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A REVIEW ON CONSTITUTIVE EQUATIONS FOR GELLED WAXY CRUDE OIL MODELING

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Abstract. *Constitutive equations applied to gelled waxy crude oils must be able to predict the different characteristics these materials exhibit, such as time-dependency, elasticity and yield stress. Even though several models have been developed over the years, no review has stated the particular characteristics of each model and how in general these equations have evolved to better represent the fluid. This paper aims to point out the main aspects of some constitutive equations developed and used for waxy crude oils. To do so, a chronological approach was used, showing how the oldest models have changed and gotten more complex as time passed by, also including different characteristics of the material. The main concern of the first papers was to model the yield stress and some sort of thixotropy, trying to evaluate a time-dependent yield stress as the gel breaks down. Newer models differ from that approach as other aspects are incorporated into the constitutive equations, such as non-ideal thixotropy, elasticity and plasticity theory concepts.*

Keywords: *Waxy crude oil, Constitutive equations, Mathematical modeling.*

1. INTRODUCTION

Production of waxy crude oils is a difficult task, due to the complex characteristics of those materials and the challenging environment of extraction and transportation. Starting from the offshore platforms, the transport is usually done through long pipelines (Martínez-Palou *et al*., 2011) placed at the sea bottom, where the temperature may be as low as 4 ºC. When removed from the reservoirs, these oils are at high temperatures, in the range of 70-150 ºC (Venkatesan *et al*., 2005) and exhibit a Newtonian behavior. However, as temperature decreases, a transition to a non-Newtonian behavior occurs (Farina and Fasano, 1997; Rønningsen, 1992) due to the lowering in paraffins solubility and their consequent precipitation (Venkatesan *et al*., 2003). As a result, the viscosity of the oil increases (Wardhaugh and Boger, 1991) and a gelled structure may build up, especially when production is shut down (Smith and Ramsden, 1978). To restart the flow under these conditions, higher pressures than the usual transport pressures can be required, which may damage the pipeline, bringing more risks to the operation. However, the prediction of these restart pressures is still a challenge (Hénaut *et al*., 1999; Lee *et al*., 2008).

To predict the start-up flow of a gelled oil properly, a mathematical model must be able to reproduce the various characteristics exhibited by the material, such as time dependency and elasto-viscoplasticity (El-Gendy *et al*., 2012; Kané *et al*., 2004; Zhang and Liu, 2008). The time dependency of waxy crudes is usually accounted for as thixotropy, which is defined as a continuous decrease of viscosity with time when flow is applied to a sample at rest, and a reversible recovery of the viscosity when this flow is ceased (Mewis and Wagner, 2009). Unfortunately, waxy crude oils display some sorts of irreversibilities (Mendes, 2015; Mendes *et al*., 2015; Tarcha *et al*., 2014), which increases even more the difficulty of modeling them, as thixotropy models cannot fully reproduce their characteristics.

Although several constitutive equations have been proposed along the years to model waxy crude oils, a thorough analysis and understanding of their evolution is required to move even further. The aim of this paper is to comprehensibly present several constitutive equations proposed to model waxy crude oils and show how the concepts behind them have evolved. Therefore, a chronological approach to the evolution of the models is employed, showing how the models have changed until nowadays. Understanding this evolution is of great importance due to the aforementioned complexity of the studied material, and, therefore, the first models may be simpler and easier to understand. Souza Mendes and Thompson (2012) have written a review on a close subject, but generally applied to elasto-viscoplastic materials and, also, many models have risen since then. In addition, no review about constitutive equations applied specifically to waxy crudes was found in the literature.

2. WAXY CRUDE MODELING EVOLUTION

One of the first relevant models is the one proposed by Petrellis and Flumerfelt (1973). Even though it is the first analyzed model, ideas behind its development are employed until nowadays. In their constitutive equation, the total shear stress (τ) was assumed to be a function of two components: a Newtonian stress (τ_μ), which arises from a Newtonian solvent for the oil, and a structural stress (τ_s), which is a function of the instantaneous structure of the fluid.

