

A NUMERICAL STUDY OF THE SIMULTANEOUS NATURAL CONVECTIVE HEAT TRANSFER FROM THE TOP AND BOTTOM SURFACES OF A THIN HORIZONTAL CIRCULAR DISK IMBEDDED IN AN ADIABATIC CONCENTRIC DISK

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Abstract. A numerical study of natural convective heat transfer from a thin, two-sided, horizontal circular disk having a uniform surface temperature that is imbedded in an adiabatic concentric disk has been undertaken. Attention has been given to the effects of the size of the adiabatic concentric disk relative to the diameter of the heated circular diameter on the natural convective heat transfer rate from the top and bottom surfaces of the circular heated disk and on the total heat transfer rate from the heated circular surface. The temperature of the top and bottom surfaces are higher than the temperature of the surrounding fluid. While there have been previous studies of natural convective heat transfer from thin, two-sided horizontal circular disks, these studies have only considered the case where the circular disk is not imbedded in an adiabatic concentric disk. The conditions considered in the present study are such that laminar flow, transitional flow and turbulent flow can occur over the circular disk. The flow around the entire circular disk has been considered and the heat transfer from the top and bottom surfaces of the disk have been studied. The flow has been assumed to be axisymmetric and steady and the Boussinesq approach has been adopted. The k -epsilon turbulence model has been used with full account being taken of buoyancy force effects. This turbulence model was applied under all conditions considered. The commercial CFD solver ANSYS FLUENT[®] has been used to obtain the solution. The mean heat transfer rate from the top and bottom surfaces of the circular disk has been expressed in terms of a mean Nusselt numbers based on the circular disk diameter. These Nusselt numbers are dependent on the Rayleigh number, on the Prandtl number and on the relative size of the adiabatic concentric disk. Results have been obtained only for a Prandtl number of 0.74, i.e., essentially the value for air. Values of the mean Nusselt number averaged over the top and bottom surfaces of the circular disk have been considered. Results have been obtained for a wide range of adiabatic concentric disk relative sizes and used to determine its influence on the natural convective heat transfer rates from the top and bottom surfaces of the circular disk and on the total heat transfer rate from this disk.

Keywords: natural convection, circular disk, adiabatic edge, k -epsilon turbulence model

NOMENCLATURE

A	[m ²]	Area of heated surface
D	[m]	Diameter of heated circular disk
g	[m/s ²]	Gravitational acceleration
k	[W/(m.K)]	Thermal conductivity
\overline{Nu}	[-]	Mean Nusselt number based on D and on the mean heat transfer rate
Pr	[-]	Prandtl number
Q	[W]	Mean heat transfer rate
Ra	[-]	Rayleigh number based on D
T_f	[K]	Undisturbed fluid temperature
T_w	[K]	Heated surface temperature
Greek symbols		
α	[m ² /s]	Thermal diffusivity
β	[1/K]	Bulk coefficient of thermal expansion
Δ	[m]	Difference between external and internal radius of the adiabatic concentric disk
ν	[m ² /s]	Kinematic viscosity

1. INTRODUCTION

Natural convective heat transfer occurs in many practical situations and remains an area of considerable basic and applied interest. In the present article attention will be restricted to an external natural convective flow, that is, to a flow situation in which there are no constraining boundary surfaces near enough to the surface being considered to have any significant influence on the natural convective flow over this surface.

Engineering situations involving external natural convective heat transfer of the general type here being considered occur in electrical, electronic and measurement systems. These devices may have different geometries, being common the shape of a circular disk. Usually, such devices are attached to some type of concentric disk, and in some cases, heat transfer in such concentric disks can be considered negligible with respect to the heat transfer in the device. However, the external radius of this concentric disk can have a considerable influence on the convective heat transfer from the circular disk.

While there have been previous studies of natural convective heat transfer from thin, two-sided circular disks, these studies have considered only the case where the circular disk is not mounted in an adiabatic concentric surrounding surface. The conditions considered in the present study are such that laminar flow, transitional flow and turbulent flow can occur over the circular disk. Numerical studies of natural convective heat transfer from horizontal plane surfaces of relatively simple shape for conditions under which laminar, transitional, and turbulent flow exist were described in the works of Oosthuizen (2014), Oosthuizen (2015a), Oosthuizen (2015b), Oosthuizen and Kalendar (2015a), Oosthuizen and Kalendar (2015b) and Oosthuizen (2016).

