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MODELING OF THE DRYING PROCESS OF AGRICULTURAL PRODUCTS: LUIKOV'S APPROACH CONSIDERING VARIABLE PARAMETERS AND USE OF GITT

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Abstract. *The drying of agricultural foods is one of the first techniques used by humans in order to preserve food without changing its nutritional value, taste, texture, etc. Drying consists of removing moisture from a product, which is made through a combination of heat and ventilation. The physical phenomena present in the drying process are heat and mass transfer, which are manifested through the conduction, convection, diffusion and evaporation mechanisms, respectively. The objective of this work was to research about the simultaneous transfer of heat and mass during a drying process of an agricultural product, through the approach of a mathematical model with an hybrid solution. For this purpose, the Luikov's model was used, which considers heat and mass transfer as a coupled problem. In the Luikov's approach, certain thermophysical parameters are defined, associated with the dimensionless numbers Lu,* Bi_q *and* Bi_m*, which in this work were considered as functions of time. The Generalized Integral Transformation Technique was used to solve the differential equations. Six cases were studied, varying the Lu,* Bi_q *and* Bi_m *parameters, following increasing and decreasing linear functions. The results showed a strong dependence of the model in relation to the studied parameters, making interesting the experimental study of the phenomenon.*

Keywords: *Solar drying, Luikov, GITT*

INTRODUCTION

The operation of drying has been widely used to preserve food product, and fruits in particular, since the reduction of their water content to certain levels inhibits microbial growth and enzymatic modifications. Additionally, food drying presents other advantages like avoiding the need of using expensive cooling systems for preservation, or facilitating transport and storage due to the reduction in size. Drying is a simple process of extracting excess of water present in a particular product. Dried fruit and vegetables have gained commercial importance and their growth on a commercial scale has given rise to an important sector of the agriculture industry (Karim and Hawlader, 2005). Therefore, over the last few years, numerous researchers have focused on food drying processes (Silveira, 2016; Reyes *et al*., 2014; Silva, 2010; Silva *et al*., 2010).

Solar drying is one of the most ancient techniques to dry bricks and food used by man. The oldest solar drying installations were found in the south of France and are estimated to be from 8.000 B.C. (Kroll *et al.*, 1989). There are two kinds of solar drying (Belessiotis and Delyannis, 2011): direct, in which the product is exposed directly to the Sun or through a transparent covering; indirect, in which the object is dehydrated by hot air. The second kind is more sophisticated, allowing temperature and air speed control, among other parameters (Belessiotis and Delyannis, 2009) There are numerous technologies for solar drying, all of which can be divided into two groups: active and passive dryers (Belessiotis and Delyannis, 2011):. In the active dryers, there is forced air circulation, meanwhile, in the passive dryers, air circulation is natural.

The physical mechanisms of drying are heat and mass transfer. The mass transfer occurs by diffusion from the interior of the food to its surface, and then by evaporation, when it comes in contact with the hot air. The heat transfer occurs by convection between moving hot air and the surface of the food, and by conduction inside the food. The physical phenomena can be represented by Luikov's model (Luikov, 1980), which considers both heat and mass transfer in a coupled system of equations. Food is considered as a porous media, which free regions contain water or vapour.

Solutions of problems involving simultaneous heat and mass transfer in porous media are presented in the literature through many methods and techniques. Within this context, hybrid solution techniques (analytic/numeric) have been developed and used. Among these techniques, the General Integral Transform Technique (GITT) is highlighted and has been widely used in heat and mass transfer problems. Nevertheless, its application in drying food processes is still scant (Silva, 2010).

The objective of this work is to research about heat and mass transfer during the drying process, through a mathematical model with hybrid solution.

2. MATERIAL AND METHODS

2.1 Phisical problem

The product was considered as an infinite one-dimensional porous plate. Inside the drying chamber, both sides of the product are exposed to a flux of dry hot air, as shown in Fig. 1.