To account for the structure of the fluid, the authors used the structural kinetics approach with one structural parameter for a thixotropic model. This means that the degree of structure of the material is fully described by a single scalar parameter, the structural parameter λ. This parameter varies from a minimum, usually zero, to represent the completely unstructured state of the material, to a maximum, usually one, which represents the completely structured material (Dullaert and Mewis, 2006). As the structure of the material changes with its breakdown due to shear, a constitutive equation is not enough to characterize the model completely. Therefore, a kinetic equation to model the changes of the structural parameter is required. Kinetic equations are not the main object of this work, but in some papers they will be analyzed, as important features are present that affect the development of the constitutive one.

The Petrellis and Flumerfelt (1973) constitutive equation is shown in Eq. (1), where μ represents the Newtonian viscosity, $\dot{\gamma}$ the shear rate, τ_y the yield stress, λ the structural parameter and B, n and $\hat{\mu}$ are constants, which depend on the material but not on the structure. Also, it is interesting to note that the term in brackets of Eq. (1) is a Herschel-Bulkley equation with the addition of another viscous term.

$$\tau = \mu\dot{\gamma} + \lambda\left[\tau_y + \left(\hat{\mu} - \mu\right)\dot{\gamma} + B\dot{\gamma}^n\right] \tag{1}$$

During a start-up problem, which is very common in the oil industry, the oil is initially gelled, meaning that the structural parameter has a high value. Therefore, the main change when restarting the flow is the breakdown of the structure, which means that λ shall decrease. This is the main feature of the kinetic equation from Petrellis and Flumerfelt (1973) presented in Eq. (2), where β and k represent functions of the studied material and λ_e is the equilibrium value for λ, being a function of the equilibrium viscosity (η_e), the shear rate, and the constants μ, $\hat{\mu}$, τ_y, B and n. If the material is sheared until the equilibrium state, no structural change is considered after that to assure some kind of irreversibility to the process.

$$\frac{d\lambda}{dt} = -\beta\dot{\gamma}^k\left(\lambda - \lambda_e\right)^2, \text{ if } \lambda > \lambda_e \tag{2}$$

Eq. (1) shows that the Newtonian part is always present on the shear stress and is the only component when the material is completely unstructured ($\lambda = 0$). Also, it is clear that the gelled waxy crude is taken to be a yield stress material, with this property being a fixed parameter. Eq. (2) can be understood as the rate of breakdown of the gel and, in this equation, the authors disregard any kind of structure build-up, meaning that their model was created to evaluate only breakdown conditions. From this equation, the equilibrium structural parameter is the minimum value that can be achieved and, once the gel is broken, it will not rearrange its structure.

Another classical mathematical model that relies on the structural kinetics approach is the one developed by Sestak *et al.* (1987) and used on the same year by Cawkwell and Charles (1987) and similarly later on by Wachs et al. (2009). In their model, the authors consider the waxy crude oil to be a thixotropic material, and evaluate its characteristics by using a mathematical definition based on the work by Houska (1981), as described in Eq. (3):

$$\begin{cases} \tau = \tau_{y0} + \tau_{y1}\lambda + \left(k + \lambda\Delta k\right)\dot{\gamma}^n, & if \quad \tau > \tau_{y0} + \lambda\tau_{y1} \\ \dot{\gamma} = 0, & if \quad \tau < \tau_{y0} + \lambda\tau_{y1} \end{cases}, \tag{3}$$

where τ_{y0} is a constant yield stress value for a specific temperature and τ_{y1} is a structure (or time) dependent yield stress. k, Δk and n are constant values for a specific temperature.

