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A NUMERICAL STUDY OF THE NATURAL CONVECTIVE HEAT TRANSFER FROM THE SURFACE OF A THIN HORIZONTAL PLATE HAVING A WAVY SURFACE WITH VARIABLE HEIGHT

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Abstract. *A numerical study of natural convective heat transfer from a one-sided, two-dimensional horizontal plate having a uniform surface temperature has been undertaken. The surface shape is wavy, and attention has been given to both the case where the surface waves have a triangular shape with variable height and to the case where the surface waves have a rectangular shape with variable height. Three profiles were considered to define the variable height: linear, parabolic and exponential. The temperature of the plate surface is higher than the temperature of the surrounding fluid. While there have been some limited previous studies of natural convective heat transfer from a one-sided horizontal plates, these studies have mainly considered only the case where the plate is flat (non-wavy) and where the flow over the plate is laminar. Here, the conditions considered are such that laminar flow, transitional flow and turbulent flow can occur over the plate. The flow has been assumed to be two-dimensional and steady and the Boussinesq approach has been adopted. The k-epsilon turbulence model has been used with full account being taken of buoyancy force effects. This turbulence model was applied under all conditions considered. The commercial CFD solver ANSYS FLUENT© has been used to obtain the solution. The mean heat transfer rate from the wavy surface has been expressed in terms of a mean Nusselt number based on the plate width. The Nusselt number is dependent on the Rayleigh number, on the Prandtl number and on the width of the plate. Results have been obtained only for a Prandtl number of 0.74, i.e., essentially the value for air. Values of the mean Nusselt number averaged over the wave surface and of the local Nusselt number for the wave surface have been considered. Results for a wide range of dimensionless surface wave height profiles for both triangular and rectangular surface waves have been obtained and used to determine which profile produces the best enhancement in the natural convective heat transfer rate compared to that from a plane (non-wavy) horizontal surface.*

Keywords: *natural convection, wavy surface, triangular waves, rectangular waves, variable height*

NOMENCLATURE

d	[m]	Width of triangular/rectangular waves
D	[-]	Dimensionless width of triangular/rectangular waves, d/L
g	[m/s^2]	Gravitational acceleration
$\overline{h}$	[W/(m^2.K)]	Mean heat transfer coefficient
H	[-]	Dimensionless height of triangular/rectangular waves, t/L
k	[W/(m.K)]	Thermal conductivity
L	[m]	Length of the heated surface
$\overline{\mathrm{Nu}}$	[-]	Mean Nusselt number based on L and on the mean heat transfer rate
Pr	[-]	Prandtl number
Q	[W]	Mean heat transfer rate
Ra	[-]	Rayleigh number based on L
t	[m]	Height of triangular/rectangular waves
T_f	[K]	Undisturbed fluid temperature
T_w	[K]	Heated surface temperature
w	[m]	Width of the heated surface
x	[m]	Horizontal coordinate

X	[-]	Dimensionless horizontal coordinate, x/L
Greek symbols		
α	[m^2/s]	Thermal diffusivity
β	[1/K]	Bulk coefficient of thermal expansion
ν	[m^2/s]	Kinematic viscosity

1. INTRODUCTION

Natural convection heat transfer occurs in many practical situations and remains an area of considerable basic and applied interest. In the present article attention will be restricted to an external natural convective flow, that is, a flow situations in which there are no constraining boundary surfaces near enough to the surface being considered to have any significant influence on the natural convective flow over this surface. Increasing the heat transfer rate in a given situation involving natural convective flows is often difficult to accomplish. Using a wavy surface is one method of attempting to enhance natural convective heat transfer rates.

The enhancement of the heat transfer rate produced by using a wavy surface arises from the increase in the surface area exposed to the fluid to which the heat is being transferred and, in some cases, to the changes in the near surface flow produced by the presence of the surface waves. The total enhancement of the heat transfer rate will depend on the shape and relative size of the surface waves. Many wavy shapes have been considered is past studies but the most common shapes considered remain rectangular, triangular and sinusoidal waves. The enhancement of the heat transfer rate produced by using a wavy surface will also depend on the flow situation being considered, for example, flow over a plane surface or flow over a cylinder, and on the thermal boundary conditions at the surface. The two surface boundary conditions most commonly considered are those in which there is a uniform temperature over the surface and those in which there is a uniform heat flux over the surface. Another factor that influences the natural convective heat transfer rate from a surface is its orientation, that is, is it horizontal or is it vertical, or is it inclined to the vertical and whether, when inclined, it is facing upward or downward (Oosthuizen, 2016).

While there have been some previous studies of natural convective heat transfer from a one-sided horizontal plates, these studies have mainly considered only the case where the plate is flat (non-wavy) and where the flow over the plate is laminar. Here, the conditions considered are such that laminar flow, transitional flow and turbulent flow can occur over the plate. Numerical studies of heat transfer from a horizontal surface having triangular waves and rectangular waves with constant height for conditions under which laminar, transitional, and turbulent flow exist are described in the works of Oosthuizen, (2016a) and Oosthuizen, (2016b), respectively. Other studies of natural convective heat transfer from horizontal wavy surfaces with constant height are described in the works of Prétot, *et al.* (2000), Prétot, *et al.* (2003), Siddiqa and Hossain (2013) and Siddiqa, *et al.*, (2015). In all of these studies, the natural convective heat transfer rate was obtained from a one-side, two-dimensional horizontal plate having a uniform surface temperature.

The purpose of the present article is to develop a numerical study of natural convective heat transfer from a one-sided, two-dimensional horizontal plate having a uniform surface temperature. The surface shape is wavy, and attention has been given to both the case where the surface waves have a triangular shape with variable height and to the case where the surface waves have a rectangular shape with variable height. It is expected that waves with variable height can produce changes in the near surface flow and then promote and enhancement of the natural convective heat transfer rate. The temperature of the horizontal plate surface is higher than the temperature of the surrounding fluid. Results for three dimensionless surface wave profiles (linear, parabolic and exponential) for both triangular and rectangular surface waves will be obtained and used to determine whether the presence of the waves with variable height can produce an enhancement in the natural convective heat transfer rate compared to that from a plane (non-wavy) horizontal surface.