The present article describes a numerical study of natural convective heat transfer from a thin, two-sided, horizontal circular disk having a uniform surface temperature and imbedded in an adiabatic concentric disk. Attention has been given to the effects of the relative size of the external adiabatic concentric surface on the natural convective heat transfer from the top and bottom surfaces and on the total surface of the heated circular disk. The temperature of the top and bottom surfaces of the circular disk are the same and higher than the temperature of the surrounding fluid. Results for a wide range of relative concentric disk sizes have been obtained and used to determine its influence in the natural convective heat transfer from the top and bottom surfaces and from the total surface area of the circular heated disk.

2. PHYSICAL SITUATION

The physical situation considered in this study is shown in Fig. 1:

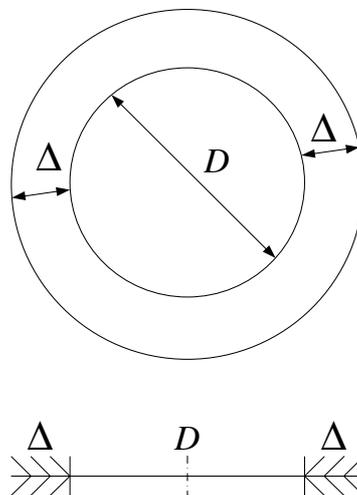


Figure 1. Horizontal thin isothermal circular disk imbedded in an adiabatic concentric disk.

The situation here being considered consists of a thin, two-sided, horizontal circular disk with diameter D having a uniform surface temperature T_w . This disk is imbedded in an adiabatic concentric disk having size Δ which is the difference between external and internal radii of the adiabatic concentric disk. The circular disk is in contact with a surrounding fluid at constant temperature T_f . For a heated surface, $T_w > T_f$, and the top and bottom heated surfaces will exchange energy to the surrounding fluid by natural convective heat transfer. The purpose of this study is to study the mean heat transfer rate by natural convection between the heated surfaces and the surrounding fluid. The mean heat transfer rates have been expressed in terms of mean Nusselt numbers based on the difference between the circular disk and surrounding fluid temperatures and on the diameter D of the circular disk, that is:

$$\overline{\text{Nu}}_{D,\text{top/bottom}} = \frac{4Q_{\text{top/bottom}}}{\pi k D (T_w - T_f)} \quad (1)$$

and

$$\overline{\text{Nu}}_{D,\text{total}} = \frac{2Q_{\text{total}}}{\pi k D (T_w - T_f)} \quad (2)$$

It should be noted that the mean total Nusselt number is the average between the mean top Nusselt number and the mean bottom Nusselt number.

3. SOLUTION PROCEDURE

In obtaining the numerical results discussed here the mean flow has been assumed to be steady. The Boussinesq approximation has been used, i.e., fluid properties have been assumed to be constant except for the density change with temperature that gives rise to the buoyancy forces, the density change being assumed to be proportional to the temperature change. Radiation heat transfer effects have been neglected. Allowance has been made for the possibility that turbulent flow can occur in the system. In order to deal with this the basic *k-epsilon* turbulence model with standard wall functions and with full account being taken of buoyancy force effects has been used. The governing equations subject to the boundary conditions have been solved numerically using the commercial CFD solver ANSYS FLUENT®. In all cases extensive grid independence and convergence-criteria independence testing was undertaken. The numerical approach used here in order to determine when turbulence develops which involves solving the Reynolds averaged governing equations together with a turbulence model in which the effects of buoyancy forces are taken into account for all conditions considered and then monitoring the results obtained with increasing Rayleigh numbers to determine when significant turbulence effects develop. This approach has been used quite extensively in the study of forced convective flows, e. g., see Schmidt and Patankar (1991) and Zheng *et al.* (1998). The solutions presented in the next section all have the following parameters:

1. The Rayleigh number, Ra_D , based on the diameter D of the heated surface and the difference between the temperature of the heated isothermal surface, T_w , and the temperature of the undisturbed fluid well away from the system, T_f , i.e.:

$$\text{Ra}_D = \frac{g \beta (T_w - T_f) D^3}{\nu \alpha} \quad (3)$$

2. The dimensionless relative size of the adiabatic concentric disk, Δ/D .
3. The Prandtl number, Pr .

Results have only been obtained for a Prandtl number of 0.74, i.e., effectively the value for air.

