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EVALUATION OF REENTRY DYNAMICS OF SARA PLATFORM CONSIDERING THE EFFECTS OF ABLATION IN THE THERMAL SHIELD

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Abstract. In this work, the dynamic analysis of the reentry of the SARA Sub-orbital Platform is performed, considering the effects of geometry change in the ablative thermal protection shield due the material consumption. A dynamic model with two degrees of freedom is coupled with an engineering model for the aerodynamic heating and surface recession due ablation in the thermal shield. Results show the mutual effects in the geometry change and impact zone after the flight, allowing concluding that, although trajectory strongly influences the ablation, only under severe reentry conditions the thermal shield ablation will affect the vehicle trajectory.

Keywords: Ablation, Computational Simulation, Trajectory

1. INTRODUCTION

Along the years ablative materials have been effectively used as TPS (Thermal Protection System) of space vehicles (Rogan and Hurwicz, 1973). Such kind of TPS absorbs the heat resulting from aerodynamic heating through the consumption of its own material. If composite materials are employed as ablative TPS, the resulting ablation is a complex phenomenon, related to several physical processes happening simultaneously (Duffa, 2013). The modification of external geometry is an important effect of ablation in a TPS. In this case there is an interaction between the mass loss and the aerodynamic coefficients, since the geometry affects trajectory, which affects the ablation (Zimmerman et al, 2003). Essentially, five figures of merit must be considered in the analysis of effect of ablation process on the atmosphere reentry of the vehicle, which are: (i) Maximum Load Factor, (ii) Altitude in which occur the maximum load factor, (iii) Critical Convective Flux, (iv) Maximum velocity in which recovery system act on the payload and (v) Dispersion of payload with recovery system. In this work, it is emphasized the analysis of maximum load factor, altitude in which occur the maximum load factor and the critical convective flux. The numerical algorithm does not consider the coupling between TPS ablation and trajectory. It is used a reduced order model to estimate the influence of the ablation in the trajectory. It was chosen the SARA Suborbital platform, in development at the Institute of Aeronautics and Space (IAE, São José dos Campo, Brazil), as a case study in order to estimate the mutual influence of these processes. SARA sub-orbital platform, Fig. 1, is being developed by IAE/CTA for such application. It has a total mass of 250 kg for a payload of about 25 kg, and is designed to provide 6 minutes of micro-gravity environment. SARA reaches the speed of 9300 km/h in atmospheric flight (Moraes, 1998).

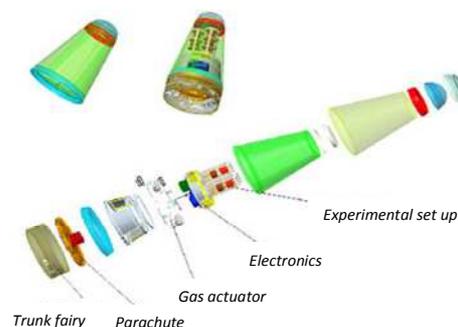


Figure 1. SARA sub orbital and its internal systems.

2. METHODOLOGY

It is possible to obtain an analytic solution for flight dynamics equation considering a reduced order model (Sikharulidze, 2001). The vehicle geometry is the principal parameter of aerothermodynamics design since the most of figures of merit of mission are based on drag coefficient, maximum load factor and maximum heat flux during the atmosphere reentry. In order to illustrate the methodology of analysis, a cone-cylinder vehicle is used as payload aerodynamic shape, in order to represent SARA suborbital platform.

2.1 Ablation

The aerodynamic warming was accounted as an external boundary condition for the payload wall. The heat transfer parameters were estimated through the Zoby method (Zoby et al., 1981) and their values at the stagnation point are showed in Fig. 2. The surface ablation is simulated using a 1-D Lagrangean procedure (Machado, 2009), according with the coordinate system presented in Fig. 3. The heat conduction and ablation estimatives allows to obtain the geometry change along the trajectory. The coupling between both calculations allows finding figures of merit to design of TPS.

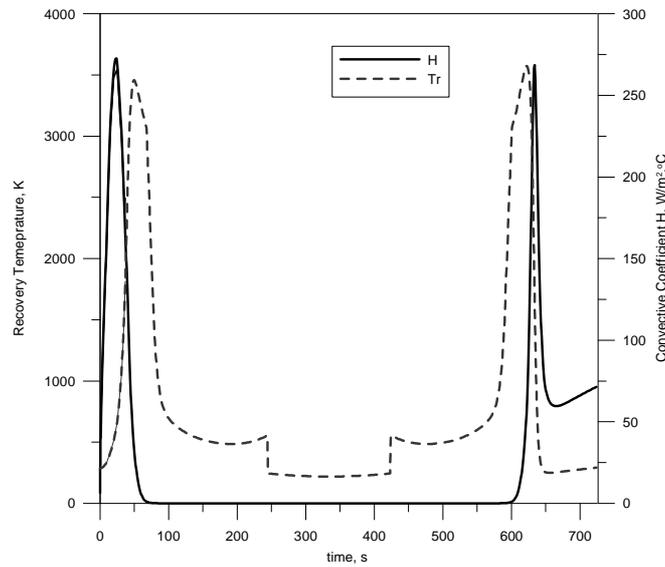


Figure 2. Recovery temperature and convective heat transfer coefficient at stagnation point, during SARA trajectory.