2. PHYSICAL SITUATION

The two physical situations considered in this study are shown in Fig. 1:

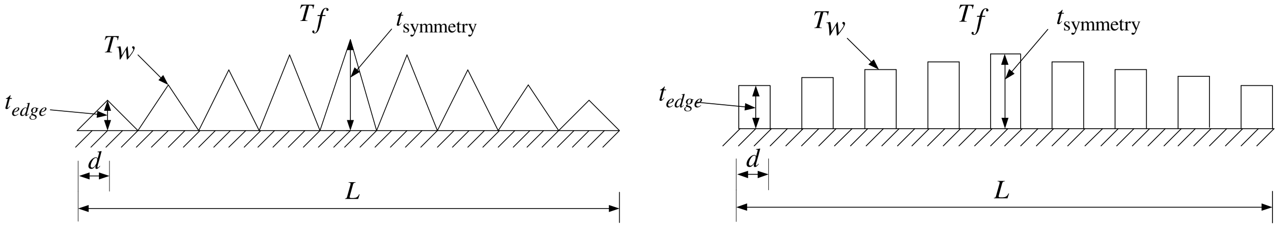


Figure 1. (a) Horizontal surface with nine triangular waves with variable height (b) Horizontal surface with nine rectangular waves with variable height

The situation here being considered consists of a two-dimensional horizontal plate having a uniform surface temperature T_w. The surface shape is wavy, and attention has been given to both the case where the surface waves have a triangular shape with variable height $t(x)$ and length *d* and to the case where the surface waves have a rectangular shape with variable height $t(x)$ and length *d*. In both cases, the nine waves are equally spaced. The surface is in contact with a surrounding fluid at constant temperature T_f. For a heated surface, $T_w > T_f$, and the heated surface will exchange energy with the surrounding fluid by natural convection. The horizontal plate has width L and depth w. The purpose of this study is to calculate the mean heat transfer rate by natural convection between the heated surface and the surrounding fluid. Because the flow is assumed symmetrical about the centerline, only half of the horizontal plate length has been considered. The mean heat transfer rate have been expressed in terms of mean Nusselt number based on the difference between the horizontal plate and surrounding fluid temperatures and on the width of the horizontal plate, that is:

$$\overline{\mathrm{Nu}}_L = \frac{2Q}{kw(T_w - T_f)} = \frac{2Q'}{k(T_w - T_f)} \quad (1)$$

In the initial situation considered, the shortest wave height, in dimensionless form, located on the edge of the horizontal surface, is $H_{\mathrm{edge}} = t_{\mathrm{edge}}/L = 0.02$, and the tallest height, in dimensionless form, located on the half of the horizontal surface (on symmetry line), is $H_{\mathrm{symmetry}} = t_{\mathrm{symmetry}}/L = 0.10$. To study the effect of the wave variable height, four additional geometric situations were considered, increasing the height of the wave located at the edge of the horizontal surface and decreasing the height of the wave located on the half of the horizontal surface.

In the first situation, it was considered $H_{\mathrm{edge}} = 0.04$ and $H_{\mathrm{symmetry}} = 0.08$. In the second situation, it was considered $H_{\mathrm{edge}} = H_{\mathrm{symmetry}} = 0.06$, a horizontal wave surface with constant height. In the third situation it was considered $H_{\mathrm{edge}} = 0.08$ and $H_{\mathrm{symmetry}} = 0.04$. Lastly, in the fourth situation it was considered $H_{\mathrm{edge}} = 0.10$ and $H_{\mathrm{symmetry}} = 0.02$. For each situation, the wave height profile was varied. With the values of H_{edge} and H_{symmetry} and its respective positions, $X_{\mathrm{edge}} = x_{\mathrm{edge}}/L$ and $X_{\mathrm{symmetry}} = x_{\mathrm{symmetry}}/L$, three different wave profiles were used to calculate the height of the three internal waves: a linear profile, a parabolic profile and an exponential profile. For a linear profile, a general expression for the wave profile can be written in a dimensionless form as:

$$H(X) = AX + B \quad (2)$$

where *A* and *B* are coefficients that can be determined from the knowledge of $H_{\mathrm{edge}}(X_{\mathrm{edge}})$ e $H_{\mathrm{symmetry}}(X_{\mathrm{symmetry}})$:

$$A = \frac{H_{\mathrm{edge}} - H_{\mathrm{symmetry}}}{X_{\mathrm{edge}} - X_{\mathrm{symmetry}}} \quad (3)$$

$$B = H_{\mathrm{edge}} - \left(\frac{H_{\mathrm{edge}} - H_{\mathrm{symmetry}}}{X_{\mathrm{edge}} - X_{\mathrm{symmetry}}} \right) X_{\mathrm{edge}} \quad (4)$$

Substituting Eqs. (3-4) into Eq. (2) and rearranging:

$$H(X) = H_{\mathrm{edge}} + \left(\frac{H_{\mathrm{edge}} - H_{\mathrm{symmetry}}}{X_{\mathrm{edge}} - X_{\mathrm{symmetry}}} \right) (X - X_{\mathrm{edge}}) \quad (5)$$

For a parabolic profile, a general expression for the wave profile can be written in a dimensionless form as:

$$H(X) = AX^2 + BX + C \quad (6)$$

where *A*, *B* and *C* are coefficients that can be determined from the knowledge of $H_{\mathrm{edge}}(X_{\mathrm{edge}})$, $H_{\mathrm{symmetry}}(X_{\mathrm{symmetry}})$ and $(dH/dX)_{X=X_{\min}} = 0$, where $X_{\min} = X_{\mathrm{edge}}$ if $X_{\mathrm{edge}} < X_{\mathrm{symmetry}}$ and $X_{\min} = X_{\mathrm{symmetry}}$ if $X_{\mathrm{edge}} > X_{\mathrm{symmetry}}$:

$$A = \frac{H_{\text{edge}} - H_{\text{symmetry}}}{X_{\text{edge}}^2 - X_{\text{symmetry}}^2 - 2X_{\min}(X_{\text{edge}} - X_{\text{symmetry}})} \tag{7}$$

$$B = \frac{H_{\text{edge}} - H_{\text{symmetry}}}{X_{\text{edge}}^2 - X_{\text{symmetry}}^2 - 2X_{\min}(X_{\text{edge}} - X_{\text{symmetry}})} \tag{8}$$

$$C = H_{\text{edge}} + \left[\frac{H_{\text{edge}} - H_{\text{symmetry}}}{X_{\text{edge}}^2 - X_{\text{symmetry}}^2 - 2X_{\min}(X_{\text{edge}} - X_{\text{symmetry}})}\right](2X_{\min}X_{\text{edge}} - X_{\text{edge}}^2) \tag{9}$$

Substituting Eqs. (7-9) into Eq. (6) and rearranging:

$$H(X) = H_{\text{edge}} + \left[\frac{H_{\text{edge}} - H_{\text{symmetry}}}{X_{\text{edge}}^2 - X_{\text{symmetry}}^2 - 2X_{\min}(X_{\text{edge}} - X_{\text{symmetry}})}\right](X^2 - 2X_{\min}X + 2X_{\min}X_{\text{edge}} - X_{\text{edge}}^2) \tag{10}$$

For an exponential profile, a general expression for the wave profile can be written in a dimensionless form as:

$$H(X) = AX^B \tag{11}$$

where *A* and *B* are coefficients that can be determined from the knowledge of $H_{\text{edge}}(X_{\text{edge}})$ and $H_{\text{symmetry}}(X_{\text{symmetry}})$:

$$A = \frac{H_{\text{edge}}}{\left(\frac{H_{\text{edge}}}{H_{\text{symmetry}}}\right)^{\left(\frac{X_{\text{edge}}}{X_{\text{edge}} - X_{\text{symmetry}}}\right)}} \tag{12}$$

$$B = \left(\frac{H_{\text{edge}}}{H_{\text{symmetry}}}\right)^{\left(\frac{1}{X_{\text{edge}} - X_{\text{symmetry}}}\right)} \tag{13}$$