4. RESULTS AND DISCUSSION

The horizontal circular disk is assumed to be thin, i.e., is assumed to have a negligible thickness with diameter D and maintained at a uniform surface temperature. The surrounding fluid is air at atmospheric pressure. Numerical simulations were performed for Rayleigh numbers varying between 10^4 to 10^{14} . Typical variations of the mean top Nusselt number with the Rayleigh number for various values of Δ/D are shown in Fig. 2. $\Delta/D = 0$ corresponds to the case of a circular disk without an adiabatic concentric disk. As expected, the mean top Nusselt number increases with increasing of the Rayleigh number. However, it can be seen that the mean top Nusselt number at a given Rayleigh number decreases with increasing of Δ/D . Variations of the mean bottom Nusselt number with the Rayleigh number for various values of Δ/D are shown in Fig. 3. Again, as expected, the mean bottom Nusselt number increases with increasing Rayleigh number. It will be seen that the mean bottom Nusselt numbers at a given Rayleigh number change only by a very small amount with an increasing of Δ/D . It will be noted from the results given in Figs. 2 and 3 that the mean Nusselt numbers for the bottom surface are mainly significantly higher than those for the top surface.

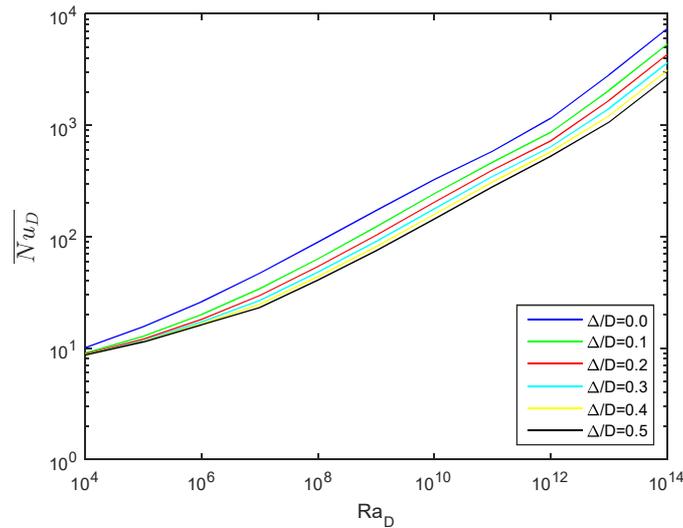


Figure 2. Variations of mean top Nusselt number with Rayleigh number for various Δ/D .

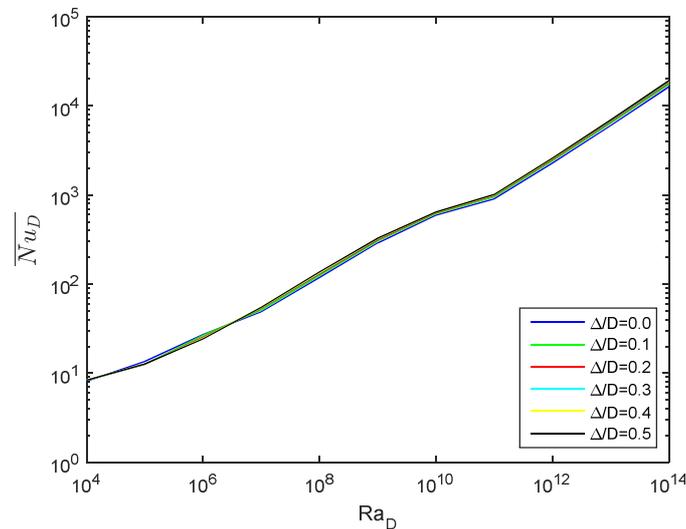


Figure 3. Variations of mean bottom Nusselt number with Rayleigh number for various Δ/D .