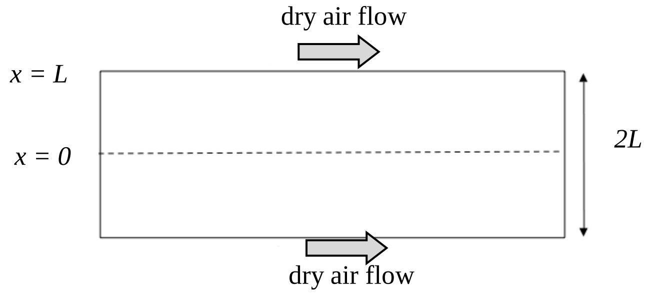


Figure 1. One-dimensional porous media

2.2 Modelling

The linear system of equations proposed by Luikov (1966) with associated initial and boundary conditions, for the modeling of such physical problem involving heat and mass transfer in a capillary porous media, can be written in dimensionless form as:

$$\frac{\partial\Theta(X,\tau)}{\partial\tau} = (1+\varepsilon LuKoPn)\frac{\partial^2\Theta(X,\tau)}{\partial X^2} - \varepsilon LuKo\frac{\partial^2\Phi(X,\tau)}{\partial X^2} \tag{1}$$

$$\frac{\partial\Phi(X,\tau)}{\partial\tau} = -LuPn\frac{\partial^2\Theta(X,\tau)}{\partial X^2} + Lu\frac{\partial^2\Phi(X,\tau)}{\partial X^2} \tag{2}$$

Initial conditions: For $0 < X < 1$

$$\Theta(X,0) = 0; \quad \Phi(X,0) = 0 \tag{3,4}$$

Boundary conditions: For $\tau > 0$

$$\frac{\partial\Theta(0,\tau)}{\partial X} = 0; \qquad \frac{\partial\Phi(0,\tau)}{\partial X} = 0 \tag{5,6}$$

$$\frac{\partial\Theta(1,\tau)}{\partial X} - Bi_q\big(1-\Theta(1,\tau)\big) + (1-\varepsilon)Bi_mKoLu\big(1-\Phi(1,\tau)\big) = 0 \tag{7}$$

$$-\frac{\partial\Phi(1,\tau)}{\partial X} + Pn\frac{\partial\Theta(1,\tau)}{\partial X} + Bi_m\big(1-\Phi(1,\tau)\big) = 0 \tag{8}$$

Where:

$\Theta(X,\tau)=\frac{T(x,t)-T_0}{T_s-T_0}$ is dimensionless temperature, $\Phi(X,\tau)=\frac{u(x,t)-u_0}{u_s-u*}$ is dimensionless moisture content, $\mathrm{X}=\frac{x}{L}$ is dimensionless length, $\tau=\frac{at}{L^2}$ is dimensionless time, $Bi_q=\frac{h_c L}{K}$ is Biot´s number for heat transfer, $Bi_m=\frac{h_m L}{k_m}$ is Biot´s number for mass transfer, $\mathrm{Lu}=\frac{a_m}{a}$ is Luikov´s number, $\mathrm{Ko}=\frac{\lambda}{c}\frac{u_o-u*}{T_s-T_0}$ Kossovich´s number and $\mathrm{Pn}=\delta\frac{T_s-T_0}{u_0-u*}$ Posnov´s number.

2.3 Solution

The equation system was solved using the General Integral Transform Technique (GITT). The GITT is a powerful hybrid numerical-analytical approach, which has been successfully applied to obtain benchmark solutions for different classes of linear and non-linear diffusion/convection problems.

The following filter was used (Ozisik, 1993), in order to accelerate the convergence:

$$\Theta(X,\tau)=1+\Theta_h(X,\tau);\qquad \Phi(X,\tau)=1+\Phi_h(X,\tau)\tag{9,10}$$

Thus, the system is written as follows:

$$\frac{\partial\Theta_h(X,\tau)}{\partial\tau}=K_{11}\frac{\partial^2\Theta_h(X,\tau)}{\partial X^2}+K_{12}\frac{\partial^2\Phi_h(X,\tau)}{\partial X^2}\tag{11}$$

$$\frac{\partial\Phi_h(X,\tau)}{\partial\tau}=K_{21}\frac{\partial^2\Theta_h(X,\tau)}{\partial X^2}+K_{22}\frac{\partial^2\Phi_h(X,\tau)}{\partial X^2}\tag{12}$$

Initial conditions: For $0 < X < 1$

$$\Theta_h(X,0)=-1;\quad \Phi_h(X,0)=-1\tag{13,14}$$

Boundary conditions: For $\tau > 0$

$$\frac{\partial\Theta_h(0,\tau)}{\partial X}=0;\qquad \frac{\partial\Phi_h(0,\tau)}{\partial X}=0\tag{15,16}$$

$$\frac{\partial\Theta_h(1,\tau)}{\partial X}+Bi_q\Theta_h(1,\tau)=Bi_m{}^{**}\Phi(1,\tau)\tag{17}$$

$$\frac{\partial\Phi_h(1,\tau)}{\partial X}+Bi_m{}^{*}\Phi_h(1,\tau)=-Bi_q\mathrm{Pn}\Theta_h(1,\tau)\tag{18}$$