Substituting Eqs. (12-13) into Eq. (11) and rearranging:

$$H(X) = H_{\text{edge}}\left(\frac{H_{\text{edge}}}{H_{\text{symmetry}}}\right)^{\left(\frac{X - X_{\text{edge}}}{X_{\text{edge}} - X_{\text{symmetry}}}\right)} \tag{14}$$

Equations (5), (10) and (14) can be used to determine the height of both the triangular and the rectangular waves. For rectangular waves, the point of height considered is located in the center of each horizontal segment of the corresponding height. According to the dimensions considered, the wave profile equations used for triangular waves can be seen in Tab. 1. In a similar way, according to the dimensions considered, the wave profile equations used for rectangular waves can be seen in Tab. 2. To verify the profile that is most suitable for the increase of the heat transfer rate by natural convection, it is convenient to name the wave height profiles as follows:

- For $H_{\text{edge}} = 0.00$ and $H_{\text{symmetry}} = 0.00$: PROFILE 0 (non-wavy flat plate)
- For $H_{\text{edge}} = 0.02$ and $H_{\text{symmetry}} = 0.10$: PROFILE 1
- For $H_{\text{edge}} = 0.04$ and $H_{\text{symmetry}} = 0.08$: PROFILE 2
- For $H_{\text{edge}} = 0.06$ and $H_{\text{symmetry}} = 0.06$: PROFILE 3 (constant wave height)
- For $H_{\text{edge}} = 0.08$ and $H_{\text{symmetry}} = 0.04$: PROFILE 4
- For $H_{\text{edge}} = 0.10$ and $H_{\text{symmetry}} = 0.02$: PROFILE 5

It should be noted that the expressions provided in Tab. 1 and 2 were developed for the specific case of nine triangular/rectangular waves.

Table 1. Wave profile equations for triangular waves

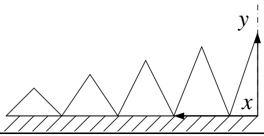
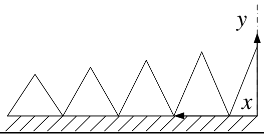
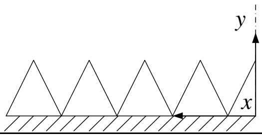
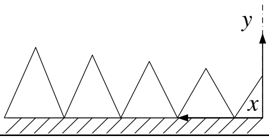
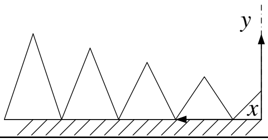
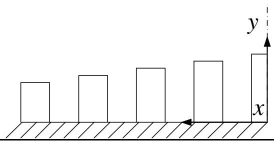
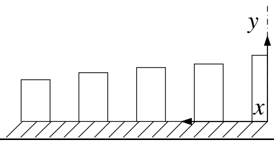
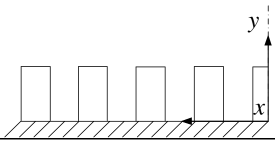
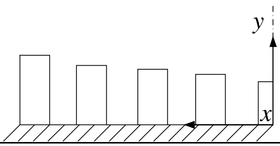
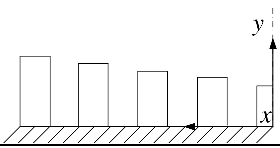
GEOMETRY	PROFILE EQUATION
	$H(X) = -0.18X + 0.1$ $H(X) = 0.405X^2 - 0.36X + 0.1$ $H(X) = 0.1(0.02675^X)$
	$H(X) = -0.09X + 0.08$ $H(X) = 0.2025X^2 - 0.18X + 0.08$ $H(X) = 0.08(0.210224^X)$
	$H(X) = 0.06$
	$H(X) = 0.09X + 0.04$ $H(X) = -0.2025X^2 + 0.18X + 0.04$ $H(X) = 0.04(4.756836^X)$
	$H(X) = 0.18X + 0.02$ $H(X) = -0.405X^2 + 0.36X + 0.02$ $H(X) = 0.02(37.383855^X)$

Table 2. Wave profile equations for rectangular waves

GEOMETRY	PROFILE EQUATION
	$H(X) = -0.18X + 0.1$ $H(X) = 0.405X^2 - 0.36X + 0.1$ $H(X) = 0.105(0.029^X)$
	$H(X) = -0.09X + 0.08$ $H(X) = 0.2025X^2 - 0.18X + 0.08$ $H(X) = 0.082(0.219^X)$
	$H(X) = 0.06$
	$H(X) = 0.09X + 0.04$ $H(X) = -0.2025X^2 + 0.18X + 0.04$ $H(X) = 0.039(4.574^X)$
	$H(X) = 0.18X + 0.02$ $H(X) = -0.405X^2 + 0.36X + 0.02$ $H(X) = 0.019(34.137^X)$

3. SOLUTION PROCEDURE

In obtaining the numerical results the mean flow has been assumed to be steady and the Boussinesq approximation has been used, i.e., fluid properties have been assumed to be constant except for the density change with temperature that gives rise to the buoyancy forces, the density change being assumed to be proportional to the temperature change. Radiation heat transfer effects have been neglected. Allowance has been made for the possibility that turbulent flow can occur in the system. In order to deal with this the basic *k-epsilon* turbulence model with standard wall functions and with full account being taken of buoyancy force effects has been used. The governing equations subject to the boundary conditions have been solved numerically using the commercial CFD solver ANSYS FLUENT©. In all cases extensive grid independence and convergence-criteria independence testing was undertaken. The numerical approach used here in order to determine when turbulence develops which involves solving the Reynolds averaged governing equations together with a turbulence model, in which the effects of buoyancy forces are taken into account, for all conditions considered and then monitoring the results obtained with increasing Rayleigh numbers to determine when significant turbulence effects develop. This approach has been used quite extensively in the study of forced convective flows, e. g., see SCHMIDT and PATANKAR (1991) and ZHENG *et al.* (1998). The Nusselt number in any situation will depend on:

1. The Rayleigh number, Ra_L, based on the reference length scale L of the heated surface and the difference between the temperature of the heated isothermal surface, T_w, and the temperature of the undisturbed fluid well away from the system, T_f, i.e.:

$$\mathrm{Ra}_L = \frac{g\beta(T_w - T_f)L^3}{\nu\alpha} \tag{15}$$

2. The dimensionless width of the surface waves, $D = d/L$.
3. The dimensionless height of the surface waves, $H = t/L$.
4. The Prandtl number, Pr.

Results have only been obtained for a Prandtl number of 0.74, i.e., effectively the value for air. All the results obtained for the wave surfaces were compared with results obtained for the same surface without waves, in terms of the a mean Nusselt number.

4. RESULTS AND DISCUSSION

The horizontal surfaces, wavy and non-wavy, are thin with width L and depth w maintained at a uniform surface temperature $T_w = 310$ K. The surrounding fluid is air at a temperature $T_f = 290$ K at atmospheric pressure. Numerical simulations were performed for Rayleigh numbers varying between 10^4 to 10^{14}. Numerical results for natural convective heat transfer rate from the surface with triangular waves were obtained by fixing $D = d/L = 0.0278$ and using fifteen different profiles for $H(X)$, that is, five linear, five parabolic and five exponential. Similarly, numerical results for natural convective heat transfer rate from the surface with rectangular waves were obtained by fixing $D = d/L = 0.0588$ and again using fifteen different profiles for $H(X)$, that is, five linear, five parabolic and five exponential.