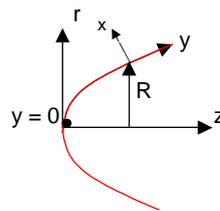


Figure 3. Coordinate system for heat transfer calculation.

2.2 Trajectory estimative

Trajectory equations for mass center of an unguided vehicle in atmosphere are:

$$\left\{ \begin{array}{l} \frac{dV}{dt} = -\frac{\rho V^2 C_D S}{2m} - g \sin \theta \\ V \frac{d\theta}{dt} = \frac{V^2}{R_E + H} \cos \theta - g \cos \theta \\ \frac{dH}{dt} = V \sin \theta \\ \frac{dL}{dt} = \frac{R_E}{R_E + H} V \cos \theta \end{array} \right. \quad (1)$$

where V is the velocity, θ is the trajectory angle, H is the altitude, L is the path over the Earth's surface, m is the mass, C_D is the drag coefficient, S is the base area, ρ is the local atmosphere density and R_E is earth's radius ($R_E = 6371$ km). Gravity acceleration is given as:

$$g = \frac{\mu}{(R_E + H)^2} \quad (2)$$

where $\mu = 398600.4 \text{ km}^3/\text{s}^2$. The load factor is defined as:

$$n_x = \frac{\rho V^2 C_D S}{2 mg} \quad (3)$$

Equation (1) becomes:

$$\frac{dV}{dt} = -g_0(n_x + \sin \theta) \quad (4)$$

It is possible to obtain an analytical solution for the flight dynamics equation considering a reduced order model. The simplified mathematical model considers the following hypothesis:

- (i) Constant gravitational acceleration, $g = g_0$;
- (ii) Relation between flight angle and load factor (n_x) approached to $\sin \theta / n_x \ll 1$ in most of the trajectory;
- (iii) Difference between reentry velocity and terminal velocity is approximately the same for different values of reentry angle.
- (iv) Density at the border of atmosphere (ρ_{en}) is assumed to be zero;
- (v) Ratio between the ballistic coefficient (σ_D) and vehicle altitude is linear: $\sigma_D = \sigma_D^{en} - \Delta \sigma_D H$;
- (vi) Ballistic coefficient (σ_D) changes only by ablative process during reentry. A small variation of $\Delta \sigma_D$ is considered;
- (vii) Trajectory angle ($\bar{\theta}$) is constant during flight.

From hypothesis (i), it is possible to rewrite Eq. (4) as:

$$\frac{dV}{dt} = -g_0 n_x \quad (5)$$

Initial and final flight conditions have to be defined in order to integrate Eq. (5). Initial condition is a function of reentry velocity, V_{en} . At the final instant the final velocity V_f is obtained from integration of Eq. (5):

$$V_{en} - V_f = g_0 \int_0^{t_f} n_x dt \quad (6)$$

According to hypothesis (iii), the left side of Eq. (6) is approximately constant. As a consequence, the greater is the reentry period, the less is the average load factor. Reentry time depend mainly on the reentry angle, in an inverse ratio. Thus, the smaller reentry angle, the smaller the average load factor. In this context a constant trajectory angle will be assumed.

$$\theta(t) = \bar{\theta} \quad (7)$$

Assuming atmospheric density as an exponential function of altitude:

$$\rho(H) = \rho_0 e^{-\lambda H} \quad (8)$$

Where ρ_0 is the density on Earth's surface and λ is the logarithmic gradient of density:

$$\lambda = -\frac{d}{dH} \ln \rho \quad (9)$$

With such hypothesis, de 1st and 3rd equations of the system of Eq. (1) can be rewritten as:

$$\frac{dV}{V} = -\frac{\rho_0 \sigma_D}{2 \sin \bar{\theta}} e^{-\lambda H} dH \quad (10)$$

where σ_D is given as:

$$\sigma_D = \frac{C_D S}{m} \quad (11)$$

According with hypothesis (iv), Eq.* é possível escrever a Eq.(10) becomes:

$$\frac{dV}{V} = -\frac{\rho_0}{2 \sin \bar{\theta}} (\sigma_D^{en} - \Delta \sigma_D H) e^{-\lambda H} dH \quad (12)$$

wherer $\Delta \sigma_D$ is the angular coefficient for the linear relationship between altitude and σ_D , σ_D^{en} is the ballistic coefficient at the reentry point. After integration of the Eq. (12) from reentry point to the current point, results:

$$V(\rho) = V_{en} e^{\frac{\rho}{2 \lambda \sin \bar{\theta}} \left(\sigma_D^{en} - \frac{1 + \lambda H}{\lambda} \Delta \sigma_D \right)} \quad (13)$$