Lastly, variations of the mean total Nusselt number with the Rayleigh number for various values of Δ/D are shown in Fig. 4. Once again, it will be seen that the mean total Nusselt number increases with increasing of the Rayleigh number. However it will be seen that the mean total Nusselt number at a given Rayleigh number almost do not change significantly with the increasing of Δ/D , suggesting that the mean bottom Nusselt number dominates the natural convective heat transfer of a thin horizontal circular disk imbedded in a adiabatic concentric disk.

It can be seen from the results given in Fig. 5 that the variations in the mean top Nusselt number at a fixed Rayleigh number over the range of Δ/D considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6 . However, the decrease in the mean top Nusselt number for Rayleigh number of 10^8 is approximately 56% for $\Delta/D = 0.5$ when compared to the value for $\Delta/D = 0$. A similar behavior can be seen in the results given in Fig. 6. The variation in the mean top Nusselt number at a fixed Rayleigh number over the range of Δ/D considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12} . However, the decrease in the mean top Nusselt number for Rayleigh number equal to 10^{14} is approximately 62% for $\Delta/D = 0.5$ when compared to $\Delta/D = 0$. Figs. 5 and 6 thus indicate that an increase in Δ causes a decrease in the natural convective heat transfer rate at the top of the horizontal circular disk.

The results presented in Figs. 7 and 8 which show typical variations of the mean bottom Nusselt number with Δ/D for various values of the Rayleigh number show basically a similar behavior to that shown by the results given in Figs. 5 and 6. Figs. 7 and 8 illustrate the variations in the mean bottom heat transfer rate, expressed in terms of the mean bottom Nusselt number, produced by different values of Δ/D .

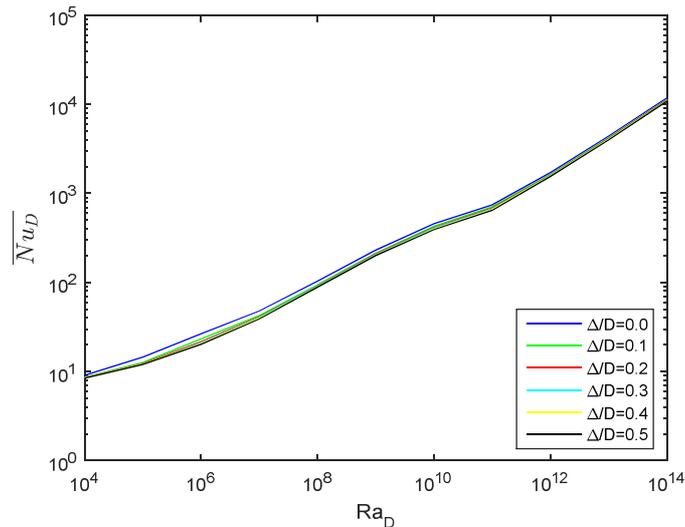


Figure 4. Variations of mean total Nusselt number with Rayleigh number for various Δ/D .

Typical variations of the mean top Nusselt number with Δ/D for various values of the Rayleigh number are shown in Figs. 5 and 6:

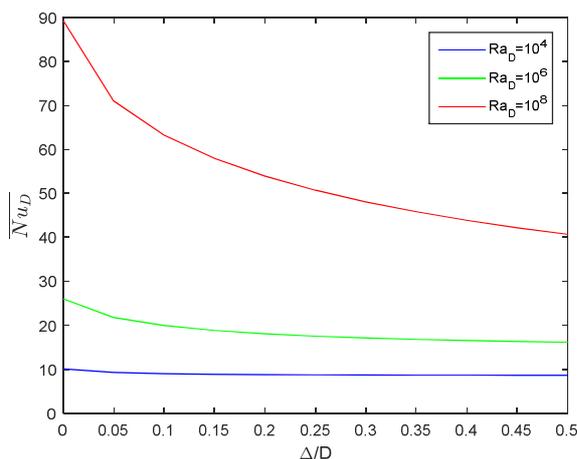


Figure 5. Variations of mean top Nusselt number with Δ/D for Rayleigh numbers of 10^4 , 10^6 and 10^8 .

