VIBRATION CONTROL USING AN INVERTED PENDULUM SYSTEM

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Abstract: This work presents a numerical-experimental study of vibration control for a system with one degree of freedom, formed by a base that slides on an air rail. An inverted pendulum tuned mass damper (IP-TMD) is designed to be installed on the top of the sliding base and, hence, to control the vibration thereof. Initially the natural frequency of the system with one degree of freedom is calculated, the initial parameters for the geometry of the IP-TMDare adopted according to an experimental model and, later, an optimization with a genetic algorithm is performed. A stability analysis of the system is made to obtain the minimum value for the rotational stiffness constant k, which connects the inverted pendulum to the sliding base. Analysis in time and frequency domain are performed for an harmonic load applied to the sliding base. Lagrange equationswere used to obtain the equations of motion for the system with one degree of freedom (sliding base) and for the controlled system with two degrees of freedom (sliding base with IP-TMD)

Keywords: Tuned mass damper; Passive control; Inverted pendulum

INTRODUCTION

Passive structural control can be used to avoid the unwanted effects of excessive vibration in a structure. This type of control modifiesthe structure stiffness and damping properties, by means of, for example, the installation of devices that absorb or transfer part of the energy that a dynamic load applies to the main structure (Avila, 2002). Inverted pendulum systems functioning as tuned mass dampers, IP-TMD (Guimarães, 2016), are examples of this type of control system.

Anh et al. (2007) compared the efficiency of the control exerted by an inverted pendulum and a simple pendulum on the main structure modeled as an inverted pendulum and found the best efficiency for the TMD-IP. Majcher&Wójcicki (2014) numerically analyzed the efficiency of a simple pendulum-type TMD as a control system for a tall building. Colherinhas et al. (2015) numerically analyzed the dynamic behavior of a slender tower before and after the installation of a simple pendulum-type TMD. Guimarães& Avila (2015) analyzed the dynamic stability of an offshore wind turbine modeled as an inverted pendulum; and Deraemaeker&Soltani (2016) started a study to obtain, in an analytical way, the solution for the optimization of a simple pendulum-type linear damped (TMD) damper.

In this study, the efficiency TMD-IP type is verified. A main system with one degree of freedom (1DOF) is analyzed, initially, without control. Subsequently, a mathematical model of an TMD-IP connected to the sliding base is presented and analyzed. The results obtained after MATLAB analysis were satisfactory.

MATHEMATICAL MODEL

The modeling of the dynamic behavior of the base sliding over the air rail consists of a system with 1DOF.There are basically two possible geometric configurations when it comes to pendulum-type structure: the simple pendulum and the inverted pendulum. A simple pendulum consists of a mass connected to a structure by means of a flexible cable or rigid rod and the position of the mass for this pendulum system is of stable equilibrium. Figure 1 illustrates in a simplified way the model of an inverted pendulum, consisting of a mass connected to a structure by means of a rigid rod, and the position of the mass for that pendulum system is of unstable equilibrium. When the oscillation of the pendulum is restricted to very small angles, ie sin (θ) θ, the system has a linear behavior.

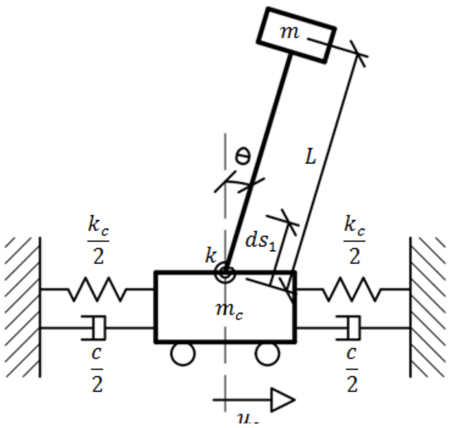


Figure 1 – Main system with a TMD-IP (adaptedfromGuimarães, 2016).