Typical variations of the mean Nusselt number with the Rayleigh number for various triangular wave height profiles are shown in Figs. 2, 3 and 4. A triangular wavy height of 0 corresponds to the case of a flat (plane) heated surface. It can be seen that the mean Nusselt number at a given triangular wave height profile generally increases with increasing of the Rayleigh number.

This is further illustrated by the results presented in Figs 5 and 6, which show typical variations of the mean Nusselt number with triangular wave linear profile for various values of the Rayleigh number. In similar way, the results presented in Figs 7 and 8 show typical variations of the mean Nusselt number with triangular wave parabolic profile for various values of the Rayleigh number. Lastly, the results presented in Figs. 9 and 10 show typical variations of the mean Nusselt number with triangular wave exponential profile for various values of the Rayleigh number. Since a triangular wave height profile equal to 0 corresponds to the case of a flat heated surface, these figures basically illustrate the variations in the heat transfer rate produced by the different profiles of the triangular waves in terms of the mean Nusselt number.

It can be seen from the results given in Fig. 5 that the variations in the mean Nusselt number at a fixed Rayleigh number over the range of triangular wave linear profiles considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6. However, the increase in the mean Nusselt number for a Rayleigh number equal to 10^8 is approximately 60% for PROFILE 5 when compared to results for PROFILE 0. A similar behavior can be seen in the results given in Fig. 6. The variation in the mean Nusselt number at a fixed Rayleigh number over the range of triangular wave linear profiles considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12}. However, the increase in the mean Nusselt number for Rayleigh number equal to 10^{14} is approximately 13% for PROFILE 5 when compared to results for PROFILE 0.

Similarly, it can be seen from the results given in Fig. 7 that the variations in the mean Nusselt number at a fixed Rayleigh number over the range of triangular wave parabolic profiles considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6. However, the increase in the mean Nusselt number for a Rayleigh number equal to 10^8 is approximately 58% for PROFILE 5 when compared to results for PROFILE 0. A similar behavior can be seen in the results given in Fig. 8. The variation in the mean Nusselt number at a fixed Rayleigh number over the range of triangular wave parabolic profiles considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12}. However, the increase in the mean Nusselt number for Rayleigh number equal to 10^{14} is approximately 13% for PROFILE 5 when compared to results for PROFILE 0.

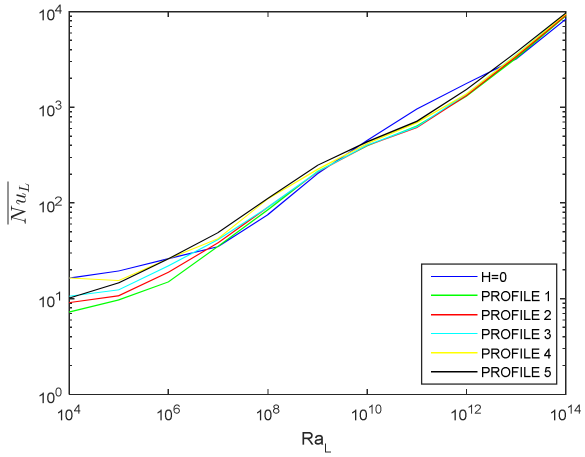


Figure 2. Variations of mean Nusselt number with Rayleigh number for various triangular wave linear profiles

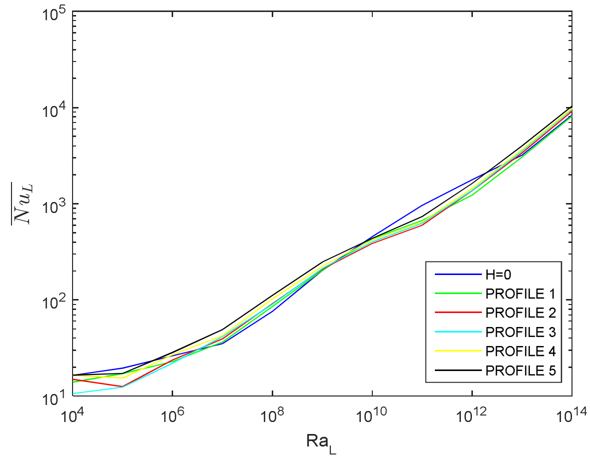


Figure 3. Variations of mean Nusselt number with Rayleigh number for various triangular wave parabolic profiles

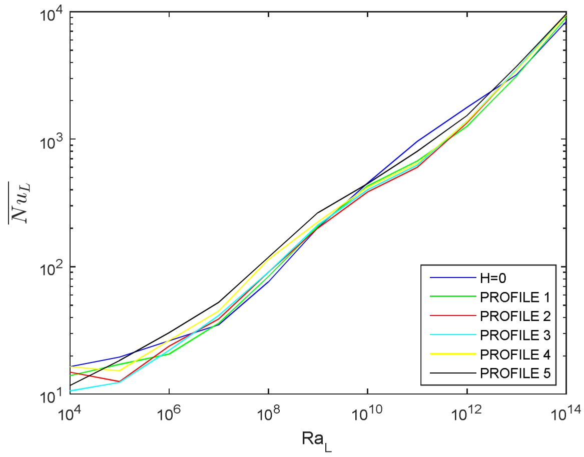


Figure 4. Variations of mean Nusselt number with Rayleigh number for various triangular wave exponential profiles

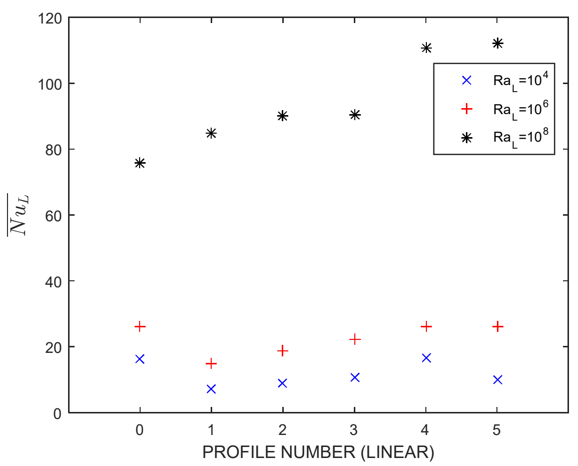


Figure 5. Variations of mean Nusselt number with triangular wave linear profiles for Rayleigh numbers of 10^4, 10^6 and 10^8

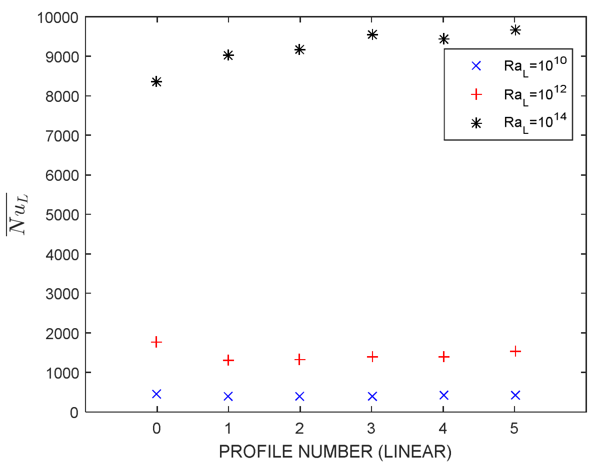


Figure 6. Variations of mean Nusselt number with triangular wave linear profiles for Rayleigh numbers of 10^{10}, 10^{12} and 10^{14}