Eq. (3) shows again the concern of the first models on the subject to take into account the yield stress of the material. In a different approach than the used by Petrellis and Flumerfelt (1973), Sestak *et al.* (1987) make use of two components for the yield stress. By doing so, the model would predict that, even for a completely unstructured state ($\lambda = 0$), the oil is going to behave as a yield stress material. However, this may not be true, as depending on the conditions the oil may behave as a Newtonian fluid (for oil reservoir temperatures, as an example). To avoid such condition, the authors state that this is a constitutive equation applied to the gelled oil. Another interesting difference from these two papers arise from the analysis of the kinetic equation of the Sestak *et al.* (1987) model:

$$\frac{d\lambda}{dt} = a\left(1-\lambda\right) - b\dot{\gamma}^m\lambda \, , \tag{4}$$

where a, b and m are constants.

Unlike Eq. (2), Eq. (4) models both the destruction and restoration of the gel. On the RHS of Eq. (4), two terms can be seen. The first one models the build-up of the structure (as λ varies from 0 to 1), and a is named the build-up coefficient, while the second term is the destruction one and b and m are destruction constants. This means that even though the material may be destroyed due to shear, some parts of it can try to reestablish a gelled structure. Also, it is interesting to note that the breakdown is favored by high shear rates, while the build-up term is independent of the flow applied. Therefore, the build-up should be slower than the breakdown in a flow for high shear rates.

Following the idea of developing a yield stress model for waxy crudes, Rønningsen (1992) modified the classical Bingham model (Bird *et al*., 1987). This modification consisted on turning the yield stress into a time-dependent function, while the plastic viscosity was assumed to be constant, i.e., the viscosity decay of the gel on a breakdown process results only from the decrease of the yield stress, as stated in Eq. (5):

$$\begin{aligned} &\tau = \tau_y\left(t\right) + \mu_p\dot{\gamma}, \text{ if } \tau > \tau_y\left(t\right) \\ &\dot{\gamma} = 0, \; \tau < \tau_y\left(t\right) \end{aligned} \quad , \tag{5}$$

where μ_p stands for the plastic viscosity of the Bingham model.

The yield stress was modeled as follows:

$$\tau_y\left(t\right) = \frac{\tau_y\left(0\right) - \tau_y\left(\infty\right)}{1+kt} + \tau_y\left(\infty\right) \tag{6}$$

where k is a constant.

As a waxy crude sample is at rest and gelled, its structure shall exhibit the yield stress τ_y (0), which is going to be the highest yield stress in this model. As time passes by in a shear induced flow, the yield stress will decay following Eq. (6) and, if the experiment time is long enough, a minimum denoted by τ_y (∞) can be hypothetically achieved. This also means that, in this model, there is no recovery of the structure.

This modified Bingham model was used by Davidson *et al.* (2004) and also by Chang *et al.* (1999), who found that the yield stress and the time-dependent rheology of the oils play a major role in order to determine the flow rate after a start-up process. To improve the yield stress concept, Chang *et al.* (1999) discuss it as three different parts, and present an important characteristic of gelled waxy crudes mostly neglected by earlier papers: elasticity. They state that when imposing flow over a sample at rest, the material will initially respond elastically, and the first yield stress is the elastic one, which represents the limit between elastic deformation and creep. By the definition of an elastic deformation, this means that if the maximum shear stress on a start-up flow experiment is lower than the elastic yield stress, the process is going to fail, as the material will return to its initial condition when the stress is relieved. The second is the static yield stress, corresponding to the minimum stress required to start the flow from a rest condition. Finally, the third yield stress is the dynamic one, which arises from an extrapolation of the flow curve to a zero shear rate condition.