It is supposed that $\bar{\theta}$ is near zero and ρ_{en} is the density at atmosphere border, by assuming $\rho_{en} \approx 0$. Sikharulidze (2001) did not consider the superficial ablation in his formulation. Thus, the solution of Eq. (10) is given as:

$$V_{Sikha}(\rho) = V_{en} e^{\frac{\rho}{2 \lambda \sin \bar{\theta}} \sigma_D^{en}} \quad (14)$$

Since the mathematical model assumes a constant flight angle ($\bar{\theta}$), it is necessary a previous evaluation for the application in the Eqs. (3, 4). Sikharulidze (2001) applied a numerical solution to obtain a preliminary result. In this work, a correction method derived from Eq. (1) will be applied:

$$\frac{d\theta}{dt} = \frac{V}{R_E + H} \cos \theta - \frac{g \cos \theta}{V} \quad (15)$$

Through the first order approximation for the variation of θ from the Taylor series, we obtain:

$$\Delta \theta = \left(\frac{V}{R_E + H} \cos \theta^{en} - \frac{g \cos \theta^{en}}{V} \right) \Delta t \quad (16)$$

The vehicle velocity (V) is calculated by the Sikharulidze (2001) formula, Eq. (4), considering $\bar{\theta} = \theta^{en}$. The time step was estimated from Eq. (3) for velocity without the gravity term:

$$\frac{dV}{dt} = -\frac{\rho V^2 C_D S_{ref}}{2 m} = -p_{dyn} \sigma_D \quad (17)$$

The first order approximation is given by:

$$\Delta t = -\frac{\Delta V}{p_{dyn} \sigma_D} \quad (18)$$

$$\Delta t = -\frac{V - V^{en}}{p_{dyn} \sigma_D^{en}} \quad (19)$$

Then, the trajectory angle is estimated by the approximation:

$$\bar{\theta} = \theta^{en} + \Delta \theta \quad (20)$$

According to Eq. (3), the maximum load factor occurs at the point of maximum dynamic pressure. Eq. (13) allows to obtain dynamics pressure:

$$q_{dyn}(\rho) = \frac{1}{2} \rho V_{en}^2 e^{\frac{\rho}{\lambda \sin \bar{\theta}} \left(\sigma_D^{en} - \frac{1 + \lambda H}{\lambda} \Delta \sigma_D \right)} \quad (21)$$

The Eq.(11) is derived from density (ρ) to determine the value when the maximum dynamic pressure occurs ($\rho_{n \max}$):

$$\rho_{n \max} = - \frac{\lambda}{\left(\sigma_D^{en} - \frac{1 + \lambda H}{\lambda} \Delta \sigma_D \right)} \sin \bar{\theta} \quad (22)$$

From Eq. (14), Sikharulidze (2001) obtained similar mathematical models for dynamic pressure equations, considering density, maximum altitude and maximum load factor, given as:

$$q_{dyn}^{Sikha}(\rho) = \frac{1}{2} \rho V_{en}^2 e^{\frac{\rho}{\lambda \sin \bar{\theta}} \sigma_D} \quad (23)$$

$$\rho_{n \max}^{Sikha} = - \frac{\lambda}{\sigma_D} \sin \bar{\theta} \quad (24)$$

$$H_{n \max}^{Sikha} = \frac{-1}{\lambda} \ln \left[\frac{\lambda \sin \bar{\theta}}{\rho_0 \sigma_D^{en}} \right] \quad (25)$$

$$n_{\max}^{Sikha} = \frac{\lambda V_{en}^2}{2 e g_0} \sin |\bar{\theta}| \quad (26)$$

It is important to mention that the formulation derived by Sikharulidze (2001) does not consider the influence of the ballistic coefficient on the maximum load factor estimation. Therefore, this parameter cannot be used to evaluate the influence of ablation on flight dynamics. Establishing the level of significance in estimating error found in nonlinear mathematical models is a difficult task. The coupling between the ablative process and the flight dynamics given by Eq. (1) is physically very complex. There are great uncertainties in estimating heat flux, mass loss, drag coefficient and vehicle attitude during flight. Even the Monte Carlo Method would not be recommended for the complete analysis of reentry, since computational costs would be excessive. Thus, the reduced order method becomes the first option for the design. The main input data in the reduced-order model is the parameter $\Delta \sigma_D$, which should be estimated by nonlinear analysis. For a first estimative of its order of magnitude it is necessary to evaluate at least three results obtained from nonlinear simulation under severe flight conditions. From there, knowing this range of variation, it is possible to proceed to statistical analysis, based on the Chebychev Inequity Theorem (Yates & Goodman, 1999). This choice is justified by the lack of knowledge in probability function (fdp) of the maximum load factor. According to the theorem:

$$P(|X - E(X)| \geq \varepsilon) \leq \frac{s(X)}{\varepsilon^2} \quad (27)$$

where X is any random variable, $\varepsilon > 0$ is a positive real number, $E(x)$ and $S(X)$ are the mean and standard deviation of the population or samples of X, respectively. The methodology consists of generating a sample for n_{MAX} from variation of $\Delta \sigma_D$ and other influence parameters, such as:

$$\Delta \sigma_D = \Delta \bar{\sigma}_D + N(0, \delta) \quad (28)$$

Where $N(0, \delta)$ is the normal distribution for the error in the vicinity of $\Delta \bar{\sigma}_D$, with zero mean and standard deviation δ . The parameter δ is estimated from the results obtained for three (for example) numerical simulations. Thus, from the Chebychev's Theorem of Inequity:

$$P(|n_{\max} - E(n_{\max})| \geq \varepsilon) \leq \frac{s(n_{\max})}{\varepsilon^2} \quad (29)$$

From this equation it is possible to establish a range of variation for a given n_{MAX} occurrence probability.

3. RESULTS

SARA structure is built with a combination of aluminium and carbon fiber. The TPS is made with a layer of cork in the conic section. The frontal ogive is protected with a layer of quartz-phenolic resin covered by a layer of cork. Table 1 shows the properties considered for each material. For this calculation, all the TPS was assumed to be made of cork, and the ablation temperature of this material were modified, in order to allow the loss of the whole ablative layer, so as to amplify the effect of geometry change. The values employed for the temperature of ablation and heat of ablation were 120° C and 20 kJ/kg, respectively.

Table 1. Properties of the materials (Da Costa et al, 1996).

Property \ material	Aluminium	Carbon fiber	Quartz-phenolic	cork
Emissivity	0.06	1	0.8	0.78
Specific heat (J/kg K)	960	800	1256	1971.8
Thermal conductivity (W/m K)	177	150	0.485	0.084
Density (kg/m ³)	2710	1750	1730	480.
Temperature of ablation (°C)	-	-	538 ⁽¹⁾	260 ⁽²⁾
Heat of ablation (J/kg)	-	-	0.78 x 10 ⁶⁽¹⁾	3 x 10 ⁶⁽²⁾

⁽¹⁾ Tick et al, 1965

⁽²⁾ Herold and Diamant, 1966

The resulting ablation in TPS is showed in Fig. 4, for the actual and modified configuration and cork ablative properties. Figure 5 shows the geometry change for both cases. It is remarkable that the geometry change for the actual configuration is very small. Although the TPS is fully consumed in the modified configuration, the geometry change seems to be also not significant. Figures 6 and 7 show that the major mass losses and surface variations of the vehicle occur during short periods of time, in two peaks in the trajectory. In figure 8, the ratio between the two parameters is presented for both configurations. It is important to know the simultaneous variation of these parameters, since it allows to estimate the coupling between the geometric change due ablation (surface) and thermal load (TPS mass loss). IN both cases this variation can be approximately represented by a linear dependence, as showed in Fig. 8.

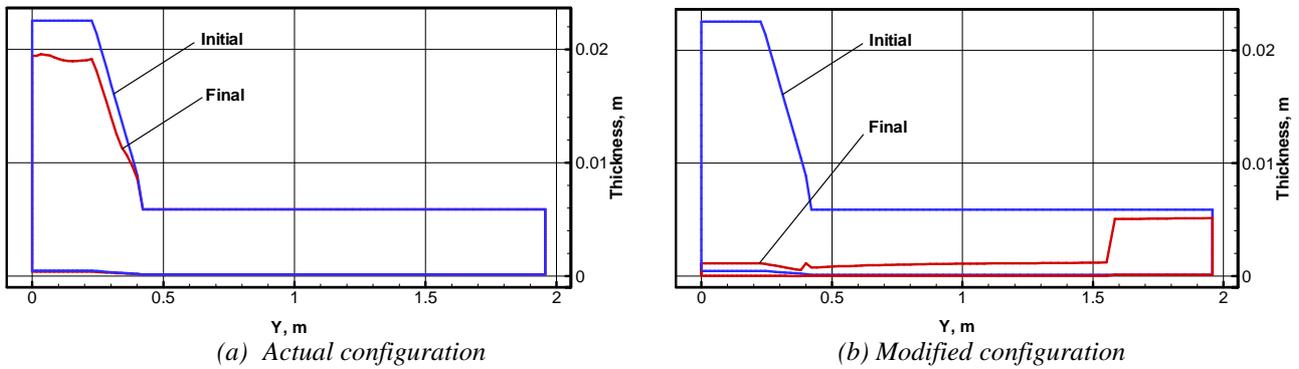


Figure 4. Variation of TPS thickness.

