{-}v_c)^2}\right] ln\left[\frac{v_w}{\left(\frac{v_w-v_c}{0.2R^0}\right)r-4v_w+5v_c}\right]\right\} \tag{14}$$

, in which σ_{cw} is the ratio of viscosities v_c/v_w and R_M $(0.8R^0)$ is the radius in which occurs the matching of the annular and core regions.

Although Vardy and Brow (2002) applied the same turbulent model, they used two different coordinates frames: cartesian in the annular region and polar in the core. To highlight the differences between these profiles, a comparison between both approaches is depicted in Figure 2. In constructing these plots, it was used the arbitrary values of v_w= 0.1 m²/s; v_c = 0.5 m²/s; R^0= 1 m; $\rho_0 = 1000$ Kg/m³, and $\frac{dp}{dx}$ =-5 Pa. The profiles exhibited good agreement, but it is notable that the completely radial profiles described herein behave smoother than the Vardy and Brown's mixed coordinated system near the matching point between the two regions.

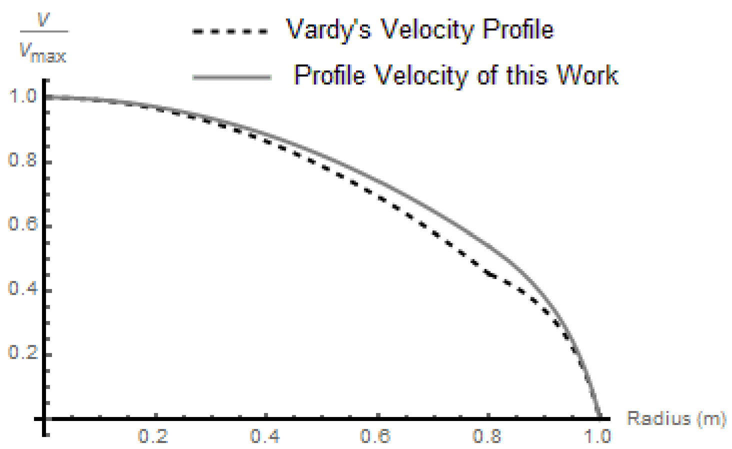

Figure 2. Comparison between the velocity profile of Vardy's work and the one proposed in this work. Both velocities are normalized by the respective maximum velocities.

By employing the strategy described at the beginning of this subsection, the material constants can be calculated analytically and written out as:

$$C = \frac{5(\nu_w-\nu_c)\,\rho_0}{R^0-R_n^0+\frac{R^0(5\nu_c-4\nu_w)}{5(\nu_w-\nu_c)}ln\left[(5-(\sigma_{cw}))\frac{R_n^0}{R^0}+5(\sigma_{cw})-4\right]}, \tag{15}$$

$$C_{j,j+1} = \frac{4\rho_0 v_c}{R_M^2}\frac{\left(\sum_{l=1}^{j} 2R_j^0\,\Delta R_j^0\right)}{R_{j+1}^0R_{j+1}^0\text{-}R_j^0R_j^0} \quad , \tag{16}$$

$$C_{j,\,j+1} = \frac{\rho_0}{\frac{R^0\left(R_{j+1}^0\text{-}R_j^0\right)}{5(v_w\text{-}v_c)}+\frac{R^0R^0(-4v_w+5v_c)}{(5v_w\text{-}5v_c)^2}ln\left[\frac{5(v_w\text{-}v_c)R_j^0+R^0(-4v_w+5v_c)}{5(v_w\text{-}v_c)R_j^0+\Delta R_j^0)+R^0(-4v_w+5v_c)}\right]}\sum_{l=1}^{j}\frac{4R_j^0\Delta R_j^0}{R_{j+1}^0R_{j+1}^0\text{-}R_j^0R_j^0} \quad . \tag{17}$$

In Eqs (15-17) R_j^0, R_n^0 , and ΔR_j^0 are, respectively, the radius of the jth and last constituent; and the thickness of each constituent. Equation (15) can be identified as the expression for the reactive material constant, and Eq. (16) and Eq. (17) stand for the interaction momentum force material constants between the constituents for the core and annular regions, respectively.

Notably, this approach uses the steady state to obtain the material constants, which at a first glance could seem inconsistent. However, it is highly recorded in the literature that the steady-state viscosity structure is assumed to be unaltered after rapid changes in the velocity after short periods of time (the well-known frozen viscosity assumption). As long as the focus is on fast transients in enclosed pipes, this hypothesis is valid (Guidaoui 2005; Vardy and Brown, 2003).