It is worth mentioning another work that used this same modified Bingham model, the one by Davidson *et al.* (2007). This paper aimed to evaluate a start-up flow under the condition of a multi-plug gelled pipeline, i.e., several gelled waxy crude oil plugs separated by gas. For each plug, Eq. (5) was used and, in addition, the Casson classical model (Chhabra and Richardson, 2008) was adapted to be compared as an alternative to the Bingham model. Their modifications to the Casson model followed the idea used for Bingham: turning the constant yield stress into a time-dependent function and, in this case, making the plastic viscosity a time-dependent function as well. In their model, the yield stress follows the concept presented on Eq. (6) and so does the plastic viscosity, with the difference that the constant k becomes a different constant, k_1 and τ_y becomes η_v in Eq. (6). The modified Casson constitutive equation is described by Eq. (7), where μ_c stands for the Casson plastic viscosity.

$$\begin{aligned} &\tau^{1/2} = \tau_y\left(t\right)^{1/2} + \left[\mu_c\left(t\right)\dot{\gamma}\right]^{1/2}, \; \tau \geq \tau_y\left(t\right) \\ &\dot{\gamma} = 0, \; \tau < \tau_y\left(t\right) \end{aligned} \tag{7}$$

By comparing the results collected from Eq. (5) and Eq. (7), the authors were able to confront the performance of both constitutive equations applied to waxy crudes. To adjust the parameters of the Bingham model, they used the experimental results from Rønningsen (1992), and suitable values for the Casson model. As a conclusion, they found out that the results from the models were very similar for the mass flow rate and the interface between the gelled oil and the incoming fluid and, therefore, no significant discrepancy when using either is expected. Even though this is an interesting result, it does not mean that both models successfully describe the whole behavior of gelled waxy crudes. In fact, they lack important features that are present on later works. However, at least for the viscoplastic behavior of the gel, it seems that both models can be satisfactorily employed.

Another important model is the one developed by Souza Mendes (2009). The author built a model that incorporated elasticity, which was mentioned earlier by Chang *et al*. (1999) as an important characteristic to be taken into account. To do so, a modified Maxwell viscoelastic model was employed, which consists on a spring associated to a dashpot in series. The difference from the classic Maxwell model comes from the fact that, for this formulation, the author makes use of the structural kinetics approach and makes both the elastic modulus and the plastic viscosity functions of the structure of the material, instead of constant values. The constitutive equation for this model is described in Eq. (8), where η_v stands for the structural viscosity and G for the elastic modulus:

$$\tau + \frac{\eta_v(\lambda)}{G(\lambda)}\dot{\tau} = \eta_v(\lambda)\dot{\gamma} \tag{8}$$

With this constitutive equation, the author is able to predict the stress overshoot during a start-up operation. This is an important feature that only exists because of elasticity. Other models that do not consider elasticity cannot reproduce this kind of behavior. Also, it is interesting to note that when λ is minimum (zero in this case), the elastic modulus tends to infinity, while the structural viscosity reaches a finite value, as can be seen on Eqs. (9) and (10).

$$G = \frac{G_0}{\lambda^m} \tag{9}$$

$$\eta_v(\lambda) = \left(\frac{\eta_0}{\eta_\infty}\right)^{\lambda} \eta_\infty \tag{10}$$

where G_0 is the completely structured elastic modulus, m is a constant parameter, η_0 is the viscosity of the completely structured material and η_∞ is the viscosity of the completely unstructured material.

The dependency of the elastic modulus and the structural viscosity on the structural parameter depicted on Eqs. (9) and (10) mean that when $\lambda = 0$, a purely viscous behavior is predicted by the model. In addition, it can be noted from Eq. (8) that if the shear stress on the material does not change with time a purely viscous behavior is predicted as well. The author also developed other models where a Jeffreys-like approach is employed, such as in Souza Mendes (2011) and Souza Mendes and Thompson (2013). These models were able to represent many of the expected phenomena, such as an elastic behavior of the gelled material and the stress overshoot during a start-up process, even though they have many parameters and are not easily calibrated.

From the evolution of the models, it can be seen that elasticity is as an important feature to be presented. Zhao *et al.* (2012), for instance, tried to understand and model the bonds between paraffin crystals when the oil is gelled. The authors evaluated their interaction as hookean springs at low deformation, as Eq. (11) depicts:

$$\tau = h(\lambda - \lambda_e)^n \gamma \tag{11}$$

where h stands for a hookean constant, λ_e is the equilibrium structural parameter, n is a constant due to the structure complexity of the material and γ is the strain.