The system parameters are: m is the lumped mass at the top of the bar, $\theta$is the angular amplitude of the bar related to the vertical, l is the bar length, k is the elastic constant of the rotational spring installed in the pendulum base,m_c is the mass of the sliding base, k_c is the spring elastic constant of the sliding base , c is the damping coefficient of the sliding base, $c_d$is the rotational damping coefficient and ρ is the linear density of the pendulum bar. These parameter values are adopted according to the experimental model constructed by Resende et al. (2018): $\frac{k_c}{2} = 164{,}71\,\mathrm{N/m}, \mathrm{c} = 0{,}0546\ \mathrm{Ns/m}$, $\mathrm{c_d} = 0{,}0463\ \mathrm{Nms/rad}$, $\mathrm{k} = 1{,}45\ \mathrm{Nm/rad}$, $\mathrm{m_c} = 1133{,}17\mathrm{g}$, $\mathrm{l} = 0{,}52\ \mathrm{m}$, $\mathrm{g} = 9{,}78\ \mathrm{m/s^2}$, $\rho = 0{,}29\ \mathrm{kg/m}$ e $\mathrm{m} = 0\mathrm{kg}$.

Applying the Lagrange equation for the generalized variables θ and u_c of the proposed system, the following equations of motion are obtained:

$$\begin{bmatrix} \frac{\rho.l^3}{3} + m.l^2 & m.l + \frac{\rho.l^2}{2} \\ m.l + \frac{\rho.l^2}{2} & m_c + m + \rho.l \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{u}_c \end{bmatrix} + \begin{bmatrix} C_d & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{u}_c \end{bmatrix} + \begin{bmatrix} k - m.g.l - \frac{\rho.g.l^2}{2} & 0 \\ 0 & K_c \end{bmatrix} \begin{bmatrix} \theta \\ u_c \end{bmatrix} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix} \tag{1}$$

whereg, f(t), θ, $u_c$are: acceleration of gravity, applied force, rotational displacement, and translational displacement.

NUMERICAL ANALYSIS

Initially the natural frequency for the main system was calculated, $\mathrm{w_n} = 17\ \mathrm{rad/s}$. Then, from a parametric analysis, the minimum value for the rotational stiffness k was obtained.The value of this stiffness parameter is obtained from the stability analysis of the system from the eigenvalues of the state matrix ***A***. A parametric analysis is performed to obtain the minimum value of the elastic constant that keeps the 2DOF system stable.

The graph of Figure 2 indicates that values equal to or greater than k = 0.3864 Nm / rad, characterize the system as stable since no positive values appear in the real parts of the eigenvalues of the state matrix ***A*** The value k = 1.45 Nm / rad, adopted, satisfies this condition. Values less than 0.3864 Nm/rad characterize the system as unstable.

A sensitivity analysis was performed from the routine developed in MATLAB by Colherinhas et al. (2016). It analyzes the influence of the parameters L (pendulum length) and μ (mass ratio), of the transfer function of the sliding base, in the response of the forced system. The 3D surface is represented in Figure 3 and the generated map is represented in Figure 4. Both suggest that, for this study, the minimum response of the controlled system is in a curve and valley region. In addition, the sensitivity analysis suggests that there are a pair of optimal values for parameters μ and L, which minimize the oscillation amplitude of the main system. The approximate values for the parameters are$L \cong 0{,}22$mand$\mu \cong 0{,}07$.

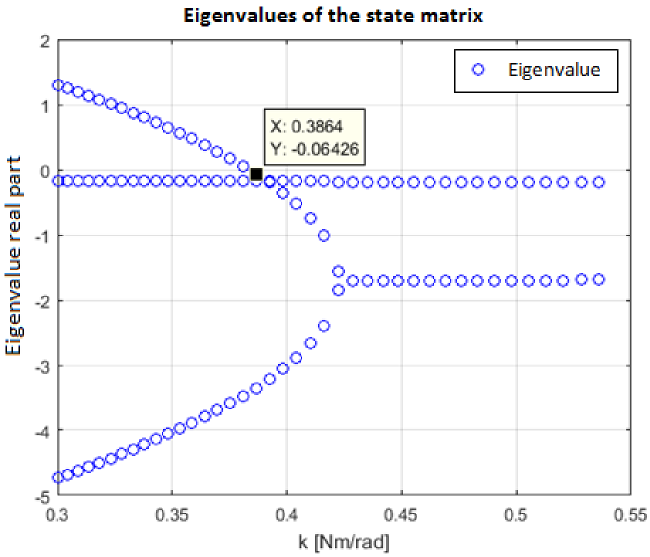
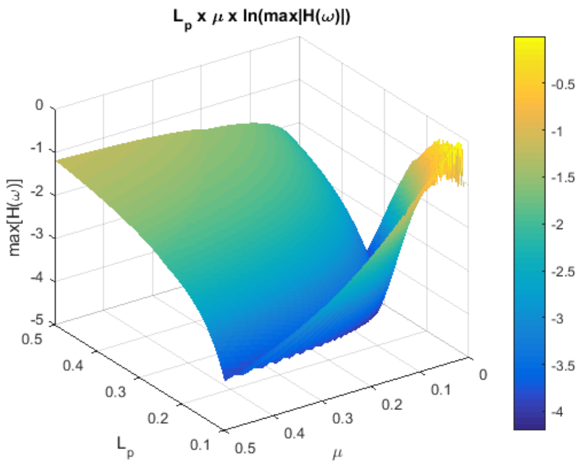