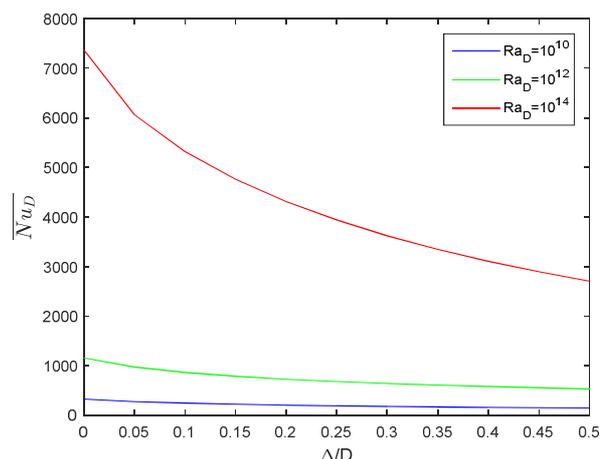


Figure 6. Variations of mean top Nusselt number with Δ/D for Rayleigh numbers of 10^{10} , 10^{12} and 10^{14} .

It will be seen from the results given in Fig. 7 that the variations in the mean bottom Nusselt number at a fixed Rayleigh number over the range of Δ/D considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6 . However, the increase in the mean bottom Nusselt number for Rayleigh number equal to 10^8 is approximately 17% for $\Delta/D = 0.5$ when compared to the value for $\Delta/D = 0$. A similar behavior can be seen in the results presented in Fig. 8. The variation in the mean bottom Nusselt number at a fixed Rayleigh number over the range of Δ/D considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12} . However, the increase in the mean bottom Nusselt number for Rayleigh number equal to 10^{14} is approximately 18% for $\Delta/D = 0.5$ when compared to the value for $\Delta/D = 0$. It will be seen that contrary to way in which the value of Δ effects the value of the Nusselt number for the top surface of the circular disk, Figs. 7 and 8 indicate that an increase in Δ causes an increase in the natural convective heat transfer rate from the bottom of the horizontal circular disk.

Lastly, results are presented for the mean total Nusselt number in Figs. 9 and 10, these figures showing typical variations of the mean total Nusselt number with Δ/D for various values of the Rayleigh number. It can be seen from the results given in Fig. 9 that the variations in the mean total Nusselt number at a fixed Rayleigh number over the range of Δ/D considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6 . However, the decrease in the mean total Nusselt number for Rayleigh number equal to 10^8 is approximately 14% for $\Delta/D = 0.5$ when compared to the value for $\Delta/D = 0$. A similar behavior can be seen in the results given in Fig. 10. The variation in the mean total Nusselt number at a fixed Rayleigh number over the range of Δ/D considered is also relatively small for Rayleigh

numbers equal to 10^{10} and 10^{12} . However, the decrease in the mean total Nusselt number for Rayleigh number equal to 10^{14} is approximately 7% for $\Delta/D = 0.5$ when compared to the value for $\Delta/D = 0$. Figs. 9 and 10, which show that the combined effect of Δ on the heat transfer rate on top and bottom surfaces of the circular disk, indicate that an increase in Δ causes an overall decrease in the natural convective heat transfer rate from the horizontal circular disk.

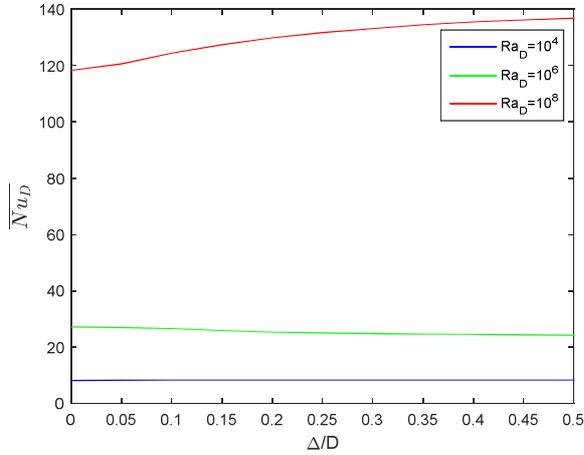


Figure 7. Variations of mean bottom Nusselt number with Δ/D for Rayleigh numbers of 10^4 , 10^6 and 10^8 .