Based on Eq. (11), which describes the material behavior under low strains, the authors developed an equation to evaluate the shear stress at both high and low strains. In order to achieve it, they had to incorporate a term to account for the slurry flow that occurs beyond the crystals bonds are broken. The final equation developed by them is presented in Eq. (12):

$$\tau = \left[\frac{1}{(n-1)k\gamma + \dfrac{1}{\lambda_0^{\,n-1}}}\right]^{n/(n-1)} \left[h\gamma - \frac{\tau_s}{\lambda_0^{\,n}}\right] + \tau_s \tag{12}$$

where k is another constant of the model, λ_0 is the initial value of the structural parameter and τ_s is a residual term to capture the slurry flow condition.

Even though the authors employ elasticity in their concept by modeling the bonds as hookean springs, it is interesting to note that, in this work, Zhao *et al.* (2012) have developed an equation that relates the shear stress of the material with the strain, not the shear rate. Also, the task of modeling the bonds between crystals before structure breakdown is not easily done. The idea of modeling it by hookean springs at low deformation seems an interesting approach at first, and some good results when compared to experimental data were found by the authors. However, the complexity of modeling these interactions can be increasingly difficult due to the various small-scale phenomena that can occur, such as a strength difference between crystals, for instance.

Guo *et al*. (2015) presented nine mathematical models applied to waxy crude oils and evaluated their accuracy on representing the material, most of them based on the Houska's model (Houska, 1981). In their analyses, the authors found out that the model of Zhao (1999) showed one of the best results. In this model, the author based his study on a possible degree of irreversibility for the waxy crudes. To improve Houska's model with this idea, the author incorporated two different structural parameters, one for the reversible part of the structure, and one for the irreversible, as shown in Eq. (13).

$$\tau = \tau_{y0} + \lambda_1 \tau_{y1} + \lambda_2 \tau_{y2} + \left(k + \lambda_1 \Delta k_1 + \lambda_2 \Delta k_2 \right) \dot{\gamma}^n \tag{13}$$

where λ_1 and τ_{y1} are respectively the structure parameter and the yield stress of the recoverable structure, and λ_2 and τ_{y2} are respectively the structure parameter and the yield stress of the unrecoverable structure.

To model the unrecoverable part of the structure, the author modeled it on the same format as Petrellis and Flummerfelt (1973), meaning that the kinetic equation for λ_2 only exhibits a decreasing term, not being able to restructure the material. Even though their model did not consider elasticity, Guo *et al.* (2015) found that it was able to predict the behavior better than the model by Houska (1981) for step increases in the shear rate.

Kumar *et al*. (2016) modeled waxy crude oils based on the Kelvin-Voigt viscoelastic approach. By doing so, the authors separated the shear stress into an elastic and a viscous parcel, leading to the constitutive equation presented on Eq. (14).

$$\tau = \mu_s \left(1 + \frac{\mu_g / \mu_s}{\left(2m\gamma + 1 \right)^{1/2}} \right) \dot{\gamma} + \frac{G_0}{\left(2m\gamma + 1 \right)^{3/2}} \gamma \tag{14}$$

where μ_s is the steady state viscosity when the gel is completely broken; μ_g is the structural viscosity of the gel; m is a rate constant and G_0 is the initial value of the elastic modulus.

In their analysis, the steady state viscosity of the oil was assumed to be of a constant value. When comparing their model to experimental results, the authors found out that their model was capable of simulating the flow of a waxy crude in some important aspects, such as the pressure propagation at low times. However, when the times evaluated increased, discrepancies were found, supposedly by non-homogeneities of the gel strength, meaning that the model has to be more developed in order to be able to reproduce all the phenomena.