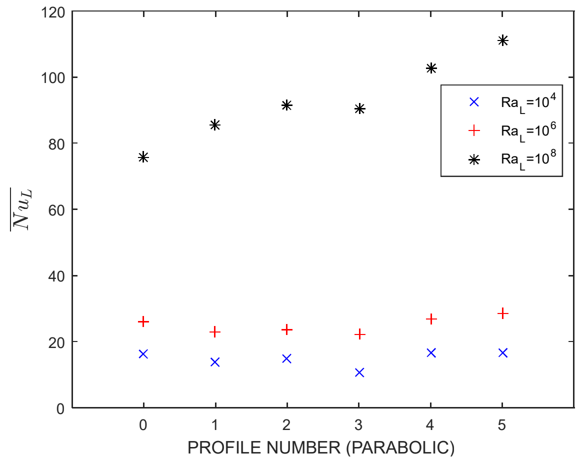


Figure 7. Variations of mean Nusselt number with triangular wave parabolic profiles for Rayleigh numbers of 10^4, 10^6 and 10^8

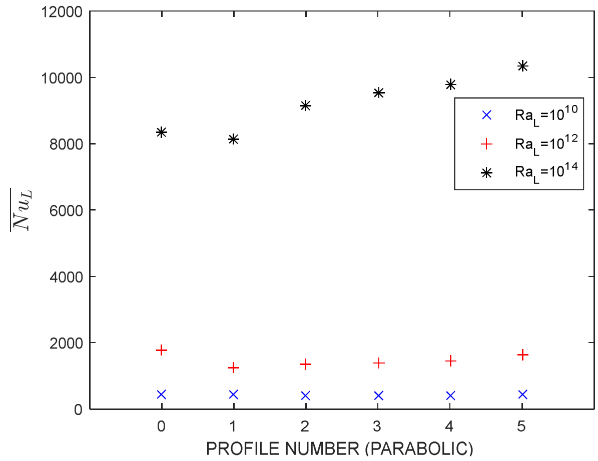


Figure 8. Variations of mean Nusselt number with triangular wave parabolic profiles for Rayleigh numbers of 10^{10}, 10^{12} and 10^{14}

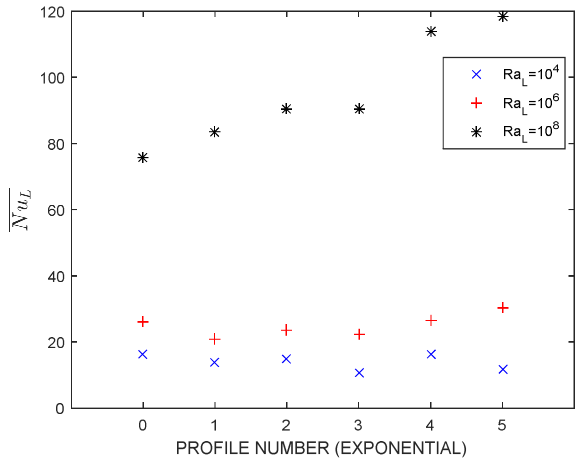


Figure 9. Variations of mean Nusselt number with triangular wave exponential profiles for Rayleigh numbers of 10^4, 10^6 and 10^8

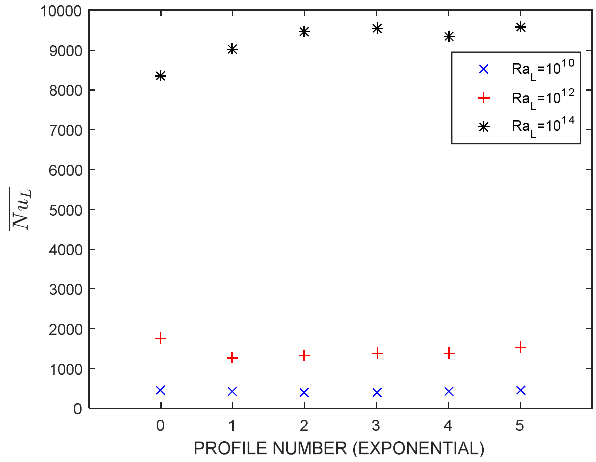


Figure 10. Variations of mean Nusselt number with triangular wave exponential profiles for Rayleigh numbers of 10^{10}, 10^{12} and 10^{14}

Lastly, it can be seen from the results given in Fig. 9 that the variations in the mean Nusselt number at a fixed Rayleigh number over the range of triangular wave exponential profiles considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6. However, the increase in the mean Nusselt number for a Rayleigh number equal to 10^8 is approximately 58% for PROFILE 5 when compared to PROFILE 0. A similar behavior can be seen in the results given in Fig. 10. The variation in the mean Nusselt number at a fixed Rayleigh number over the range of triangular wave exponential profiles considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12}. However, the increase in the mean Nusselt number for Rayleigh number equal to 10^{14} is approximately 13% for PROFILE 5 when compared to results for PROFILE 0.

It will therefore be seen that the increase in the mean Nusselt number iss more pronounced for Rayleigh numbers equal to 10^8 and 10^{14}. It was also seen that this increase is greater for the PROFILE 5 when compared to results for PROFILE 0. It was also found that the shape of the profile does not have a considerable influence in increasing the heat transfer rate, being approximately, on average, 59% higher for a Rayleigh number equal to 10^8 and approximately, on average, 13% higher for a Rayleigh number equal to 10^{14} in all cases, i.e., linear, parabolic or exponential variations of the height of the triangular waves.

An important result that can be observed from the results given in Figs. 5-10 is that for Rayleigh numbers equal to 10^4, 10^6, 10^{10} and 10^{12}, there are no significant changes in the mean Nusselt number for the profiles considered, either for a linear, parabolic or exponential variation of the triangular wave heights. However, for Rayleigh numbers equal to 10^8 and 10^{14}, the differences of the mean Nusselt number are appreciable. Firstly, it is clear that the presence of waves, regardless of the profile and the pattern of variation of the heights, causes an increase in the mean Nusselt number with respect to the surface without waves (PROFILE 0). Secondly, this increase is more significant for PROFILE 5 (tallest wave on the plate edge) when compared to results for PROFILE 1 (shortest wave on the plate edge). An explanation for this fact can be seen in Figs. 11-12, which show the temperature distribution for an exponential variation of the heights of the triangular waves for $\mathrm{Ra}_L = 10^8$:

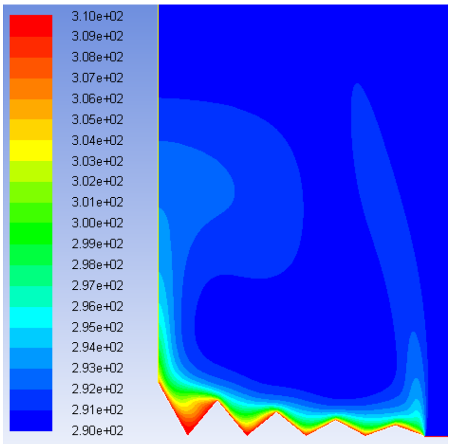


Figure 11. Temperature distribution for PROFILE 1

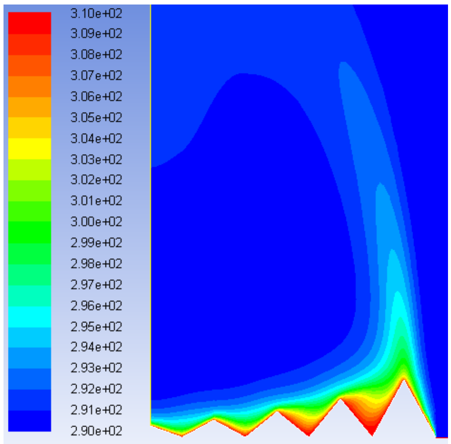


Figure 12. Temperature distribution for PROFILE 5

A vertical plume form can be seen on the edge of the wave surface in Fig. 11. However, on the edge of the wave surface shown in Fig. 12 there is also a vertical plume form, but with greater vertical extension and with some movement of the fluid to the center of the wave surface, suggesting a recirculation cell. This justifies a higher natural convective heat transfer rate in PROFILE 5 when compared to results for PROFILE 1. The same behavior was found to occur with a linear and parabolic variation of the heights of the triangular waves for $\mathrm{Ra}_L = 10^8$. In addition, similar behavior was also found for $\mathrm{Ra}_L = 10^{14}$ in all cases, i.e., linear, parabolic or exponential variations of the height of the triangular waves. Typical variations of the mean Nusselt number with the Rayleigh number for various rectangular wave height profiles are shown in Figs. 13, 14 and 15. A rectangular wavy height of 0 corresponds to the case of a flat (plane) heated surface. It can be seen that the mean Nusselt number at a given rectangular wave height profile increases with increasing of the Rayleigh number.

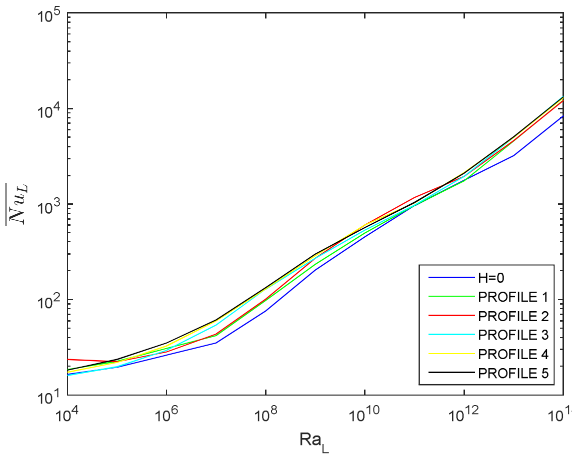


Figure 13. Variations of mean Nusselt number with Rayleigh number for various rectangular wave linear profiles

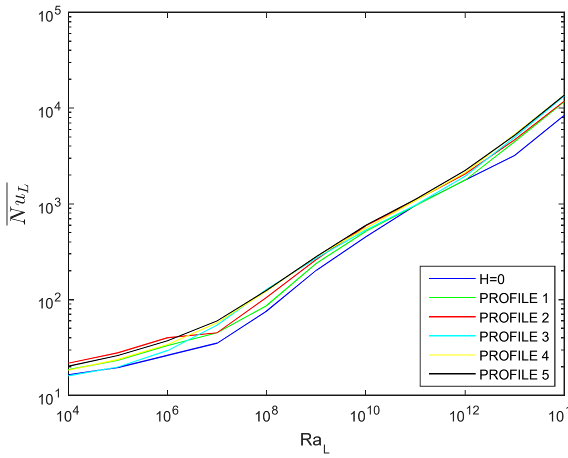


Figure 14. Variations of mean Nusselt number with Rayleigh number for various rectangular wave parabolic profiles

This is further illustrated by the results presented in Figs 16 and 17, which show typical variations of the mean Nusselt number with rectangular wave linear profile for various values of the Rayleigh number. In similar way, the results presented in Figs. 18 and 19 show typical variations of the mean Nusselt number with rectangular wave parabolic profile for various values of the Rayleigh number. Lastly, the results presented in Figs. 20 and 21 show typical variations of the mean Nusselt number with rectangular wave exponential profile for various values of the Rayleigh number. Since a rectangular wave height profile equal to 0 corresponds to the case of a flat heated surface, these figures basically illustrate the variations in the heat transfer rate produced by the different profiles of the rectangular waves in terms of the mean Nusselt number.

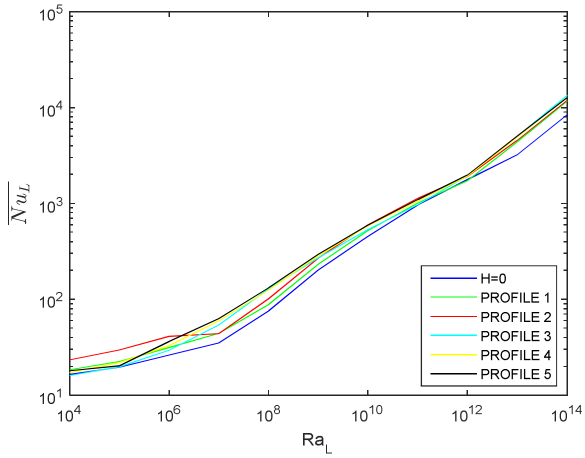


Figure 15. Variations of mean Nusselt number with Rayleigh number for various rectangular wave exponential profiles

It can be seen from the results given in Fig. 16 that the variations in the mean Nusselt number at a fixed Rayleigh number over the range of rectangular wave linear profiles considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6. However, the increase in the mean Nusselt number for a Rayleigh number equal to 10^8 is approximately 71% for PROFILES 3, 4 and 5 when compared to results for PROFILE 0. A similar behavior can be seen from the results given in Fig. 17. The variation in the mean Nusselt number at a fixed Rayleigh number over the range of rectangular wave linear profiles considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12}. However, the increase in the mean Nusselt number for Rayleigh number equal to 10^{14} is approximately 53% for PROFILES 3, 4 and 5 when compared to results for PROFILE 0.

Similarly, it can be seen from the results given in Fig. 16 that the variations in the mean Nusselt number at a fixed Rayleigh number over the range of rectangular wave parabolic profiles considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6. However, the increase in the mean Nusselt number for a Rayleigh number equal to 10^8 is approximately 68% for PROFILES 3, 4 and 5 when compared to results for PROFILE 0. A similar behavior can be seen from the results given in Fig. 19. The variation in the mean Nusselt number at a fixed Rayleigh number over the range of rectangular wave parabolic profiles considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12}. However, the increase in the mean Nusselt number for Rayleigh number equal to 10^{14} is approximately 50% for PROFILES 3, 4 and 5 when compared to results for PROFILE 0.