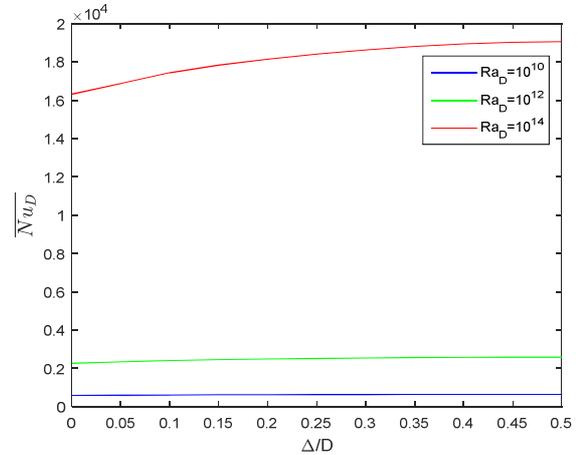


Figure 8. Variations of mean bottom Nusselt number with Δ/D for Rayleigh numbers of 10^{10} , 10^{12} and 10^{14} .

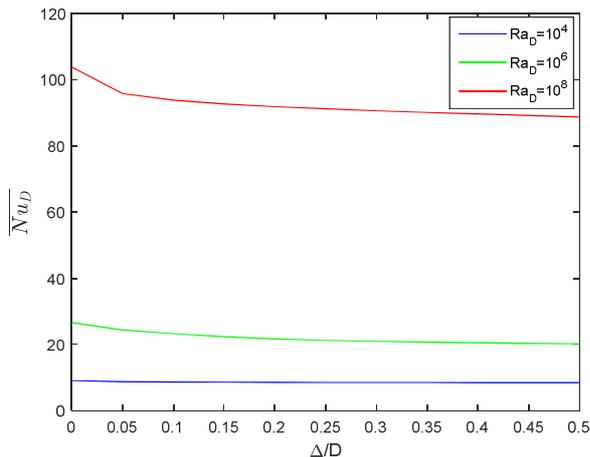


Figure 9. Variations of mean total Nusselt number with Δ/D for Rayleigh numbers of 10^4 , 10^6 and 10^8 .

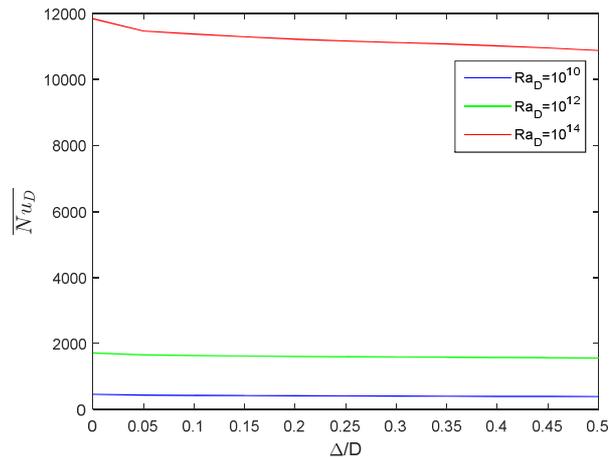


Figure 10. Variations of mean total Nusselt number with Δ/D for Rayleigh numbers of 10^{10} , 10^{12} and 10^{14} .

5. CONCLUSIONS

A study of the effect of an adiabatic concentric disk on the natural convective heat transfer rate from a horizontal heated isothermal circular disk has been undertaken. The effect of different dimensionless adiabatic concentric disk Δ/D , in particular, has been studied. Some conclusions derived from the results obtained in the present study are:

1. The mean top Nusselt number increases with increasing of the Rayleigh number. However, the mean top Nusselt number at a given Rayleigh number decreases with increasing of Δ/D .
2. The mean bottom Nusselt number increases with increasing of the Rayleigh number. However the mean bottom Nusselt number at a given Rayleigh number effectively does not change with increasing of Δ/D .
3. The mean total Nusselt number at a given Rayleigh number effectively does not change with increasing of Δ/D , because the mean bottom Nusselt number dominates the natural convective heat transfer from a thin horizontal circular disk imbedded in a concentric adiabatic disk.

Thus an increase in Δ causes a decrease in the natural convective heat transfer rate from the top of the horizontal heated circular disk, an increase in the natural convective heat transfer rate from the bottom of the horizontal heated circular disk, and decrease in the overall natural convective heat transfer rate from the horizontal circular disk.

6. ACKNOWLEDGEMENTS

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