It is also interesting to note that on this model the authors used a simple kinetic equation, as shown in Eq. (15)

$$\frac{d\lambda}{dt} = -m\dot{\gamma}^q \left(\lambda - \lambda_e \right)^n \tag{15}$$

where q is assumed to be one in their analysis, n is a constant and λ_e is the equilibrium value for the structural parameter.

Eq. (15) shows an unbalance between the equilibrium state and the instant degree of structure. It can be seen, however, that this equation does not allow for structure reconstruction, as the equilibrium state is the minimum value that can be achieved, meaning that only the decay of structure can be modeled.

A model that aims to simplify the number and complexity of parameters to be fit, while keeping a good prediction of the phenomena present during a start-up is the one of Santos and Negrão (2017). The authors presented a general constitutive equation, described by Eq. (16):

$$\tau = F_1 \left(\dot{\gamma}, \ddot{\gamma}, \dot{\tau}, \theta_1, \theta_2, \ldots, \theta_n \right) \tag{16}$$

where $\dot{\gamma}$ is the shear rate, $\ddot{\gamma}$ is the rate of the shear rate, $\dot{\tau}$ is the rate of the shear stress and θ_1, θ_2 and θ_n are time dependent properties of the material.

One of the main differences of this approach from most of the ones studied is that it does not rely on a structural parameter and a kinetic equation, but rather on the unbalance between the equilibrium state and the instantaneous properties of the material, as given by Eq. (17).

$$\frac{d\theta_k(L,t)}{dt} = F_2(L) F_3 \left[\theta_{k,e}(L) - \theta_k(L,t) \right] \tag{17}$$

where θ_k is an arbitrary property of the material, L is the imposed load, F_2 is a function of and invariant of the load and F_3 is a function of the property unbalance. Also, $\theta_{k,e}$ is the equilibrium value of the property θ_k.

By using a Jeffrey's like equation to exemplify the potential of their model, the authors showed that it is capable of predicting well experimental results for shear rate and shear stress controlled tests.

Two important works based on a concept called isotropic-kinematic hardening (or IKH) are Dimitriou and McKinley (2014) and Geri *et al*. (2017). These definitions are not commonly used in fluid mechanics, being part of plasticity theory and, as the authors define: "Kinematic hardening accounts for a movement of the center of the yield surface in stress space, while isotropic hardening accounts for an expansion or contraction of the yield surface". Dimitriou and McKinley (2014) define their model based on a combination of viscoelastic and plastic deformations. However, the authors are mostly concerned about the plastic contribution (the viscoelastic term was taken to be a Maxwell one, but can easily be changed into another classic viscoelastic model). This plastic parcel was evaluated as a modified Bingham equation, in order to take into account the isotropic-kinematic hardening.

The concept used by the authors is somewhat similar to that employed by Sestak *et al*. (1987), where the yield stress is decomposed in two parts. In this case, however, this decomposition comes from the yield surface definition. The center of the yield surface (defined as σ_{back}) can change due to the imposed plastic strain, while the yield stress (defined as σ_y) can change due to the contraction or expansion of the surface, which occurs due to the variation of the microstructure of the material. Plastic deformations, therefore, are present only if the imposed load lies out of the surface defined by $[\sigma_{back} + \sigma_y, \sigma_{back} - \sigma_y]$, which in this case is a single axis. In summary, the two main equations of the model are as follows:

$$\gamma = \gamma^{ve} + \gamma^{p} \tag{18}$$

$$\dot{\gamma}^{p} = \begin{cases} 0, & , if \left|\sigma - \sigma_{back}\right| < \sigma_y \\ \dfrac{\sigma - \sigma_{back}}{\left|\sigma - \sigma_{back}\right|}\left(\dfrac{\left|\sigma - \sigma_{back}\right| - \sigma_y}{\mu_p}\right) & , if \left|\sigma - \sigma_{back}\right| \geq \sigma_y \end{cases} \tag{19}$$

where γ^{ve} is the viscoelastic part of the deformation, γ^p is the plastic part, $\dot{\gamma}^p$ is the plastic shear rate and μ_p is the plastic viscosity of the material.