Figure 2 – Parametric analysis.



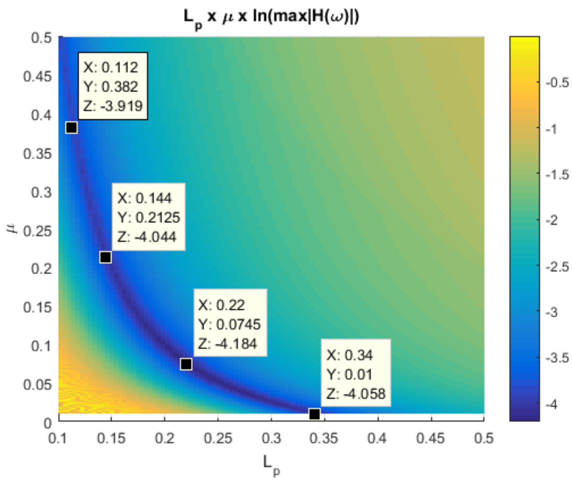


Figure 3 –Sensivity surface (max|H(w)|)xFigure 4 –Optimal points on the sensibility mapemfunçãoLandμ.

The natural frequencies associated with the two vibration modes of the 2DOF system are$w_{n1} = 8{,}6992$ rad/s and$w_{n2} = 17{,}0797$ rad/s. Figure 5 illustrates the two modes of vibration of the studied system composed by the inverted pendulum connected to the sliding base.

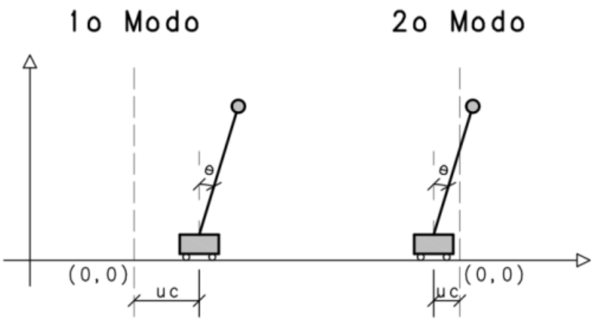


Figure 5 – Vibrationmodes of the controlled system.

Afterwards, an optimization with genetic algorithm will be performed, with the intention of confirming the values found in the sensitivity analysis. After that, a numerical analysis of the TMD-IP with the optimal values will be performed. Finally, we will calculate the time and frequency responses for the controlled and uncontrolled system, subjected to a harmonic force applied to the sliding base.

CONCLUSIONS

In this work it waspresented the mathematical models of one and two degrees of freedom, corresponding respectively to a system formed by a sliding base and to a system formed by a sliding base associated with an TMD-IP. The frequencies found for the 1DOF and 2DOF systems were lower than 18 rad / s. The coefficient of rotational stiffness adopted is higher than the limit value found in the parametric analysis, which keeps the system stable. The sensitivity map for the controlled system subjected to a harmonic force applied to the sliding base suggests that its response is minimized using the parameters$L \cong 0{,}22$m e $\mu \cong 0{,}07$ for the TMD-IP.

Based on this, the TMD-IP will be tested with three different configurations, they are: $L \cong 0{,}22$m e $\mu \cong 0{,}0$; $L \cong 0{,}22$m e $\mu \cong 0{,}07$; e $L \cong 0{,}22$m e $\mu \cong 0{,}14$. It will be verified if the configuration with the optimal parameters will really be the configuration whose system with 2DOF oscillates with the smallest magnitudes.

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