In the first step, Sturm-Liouville problems were chosen (Ozisik, 1993). Thus, for heat transfer, the auxiliary problem is:

$$\frac{d^2\Psi_i(X)}{dX^2}+\mu_i{}^2\Psi(X)=0\tag{19}$$

$$\frac{d\Psi_i(0)}{dX}=0;\qquad \frac{d\Psi_i(X)}{dX}+Bi_q\Psi_i(1)=0\tag{20,21}$$

In which the eigen function is the following (Ozisik, 1993):

$$\Psi_i(X)=\cos(\mu_i X)\tag{22}$$

Similarly, the auxiliary problem for mass transfer is:

$$\frac{d^2\varphi_i(X)}{dX^2}+\lambda_i{}^2\varphi(X)=0\tag{23}$$

$$\frac{d\varphi_i(0)}{dX}=0;\qquad \frac{d\varphi_i(X)}{dX}+Bi_m\varphi_i(1)=0\tag{24,25}$$

In which the eigen function is the following (Ozisik, 1993):

$$\varphi_i(X) = \cos(\lambda_i X) \tag{26}$$

The second step is to find the pairs transform-inverse for the equations defined on the first step. For these equations, the pairs are:

- Heat Transfer

$$\text{Transform} \rightarrow \overline{\Theta}_i(\tau) = \int_0^1 \bar{\psi}_i(X)\Theta_h(X,\tau)\,dX \tag{27}$$

$$Inverse \rightarrow \Theta_h(X,\tau) = \sum_{i=1}^{\infty} \overline{\Psi}(X)\,\overline{\Theta}_i(\tau) \tag{28}$$

- Mass Transfer

$$\text{Transform} \rightarrow \overline{\Phi}_i(\tau) = \int_0^1 \bar{\varphi}_i(X)\Phi_h(X,\tau)\,dX \tag{29}$$

$$Inverse \rightarrow \Phi_h(X,\tau) = \sum_{i=1}^{\infty} \overline{\varphi}(X)\,\Phi_i(\tau) \tag{30}$$

The third step consisted in applying the integral transform to the partial differential equations, obtaining ordinary differential equations. Thus, the operator $\int_0^1 \overline{\Psi}_i dX$ was applied to Eq. (11) and the operator $\int_0^1 \overline{\Phi}_i dX$ was applied to Eq. (12).

Thus, performing some lengthy but straightforward manipulations, we obtain the following coupled system of ordinary differential equations for the transformed variables:

- Heat Transfer

$$\frac{\partial\overline{\Theta}_i(\tau)}{\partial\tau} + K_{11}{\mu_i}^2\overline{\Theta}_i(\tau) + K_{12}{\mu_i}^2\sum_{j=0}^{\infty} a_{ij}\overline{\Phi}_j(\tau) = \overline{\Phi}_i(1)KoLu\left[\left(Bi_m - \varepsilon Bi_q\right)\Phi_h(1,\tau) + \varepsilon PnBi_q\Theta_h(1,\tau)\right] \tag{31}$$

- Mass Transfer

$$\frac{\partial\overline{\Phi}_i(\tau)}{\partial\tau} + K_{22}{\lambda_i}^2\overline{\Phi}_i(\tau) + K_{21}{\lambda_i}^2\sum_{j=0}^{\infty} b_{ij}\overline{\Theta}_j(\tau) = -\Theta_i(1)LuPn[Bi_m\Theta_h(1,\tau) + (1-\varepsilon)KoLuBi_m\Phi_h(1,\tau)] \tag{32}$$

The solution of the transformed system is obtained through the IMSL numerical library. To get the solution to the original problem, the inverse formulas must be applied.

3. RESULTS AND DISCUSSION

It was considered a base case, with constant thermophysical parameters and six cases with thermophysical parameters varying in time (Tab. 1.). To represent the variation of the dimensionless parameters, linear functions were chosen (Tab. 2), which values are similar to the ones studied in other works (Silva, 2010).