To solve the transient system of equations, the well-known and classic method of characteristics, with specified time step (Willie and streeter, 1993), was used to obtain approximate numerical solutions to the hyperbolic system of partial differential equations.

3. PRELIMINARY RESULTS

To access the capability of the proposed model of describing the unsteady friction, a numerical simulation was carried out and the result contrasted with experimental data available in Holmboe and Roleau (1967) and along with the two-dimensional numerical model of Araya and Chaudry (1997). The experimental apparatus is a reservoir-pipe-valve installation depicted in Figure 3. The reservoir is a constant pressure device from which water (ρ = 1000 Kg/m³ and viscosity μ = 0.00086 Ns/m²) flows downstream in the steady state through a constant diameter steel pipe, having a length L = 36.09 m and an internal diameter D = 0.025 m. A block valve, initially at the fully-opened position, is installed at the downstream end of this pipe, and the steady flow velocity is reported as V_0 = 0.21m/s. This initial condition is suddenly altered to a transient phenomenon by a rapid valve closure maneuver taking place in 0.04s. According to the authors, the wave speed in this system is a = 1350 m/s and the initial Reynolds Number is Re = 6166. The pressure in the transient conditions was recorded with piezoelectric transducers installed at the middle of the pipe length and at the block valve.

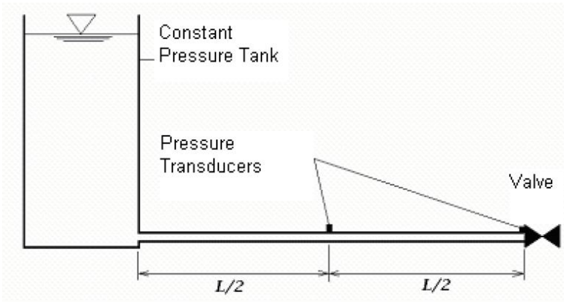

Figure 3. The experimental setup used by Holmboe and Roleau (1967).

The comparison among the pressure responses at the valve is shown in Figure 4. To achieve a better visualization, the pressure is normalized by the Joukowsky head aV_0/g while the time is normalized by the wave travel time from the reservoir to the valve *L/a*. It is clear that the model developed herein presents a good agreement with the experimental data. Furthermore, since the model is one-dimensional it is easier to implement computationally than the Araya and Chaudry's two-dimensional model. As a result, this model shows a great potential to be used in fast transient real-time applications.

In addition, it is highlighted the simplicity of the identification of the material constants of the proposed frequency-dependent friction model, in contrast with the complex procedure found in Vardy and Brown (2007).

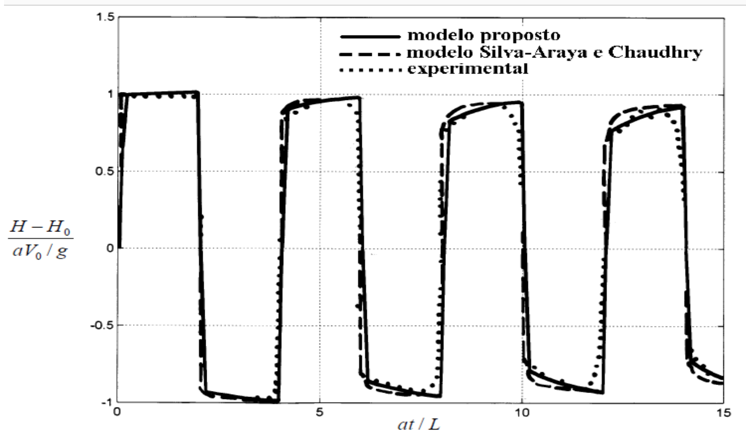

Figure 4. The normalized pressure at the valve.

Furthermore, the consistent theoretical framework in which the models are developed allows, in a simple way, the verification of its thermomechanical coherence. It can be proved that the constraints imposed by the SLT through Eqs. (11) and (12) in the virtual structured model state that all material constants must be positive, and, indeed, all of them developed herein are positive. On the contrary, as described in the Víktovský, *et al.* (2006), the up-to-date instantaneous acceleration models may fail in attending the SLT in certain types of fluid transient events. As a matter of fact, most of the models available in the literature disregard this kind of based upon the SLT, despising the importance of thermodynamical consistency of the model. Not to mention the relevant role of an accurate energy analysis to design specialized devices to mitigate water-hammer effects upon the pipes.