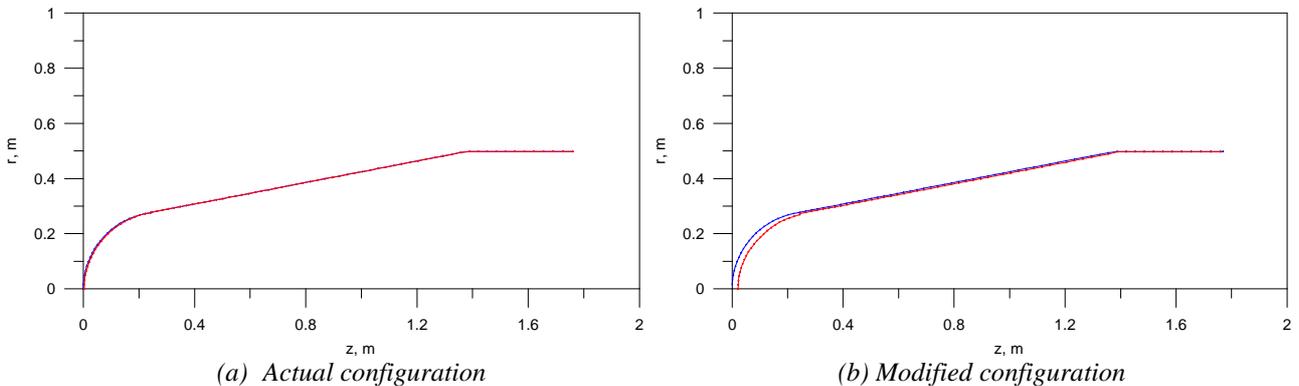


Figure 5. Variation of external geometry.

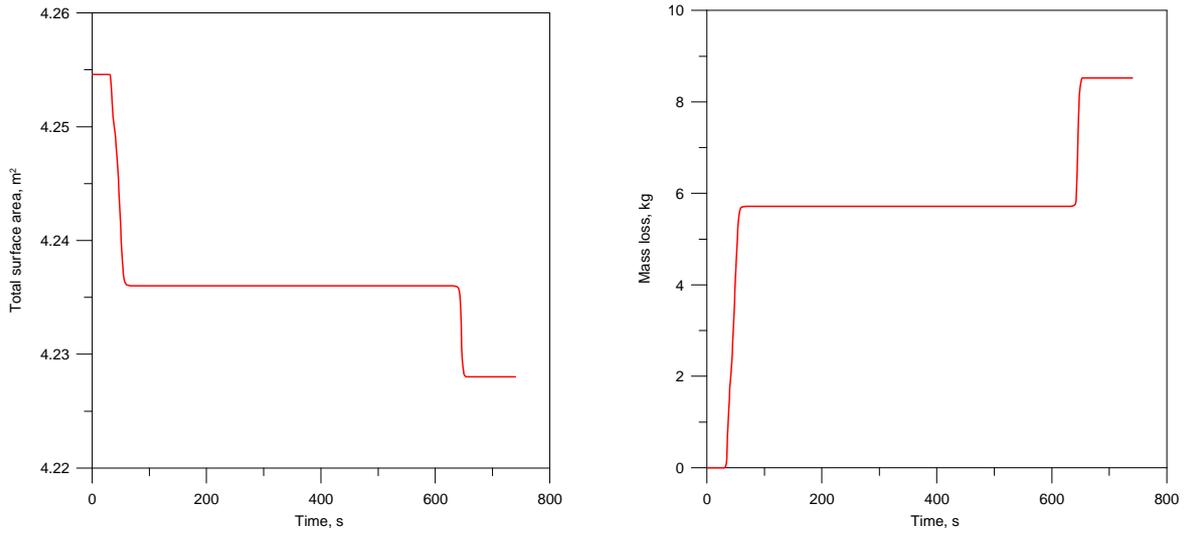


Figure 6. Surface variation and mass loss during trajectory for the actual configuration.