Table 1. Analised Cases (Silva, 2010)

Properties	Base Case
L (m)	2.0×10^{-3}
ρ (kg/m³)	980
h_c (W/m².K)	36.51
K (W/m.K)	0.5424
c (J/kg.K)	3350
λ (J/kg)	2.3586×10^{6}
v_{ar} (m/s)	0.70
u_0 (kg/kgdry)	4
u^* (kg/kgdry)	0.20
D_{ef} (m²/s)	2.41×10^{-10}
Pn	0.142
Ko	74.32
ε	0.3
Lu	0.0015
Bi_q	0.135
Bi_m	2.43

Table 2. Analyzed Cases

Case	***Lu***	***Bi$_q$***	***Bi$_m$***
1	0.0015+0.00000004t	0.135	2.43
2	0.0015-0.00000004t	0.135	2.43
3	0.0015	0.135+0.00004t	2.43
4	0.0015	0.135-0.00004t	2.43
5	0.0015	0.135	2.43+0.00004t
6	0.0015	0.135	2.43-0.00004t

3.1 Simultaneous Heat and Mass Transfer

On Figure 2a it is observed that the temperature increases asymptotically after an initial decrease. This behavior is expectable because the maximum temperature the product can achieve is the air temperature, which is constant. It can be observed that the temperature is higher in the surface of the product. This is also reasonable because heat transfer goes from the surface to the inner part.