Lastly, it can be seen from the results given in Fig. 20 that the variations in the mean Nusselt number at a fixed Rayleigh number over the range of rectangular wave exponential profiles considered is relatively small for Rayleigh numbers equal to 10^4 and 10^6. However, the increase in the mean Nusselt number for a Rayleigh number equal to 10^8 is approximately 71% for PROFILES 3, 4 and 5 when compared to results for PROFILE 0. A similar behavior can be seen from the results given in Fig. 21. The variation in the mean Nusselt number at a fixed Rayleigh number over the range of rectangular wave exponential profiles considered is also relatively small for Rayleigh numbers equal to 10^{10} and 10^{12}. However, the increase in the mean Nusselt number for Rayleigh number equal to 10^{14} is approximately 47% for PROFILES 3, 4 and 5 when compared to results for PROFILE 0.

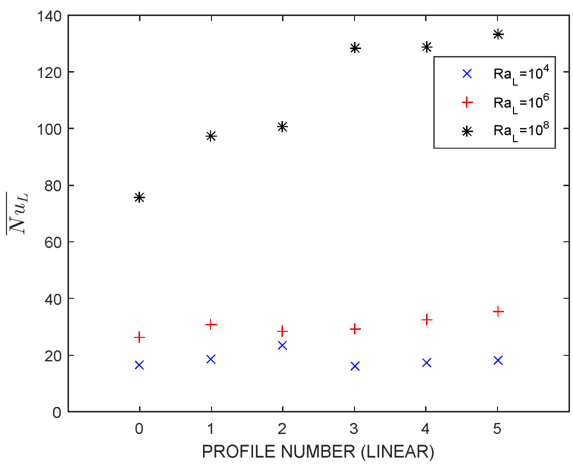


Figure 16. Variations of mean Nusselt number with rectangular wave linear profiles for Rayleigh numbers of 10^4, 10^6 and 10^8

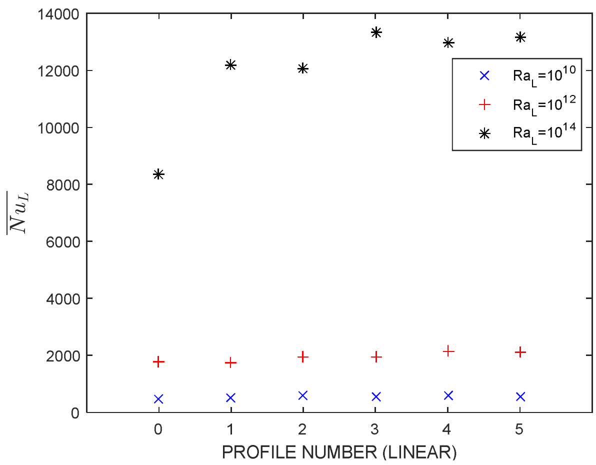


Figure 17. Variations of mean Nusselt number with rectangular wave linear profiles for Rayleigh numbers of 10^{10}, 10^{12} and 10^{14}

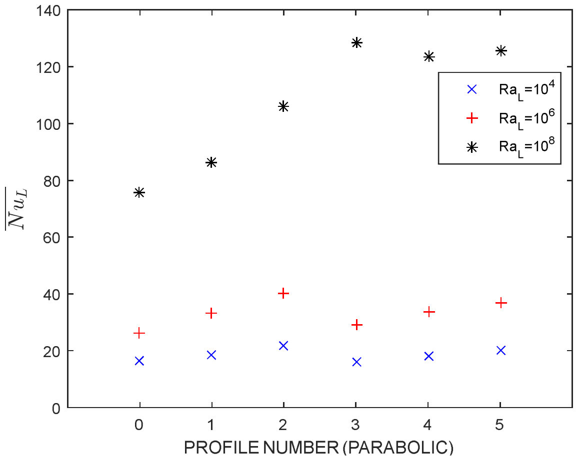


Figure 18. Variations of mean Nusselt number with rectangular wave parabolic profiles for Rayleigh numbers of 10^4, 10^6 and 10^8

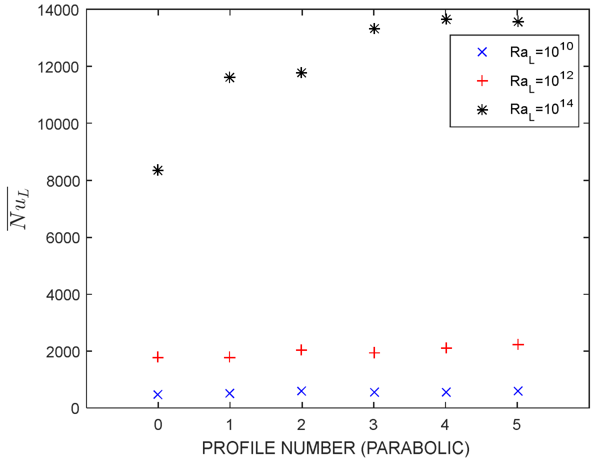


Figure 19. Variations of mean Nusselt number with rectangular wave parabolic profiles for Rayleigh numbers of 10^{10}, 10^{12} and 10^{14}

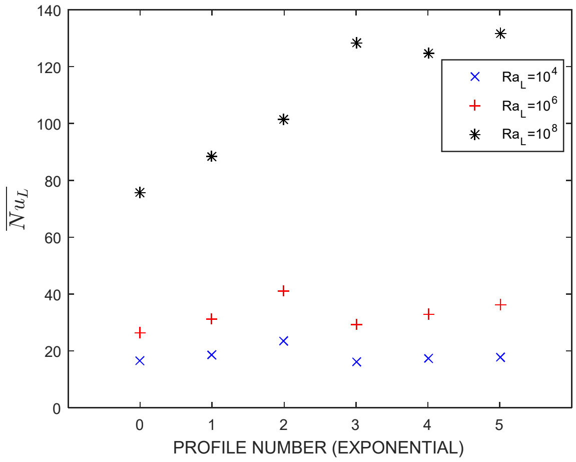


Figure 20. Variations of mean Nusselt number with rectangular wave exponential profiles for Rayleigh numbers of 10^4, 10^6 and 10^8

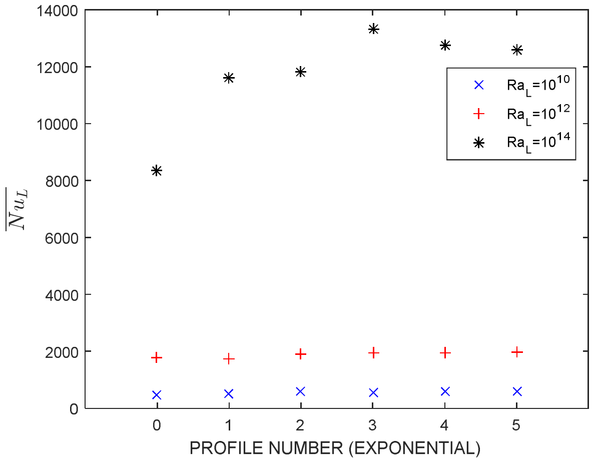


Figure 21. Variations of mean Nusselt number with rectangular wave exponential profiles for Rayleigh numbers of 10^{10}, 10^{12} and 10^{14}

It is seen therefore that the increase in the mean Nusselt number is more pronounced for Rayleigh numbers equal to 10^8 and 10^{14}. It was also seen that this increase is greater for the PROFILES 3, 4 and 5 when compared to PROFILE 0. Again, it was also found that the shape of the profile does not have a considerable influence in increasing the heat transfer rate, being approximately, on average, 70% higher for a Rayleigh number equal to 10^8 and approximately, on average, 50% higher for a Rayleigh number equal to 10^{14} in all cases, i.e., linear, parabolic or exponential variations of the height of the rectangular waves.