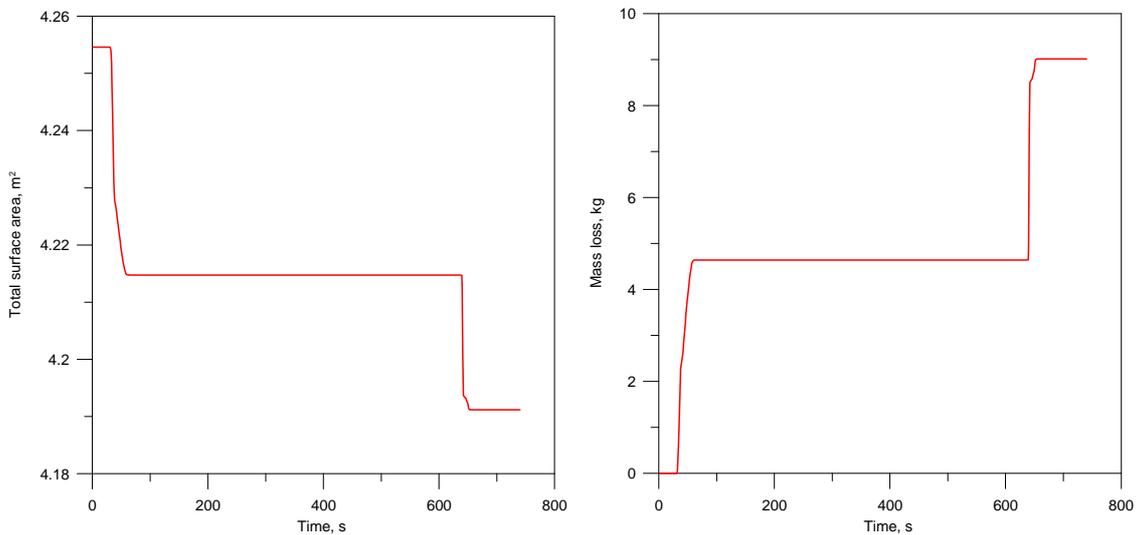
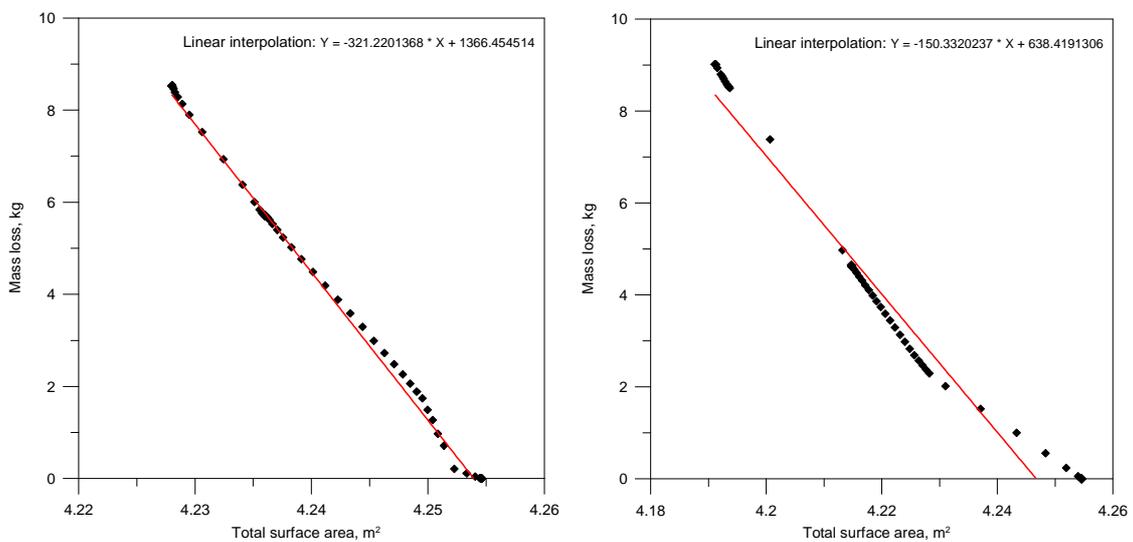


Figure 7. Surface variation and mass loss during trajectory for the modified configuration.



(a) Actual Configuration

(b) Modified Configuration

Figure 8. Mass loss as function of the external surface variation during trajectory.

The input data are presented in Tab.2. Figure 9 shows the density curve used to estimate the parameter λ and the gravitational model used in the flight dynamics numerical simulation. In ballistic reentry, the force applied is always opposite to the flight direction (drag force). The primary design parameter for ballistic reentry, ballistic coefficient σ_D , is defined as Eq. (11). Heating and acceleration are less intense for low σ_D (low weight and/or high drag due to frontal area) provided that reentry takes place in the upper part of the atmosphere, where the air is at low density. Figure 10 shows a typical trajectory of the SARA Sub-orbital Platform (Machado, 2006). The trajectory is characterized by angle, velocity, altitude and range. The performance parameter evaluated already was the load factor. From that figure it the load factor was extracted, as a non-linear mathematical factor. This parameter was used to validate the analytical model presented. It should be mentioned that trajectory was not calculated using the coupling between flight dynamic and the mathematical model for ablation. From the results, $\Delta\bar{\sigma}_D/\Delta H$ was estimated.

Table 2. Reentry conditions.

	Value	Unit
Altitude (H)	100	km
Angle (θ)	-3.2	deg
Velocity (V)	7216	m/s
σ_D	$1 \cdot 10^{-3}$ and $1 \cdot 10^{-2}$	m^2/kg

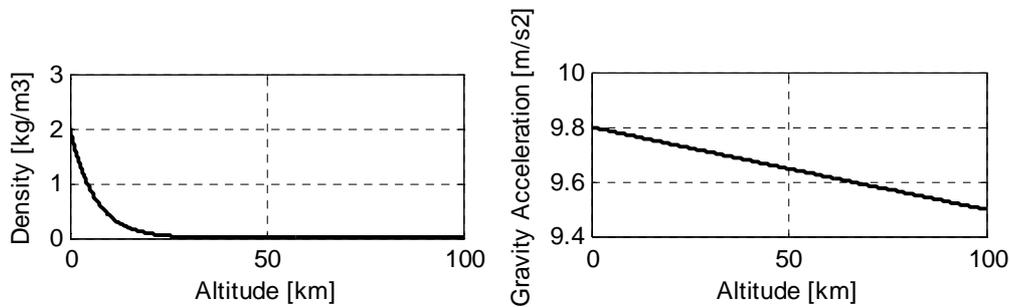


Figure 9. Atmosphere model.