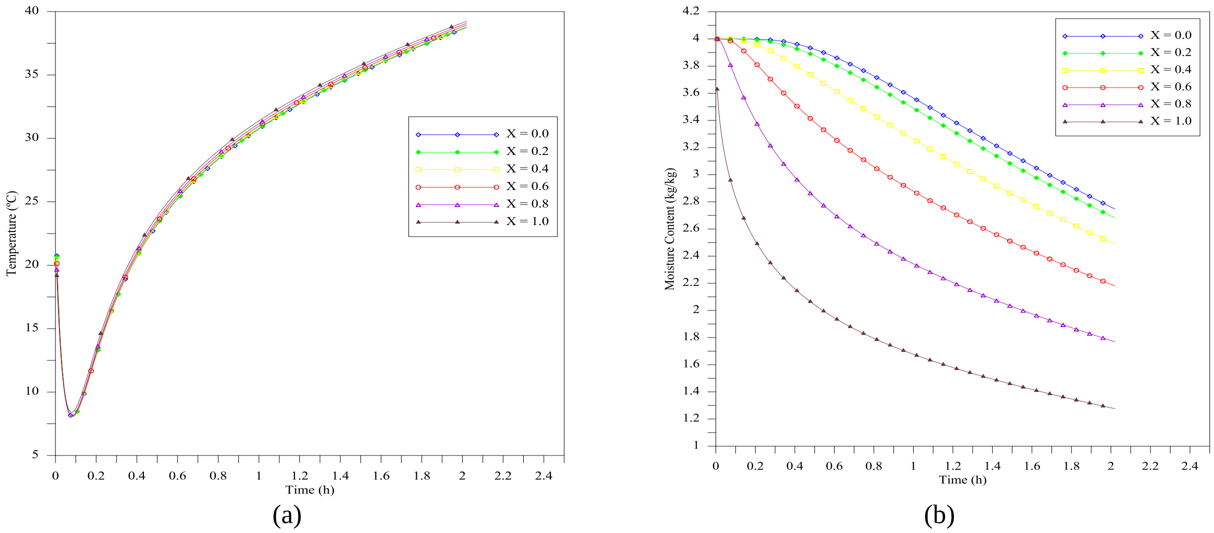


Figure 2. Results for: (a) Temperature vs time, (b) Moisture Content vs time

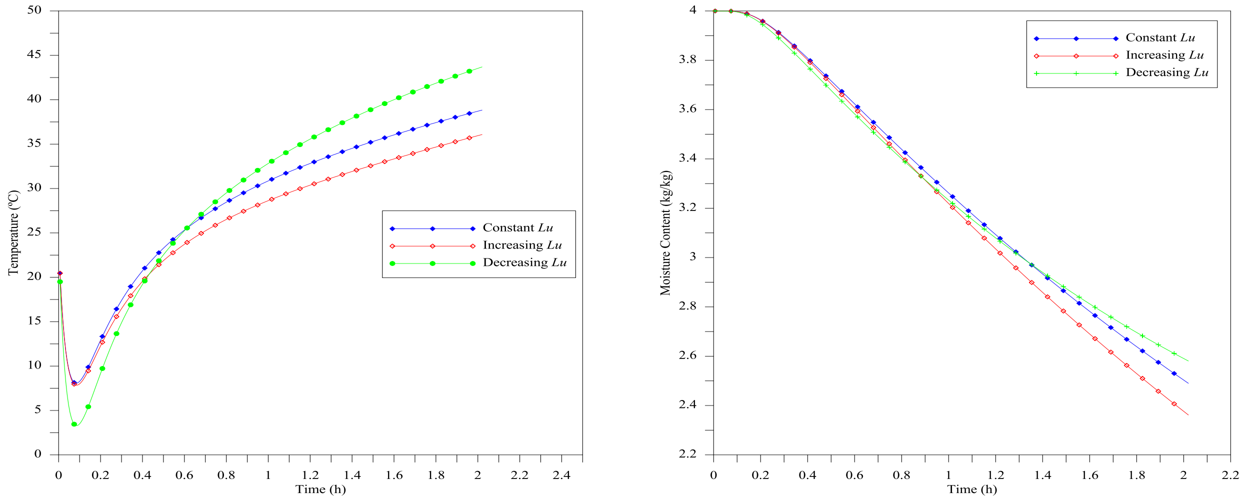
On Figure 2b it is observed an asymptotic decreasing of the moisture. This can be explained considering that the minimum moisture that the product can achieve is the air moisture, which is constant. It can be observed that moisture is higher in the inner part of the product. This makes sense, since mass transfer goes from the inner part to the surface.

3.2 Dimensionless Parameters Analysis

On Figures 3 to 5 are presented the graphs of temperature as a function of time, and moisture as a function of time, for the dimensionless position X = 0.4 , varying Lu, Bi_q and Bi_m with the according to the functions given in Tab. 2.

3.2.1 Results Varying *Lu*

In the curves of Fig. 3a it can be observed that temperatures for decreasing Lu are higher than the temperatures for constant Lu. Knowing that $Lu=a_m/a$, and that only temperature is being analysed, increasing Lu is equivalent to a decreasing thermal diffusivity, causing that temperature increases with a lower speed. On Figure 3b it can be verified that the moisture drop is more intense for increasing Lu. This is because increasing Lu equals increasing mass diffusivity. Therefore, moisture is transfered with a higher rate than in the other cases.

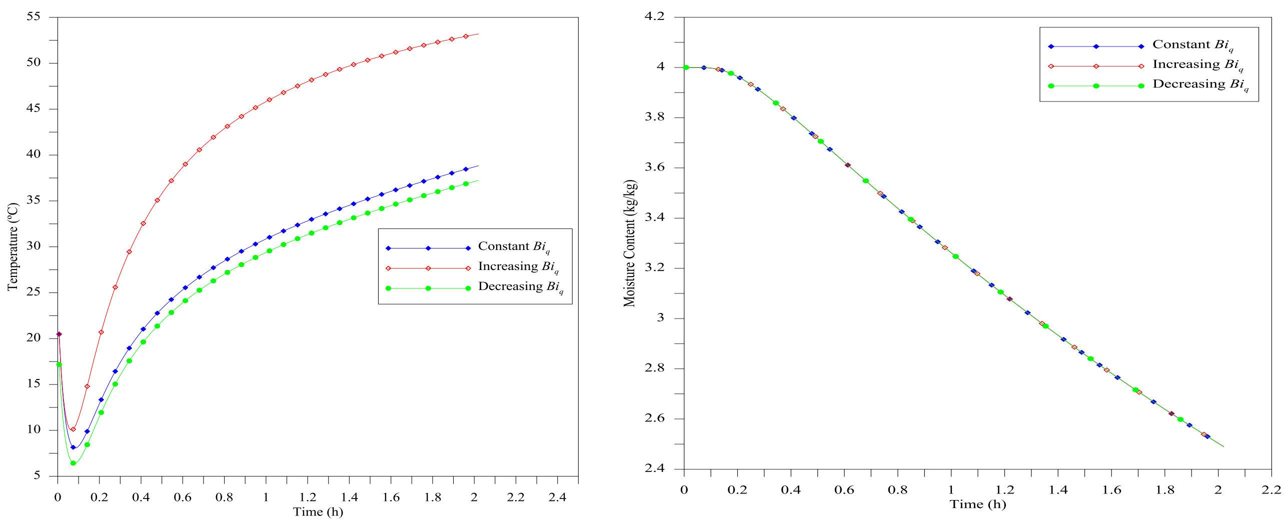


(a) (b)

Figure 3. Results varying Lu for the dimensionless position X = 0.4: (a) Temperature vs time, (b) Moisture Content vs time

3.2.2 Results Varying Bi_q

In the curves of Fig. 4a it can be observed that the temperatures for increasing B_{iq} are higher than the temperatures for constant B_{iq} and also higher than the temperature for decreasing B_{iq}. Increasing B_{iq} means that the rate of growth of the convective coefficient is higher than the rate of growth of the conductive coefficient. An increase on the convective coefficient results in an increase on the temperature gradient, causing a higher heat transfer to the inner part of the product.