4. CONCLUSION

This study has presented a model for unsteady friction in turbulent flows by employing a virtual structure model in the context of the continuum mixture theory. In contrast with other models, this new approach has the advantage of an easy identification of the material constants along with a simple computational implementation. In addition, the SLT is always satisfied, allowing it to be used for all kinds of Newtonian fluids subjected to transient flows in pipes, whatever the pipe installation might be. Because of the model simplicity, it constitutes a promising tool for real-time applications in fluid transient events in pipe flows.

5. REFERENCES

Brunone, B., Ferrante, M., and Cacciamani, M., 2004. “Decay of pressure and energy dissipation in laminar transient flow.” *J. Fluids Eng.*, Vol.126, p. 928–934.

Brunone, B., Golia, U.M., Greco, M., 1991. Some remarks on the momentum equations for fast transients. In *Proceedings of the International Meeting on Hydraulic Transients with Column Separation*. Valencia, Spain.

Drew, D. A. and Passman, S. L., 1998. *Theory of multicomponent fluids.* Springer-Verlag, New York.

Cunha, M. and Rachid, F. B. F., 2005. In *Proceedings of the 18th International Congress of Mechanical Engineering-COBEN* (2005). Ouro Preto, Brazil.

Guidaoui, M. S., Zhao, M., McIlnnis, D. A., Axworthy, D.H., 2005. “A review of water hammer theory and Practice”. *Applied Mechanics Reviews*, 58, p. 49-76.

Holmboe, E. L. and Roleau, W. T., 1967. “The effect of the viscous shear on transients in liquid lines”. *Journal of Basic Engineering, Trans. ASME,* Vol. 89, p. 49-86.

Ohmi, M., and Usui, T. _1976_. “Pressure and velocity distributions in pulsating turbulent pipe flow. Part 1: Theoretical treatment.” *Bull. JSME*, Vol. 19, p. 307–313.

Pezzinga, G.,1999. “Quasi-2D model for unsteady flow in pipe networks”. *J. Hydraul. Eng.*, Vol. 125, p.676–685.

Urbanowicz, K. and Zarzycki, Z., 2012. “New efficient approximation of weighting functions for simulations of unsteady friction losses in liquid pipe flow”. *Journal of Theoretical and Applied Mechanics*, Vol. 50, p. 487-508.

Vardy, A. E. and Brown, J. M. B., 2003. “Transient Turbulent Friction in Smooth Pipe Flows”. *Journal of Sound and Vibration*, Vol. 259(5), p. 1011–1036.

Vardy, A. E., and Brown, J. M. B., 2007. “Approximation of turbulent wall shear stress in highly transient pipe flows”. *J. Hydraul. Eng.*, Vol. 133, p. 1219-1228.

Vardy, A. E., and Brown, J. M. B., 2004. “Transient turbulent friction in fully rough pipe flows.” *J. Sound Vib.*, Vol. 270, p. 233–257.

Vardy, A. E., and Brown, J. M.B., 1995. “Transient, turbulent, smooth pipe friction.” *J. Hydraul. Res.*, Vol. 334, p. 435–456.

Vitkovský, J., Bergant, A., Simpson, A. R., and Lambert, A., 2006. “Systematic evaluation of one-dimensional unsteady friction models in simple pipelines.” *J. Hydraul. Eng.*, Vol. 132, p. 696–708.

W. F. Silva-Araya and M. H. Chaudhry, 1997.” Computation of energy dissipation in transient flow”. *Journal of Hydraulic Engineering*, Vol.123, p. 108–115.

Wylie, E. B. and Streeter, V. L., 1993. *Fluid Transient in Systems*. Prentice Hall, New Jersey.

Zarzycki, Z, 1997. “Hydraulic resistance of unsteady turbulent liquid flow in pipes”. In *Proceedings of the 3rd International Conference on Water Pipeline Systems*, The Hague, BHR Group, p. 163–178.

Zielke, W., 1968. “Frequency-Dependent Friction in Transient Pipe Flow”. *ASME Journal of Basic Engineering*, 4 Vol. 91, p. 109-115.

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