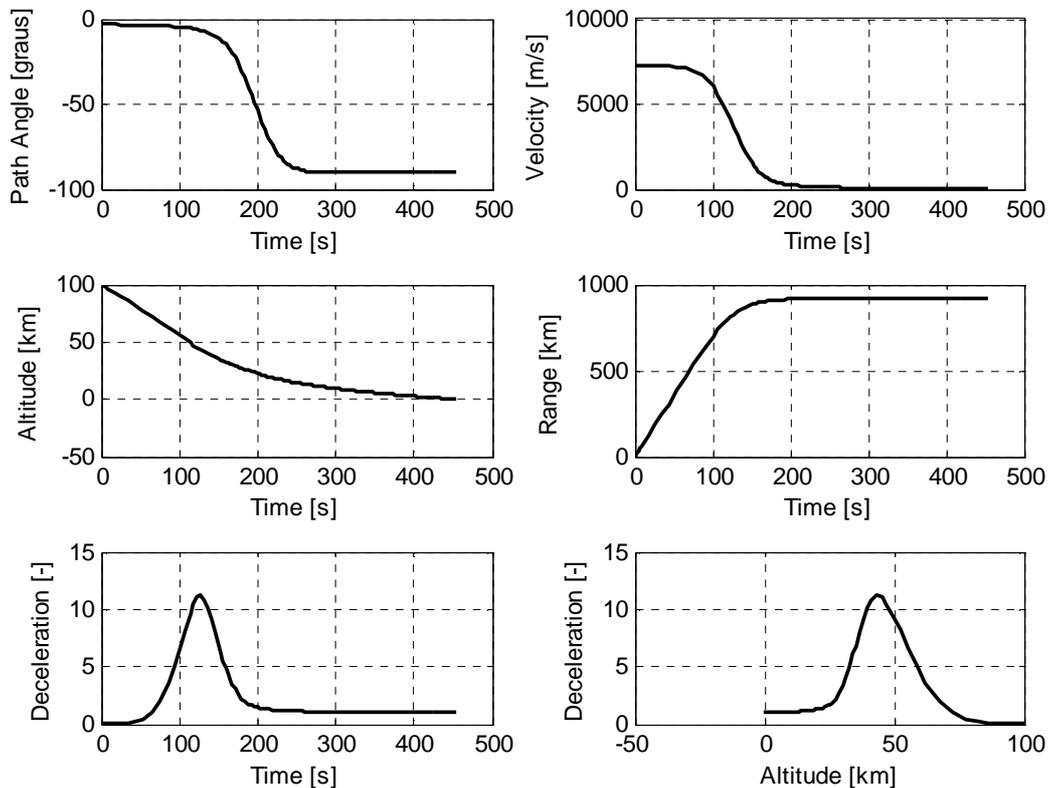


Figure 10. Nominal trajectory.

Tables 3 and 4 present the results obtained by the two methodologies presented, by Sikharulidze (2001) and the reduced order model, considering the nonlinear solution, Eq. (1). The evaluation conditions were: $\sigma_D = 1 \cdot 10^{-3} \text{ m}^2/\text{kg}$ e $\sigma_D = 1 \cdot 10^{-2} \text{ m}^2/\text{kg}$ (10-times increase). The time interval was approximately 2 seconds. Both results demonstrated satisfactory results for load factors in terms of initial engineering design error. (maximum of 10-15%). Although, the reduced order model produced better results for density, altitude and dynamic pressure in both cases. The estimation of these parameters is important to obtain a satisfactory assessment of the drag force during the flight.

Table 3. Comparative analysis with the maximum load factor condition ($\sigma_D = 1 \cdot 10^{-3} \text{ m}^2/\text{kg}$).

	Sikharulidse	Reduced Order Model
Density	38.16 %	10.05 %
Altitude	10.98 %	2.76 %
Dynamic Pressure	34.71 %	5.04 %
Load Factor	13.71 %	6.81 %

Table 4. Comparative analysis with the maximum load factor condition ($\sigma_D = 1 \cdot 10^{-2} \text{ m}^2/\text{kg}$).