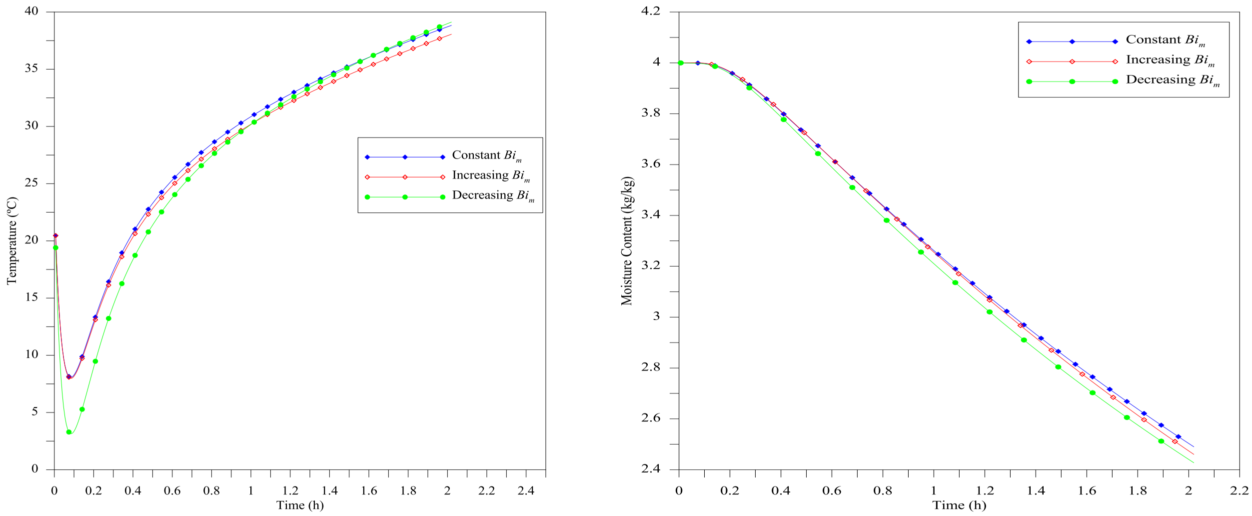
(a) (b)

Figure 4. Results varying Bi_q for the dimensionless position X=0.4: (a) Temperature vs time, (b) Moisture Content vs time

The Figure 4b shows that the temporal variation of the moisture does not depend on the function adopted for B_{iq}. This is because B_{iq} is not a mass transfer parameter.

3.2.3 Results Varying Bi_m

In the curves of Figure 5a it can be observed that the temperatures for decreasing B_{im} are higher than temperatures for constant B_{im} and also higher than the temperature for increasing B_{im}. Decreasing B_{im} means that the rate of growth of the conductive coefficient (diffusion) is higher than the rate of growth of the convective coefficient (evaporation). If the water remains in the product, its heat causes' an increase on the product's temperature.



(a) (b)

Figure 5. Results varying Bi_m for the dimensionless position X=0.4: (a) Temperature vs time, (b) Moisture Content vs time

The Figure 5bt shows that the moisture decrease is more intense for increasing B_{im}. This makes sense because a higher B_{im} means higher evaporation, which causes a higher concentration gradient. That causes a higher mass transfer.

4. CONCLUSION

Mathematical modeling could be a useful tool for optimizing the drying process. Understanding the physical phenomena is fundamental in order to build a reliable model.

Luikov's model seem to be proper for it considers coupled heat and mass transfer. Hybrid numeric-analytic methods are a powerful tool for solving complex problems, such as Luikov's model.

The effects of varying Lu, Bi_q and Bi_m were the ones expected. The model considering parameters as function of time could add more precision to Luikov's model.

Luikov's model shows high sensibility to the parameters Lu, Bi_q and Bi_m. Therefore, it would be interesting to make experimental research in order to contrast the results and choose appropriate functions to each parameter.

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