An important result that can be observed by considering Figs. 16-21 is that for Rayleigh numbers equal to 10^4, 10^6, 10^{10} and 10^{12}, there are no significant changes in the mean Nusselt number for the profiles considered, either for a linear, parabolic or exponential variation of the rectangular wave heights. However, for Rayleigh numbers equal to 10^8 and 10^{14}, the differences of the mean Nusselt number are appreciable. Firstly, it is clear that the presence of waves, regardless of the profile and the pattern of variation of the heights, causes an increase in the mean Nusselt number with respect to the surface without waves (PROFILE 0). Secondly, this increase is more significant for PROFILE 5 (tallest wave on the plate edge) when compared to results for PROFILE 1 (shortest wave on the plate edge). Again, an explanation for this fact can be seen in Figs. 22-23, which show the temperature distribution for an exponential variation of the heights of the rectangular waves for $\mathrm{Ra}_L = 10^8$.

A vertical plume form can be seen on the edge of the wave surface of the Fig. 22. However, on the edge of the wave surface shown in Fig. 23 there is also a vertical plume form, but with greater vertical extension and with the tendency of movement of the fluid to the center of the wave surface, suggesting a recirculation cell. This explains the higher natural convection heat transfer rate in PROFILE 5 when compared to results for PROFILE 1. The same behavior was found to occur with a linear and parabolic variation of the heights of the rectangular waves for $\mathrm{Ra}_L = 10^8$. In addition, similar behavior was also found for $\mathrm{Ra}_L = 10^{14}$ in all cases, i.e., linear, parabolic or exponential variations of the height of the rectangular waves.

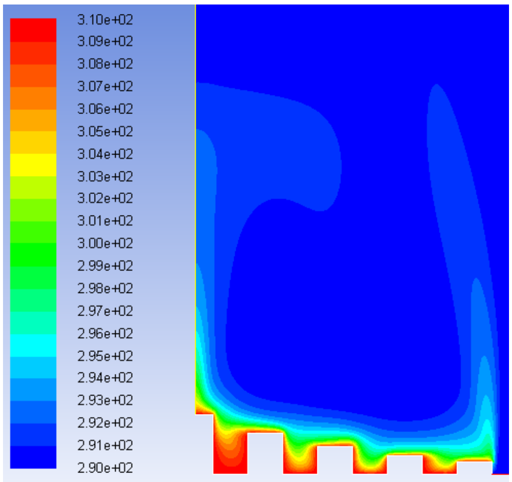


Figure 22. Temperature distribution for PROFILE 1

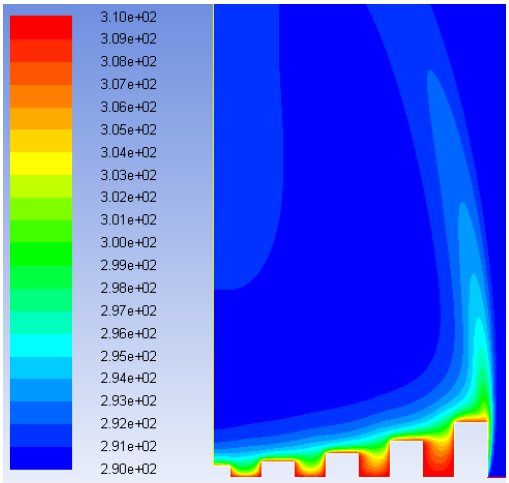


Figure 23. Temperature distribution for PROFILE 5

Finally, comparing the heat transfer rates from a horizontal plate having triangular waves with the same horizontal plate having rectangular waves, it can be seen that the natural convective heat transfer rate from a horizontal plate having rectangular waves is greater than the natural convective heat transfer rate from a horizontal plate having triangular waves. This result can be seen for all the cases studied, suggesting that the use of rectangular waves having variable height in a horizontal plate is more efficient from the viewpoint of natural convective heat transfer than the use of triangular waves having variable height. Main The reason for this is the greater surface area in the case of a horizontal plate having rectangular waves when compared to a horizontal plate having triangular waves.

5. CONCLUSIONS

A study of the effect of rectangular and triangular wavy surfaces with variable wave height on the natural convective heat transfer rate from a horizontal heated isothermal surface has been undertaken. The effect of different wave height distribution, in particular, has been studied. Some conclusions obtained from the study are:

1. It was found that the increase in the mean Nusselt number was more pronounced for Rayleigh numbers equal to 10^8 and 10^{14}. It was also found that this increase is greater for the PROFILE 5 when compared to results for PROFILE 0. It was also found that the shape of the profile does not have a considerable influence on the increase in heat transfer rate, being approximately, on average, 59% higher for a Rayleigh number equal to 10^8 and approximately, on average, 13% higher for a Rayleigh number equal to 10^{14} in all cases, i.e., linear, parabolic or exponential variations of the height of the triangular waves.
2. It was found that the increase in the mean Nusselt number was more pronounced for Rayleigh numbers equal to 10^8 and 10^{14}. It was also seen that this increase is greater for the PROFILES 3, 4 and 5 when compared to results for PROFILE 0. It was also found that the shape of the profile does not have a considerable influence in increasing the heat transfer rate, being approximately, on average, 70% higher for a Rayleigh number equal to 10^8 and approximately, on average, 50% higher for a Rayleigh number equal to 10^{14} in all cases, i.e., linear, parabolic or exponential variations of the height of the rectangular waves.
3. For triangular and rectangular waves with variable height, it has been noticed higher natural convective heat transfer rates for the PROFILE 5 when compare to results for PROFILE 1. The same behavior has been noticed for a linear and parabolic variation of the heights of the triangular/rectangular waves for $Ra_L = 10^8$. In addition, a similar behavior has also been noticed for $Ra_L = 10^{14}$ in all cases, i.e., linear, parabolic or exponential variations of the height of the triangular waves.
4. Comparing the heat transfer rates from a horizontal plate having triangular waves with the same horizontal plate having rectangular waves, it was found that the natural convective heat transfer rate from a horizontal plate having rectangular waves is greater than the natural convective heat transfer rate from the same horizontal plate having triangular waves. This result was found for all the cases studied, suggesting that the use of rectangular waves having variable height on a horizontal plate is more efficient from the viewpoint of natural convective heat transfer than the use of triangular waves having variable height. The reason for this is mainly the greater surface area in the case of a horizontal plate having rectangular waves when compared to a horizontal plate having triangular waves.

6. ACKNOWLEDGEMENTS

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