	Sikharulidse	Reduced Order Model
Density	47.36 %	7.83 %
Altitude	8.65 %	0.38 %
Dynamic Pressure	35.32 %	13.26 %
Load Factor	1.64 %	11.43 %

The following results presented consider any possible variation in flight angle (trajectory error) and ballistic coefficient (ablative process) during flight. The reduced-order model is used to estimate the performance parameters, such as the load factor. A zero mean and standard deviation of 0.01 were considered for both cases: $\Delta\bar{\sigma}_D/\Delta H$ and initial flight angle (θ^{En}). The time interval was 286 seconds for 105 Monte Carlo iterations. The load factor range based on the 95% requirement was [13.14013, 14.23683]. In other words, there is at least a 95% probability that the maximum load factor found during flight, considering ablation and angle-of-flight error, is within this range. The results are presented in Fig. 11. Figure 12 shows the results obtained for the same conditions, however adopting a uniform distribution error. The load factor range based on the 95% requirement was [12.37200, 15.00759]. This is more conservative estimate than employing a normal distribution. It should be mentioned that the computational time for nonlinear model is much higher than using the reduced order model. This type of analysis proves to be impractical when employing the mathematic model coupled for a preliminary design.

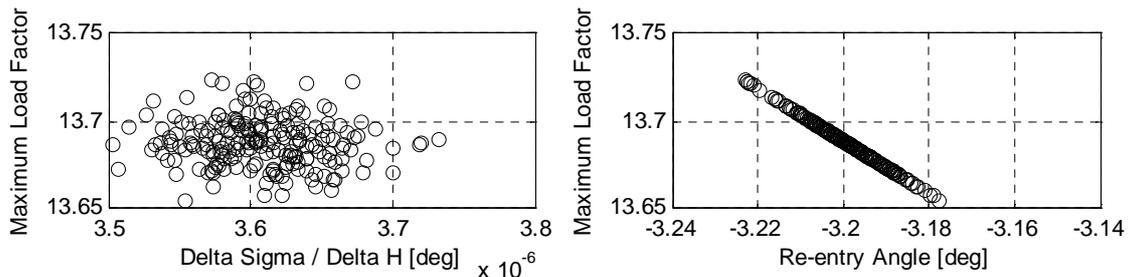


Figure 11. Correlation between load factor and parameters $\Delta\bar{\sigma}_D/\Delta H$ and initial flight angle for a normal distribution.

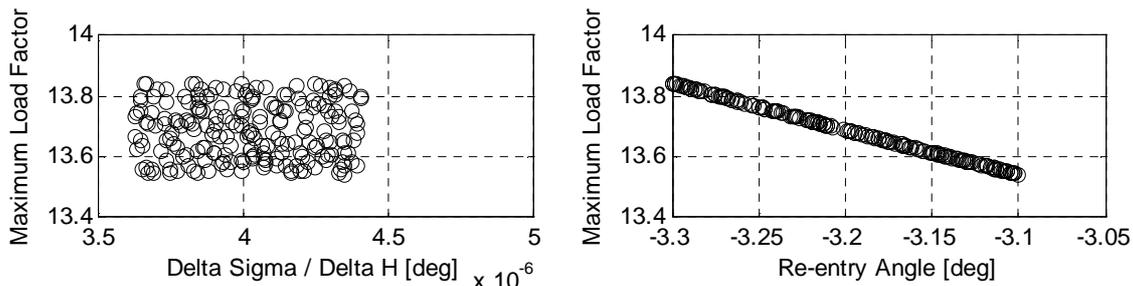


Figure 12. Correlation between load factor and parameters $\Delta\bar{\sigma}_D/\Delta H$ and initial flight angle for a uniform distribution.

4. CONCLUSION

In this work, an engineering model and a computational tool were developed to analyze the mutual effect between surface ablation of the thermal protection and the variation of the trajectory for orbital and sub-orbital vehicles. A modified nonlinear model, based on a traditional methodology, was proposed for flight dynamics, accomplishing the effect of surface variation and mass loss of the vehicle during the trajectory.

The methodology was applied to the SARA sub-orbital platform, developed, in IAE / DCTA., a case where almost all TPS material is removed due to aerodynamic heating was considered through an extrapolation of the ablative process. Even in this case, the geometric outer surface variation results discreet. In addition, it has been found that there is an approximately linear relationship between the thermal load and the variation of the surface during the trajectory.

The application of the flight dynamics methodology through a probabilistic analysis taking into account the normal and uniform distributions for the standard deviation of the performance parameters, showed that the effect of the surface ablation over the trajectory is practically negligible, since there is major uncertainties and other factors involved.

The main conclusion of this work is that the trajectory highly influences the TPS ablation of the space vehicle, but the direct influence of geometry change due to ablation surface over the trajectory is negligible. However, the ablation effects must be accounted during trajectory calculation due to the aerodynamic heating constraints. It is important to mention the computational cost applying nonlinear model is more expensive than that obtained by the reduced order analysis. This kind of analysis is impracticable using the coupled mathematical model in the preliminary design.

5. ACKNOWLEDGEMENTS

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