Geri et al. (2017) moved even further on the IKH idea by incorporating concepts of fractal aggregates of the microstructure. By doing so, the authors assumed that the existing crystals could agglomerate, forming fractal clusters, and somewhat changing the behavior of the material. Therefore, their new model was called FIKH (Fractal isotropic-kinematic hardening). Just as the model by Dimitriou and McKinley (2014) the authors did not give much attention to the viscoelastic model of choice, picking the Maxwell model for the linear viscoelastic region. The concepts of yield surface and back stress were used on this model again. In addition, the two main differences from the former one are the use of a second structural parameter that describes the evolution of the fractal connectivity between crystals, and the volume fraction of solid wax crystals, which is obtained by computing the mass fraction of crystallized wax. The authors claimed that the volume fraction of solid wax crystals is an important parameter when dealing with waxy crudes. This volume fraction can vary within the studied processes, i.e., it is not a constant value, but a function of the temperature of the oil and may also change its behavior. Another main difference arises from the modification of the plastic shear rate, which becomes as defined on Eq. (20), where *m* is a constant parameter.

$$\dot{\gamma}^{p} = \begin{cases} 0 & , if \ \left|\sigma - \sigma_{back}\right| \leq \sigma_y \\ \left(\dfrac{\left|\sigma - \sigma_{back}\right| - \sigma_y}{\mu_p}\right)^{1/m} & , if \ \left|\sigma - \sigma_{back}\right| > \sigma_y \end{cases} \tag{20}$$

It is interesting to note that the Geri et al. (2017) model has a high number of parameters to be fit, 15. The authors, however, claim that even though this number is high, most of the parameters are easily measured from experimental tests on rheometers, such as flow curves and oscillatory shear experiments. The constant *m*, for instance, can be computed from the slope of the flow curve at high shear rates,

3. CONCLUSIONS

In this work, an analysis of some papers devoted to modeling waxy crude oil was conducted. Finding equations to model such a complex material is a difficult task, which can be assisted by understanding how the ideas behind constitutive equations have evolved up to nowadays. It can be seen that the concern of the first models was mainly about including yield stress and oftenly modifying classical models to transform constant yield stresses into time or structural dependent parameters. This makes sense, as a major problem on waxy crude's industry is the start-up flow of a gelled oil, which exhibits a yield stress in practical terms. The time (or structural) dependency of the yield stress and of the material in general was mostly modeled as a thixotropy problem, which is an important feature but not exactly a reality for waxy crude oils, as mentioned earlier (Mendes *et al*., 2015; Tarcha *et al*., 2014).

Most of the models developed after Souza Mendes (2009) incorporate at least an elasticity term in their formulation. This demonstrates an important evolution from the papers that mainly cared about yield stress only, which were not able to reproduce characteristics such as pressure overshoot, for instance. However, most models do not give much attention to the phenomena under the elastic regime due to its smaller importance when compared to the structure breakdown that is led by plastic deformation.

After the incorporation of elasticity into the models, many different approaches to the problem were studied, such as Zhao *et al*. (2012) that aim to understand the links between crystals and their interactions, Santos and Negrão (2017), who seek a simpler yet representative model without relying on a structural parameter. Also, the IKH model and its variants incorporate plasticity theory into the modeling of gelled waxy crudes, while unfortunately increasing the complexity of the models and the number of parameters to be fit.

From these analyses, it can be seen that the modeling of gelled waxy crudes has evolved from a simple understanding of the steady-state flow of the material, to approaches that try to mimic with perfection the most complex experimental results. However, no general model yet exists, one that is capable of depicting every phenomena exhibited by the material while keeping it simple to fit and analyze. The different approaches seen over the last years only make it even more clear that the area still has a lot more potential to be researched.

4. ACKNOWLEDGEMENTS

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