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SOLUTION FOR THE CALCULATION OF THE DISTRIBUTION OF CONCENTRATION IN PROPAGATION OF CONTAMINANTS IN SANITARY LANDFILL

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Abstract: *Nowadays, the concern with protecting the environment and in particular with water resources is becoming more frequent. Thus contamination of soil and water table coming the processes of diffusion and convection of the concentration of pollutants that accumulate over time in sanitary landfills represents an extremely important factor. The present work uses the ideas of the Generalized integral transform technique (GITT) to provide a solution, explicit, for the distribution of the concentration in the propagation of contaminants in sanitary landfills. In the proposed mathematical modeling, it is considered a physical problem of mass transfer in transient, two-dimensional, porous medium with uniform profile of propagation velocity of the contaminant. The results presented in the form of graphs allow to evaluate the influence of the Peclet and the retardation factor on the concentration field.*

Keywords: *sanitary landfills, contamination of soil, Transport of mass in a porous medium, GITT.*

1. INTRODUCTION

The study of the control and analysis of contaminants through the use of barriers has been constituted in a research area that already demands a lot of time and interest from several researchers. With the increasing environmental demand, it is extremely important to dispose of solid waste in places that prevent or at least reduce the contact of these wastes with the environment. An important concern in the disposal process of these wastes is that the control projects ensure an adequate system to prevent the flow of contaminants, for example throughout the lifespan of a sanitary landfill and after your closure, considering that some types of waste still have quite high toxic potential for hundreds or even thousands of years. It is necessary, then, to have the capacity to quantify the amount, the distance of advance of the front and for how long the contaminants coming from such residues will be present in the soil (de ARÊAS, 2006).

The presence of pollutants in the form of gas or steam in the soil is due to several factors. The variety of polluting substances in a sanitary landfill is enormous, resulting in numerous ways of interacting with the soil. When present in the subsoil, the gases will be transported, becoming available to be adsorbed to the soil particles or to dissolution in the subsoil water, thus increasing the volume of subsoil water in contact with the pollutant (de IGNATIUS, 1999), or still be released into the atmosphere, increasing their concentration in the local. In this way, it is necessary to know in detail the conditions of the medium, contamination and flow, as well as the properties of pollutants and soil, as these are factors that have a great influence on the transport of substances. With this information it is possible to estimate the area of the contaminated region and predict the advance of the contamination plume, so that treatment and decontamination methods can be developed (de BORGES, 1996).

Infiltration of percolate into the soil of a sanitary landfill (a liquid consisting of rainwater and leachate produced by the degradation of landfill) may lead to a number of contaminants into the water table, thereby spreading contamination of the soil and groundwater surrounding the sanitary landfill, were the motivators that led us to develop this work.

A wide-ranging bibliographical review of the subject matter can be found in the specialized literature, particularly in the works of (NOBRE, 1987, COTTA, 2002 and 2003, BARROS, 2003 and 2004, ANTÔNIO, 2005 and GENUCHTEN, 2013).

Therefore, in this work we simulate the distribution of the concentration field in the propagation of pollutants in sanitary landfills through the proposed mathematical model, evaluating the influence of the parameters for analysis and prevention of contamination of the soil and water table below the landfill, as shown in fig. 1.

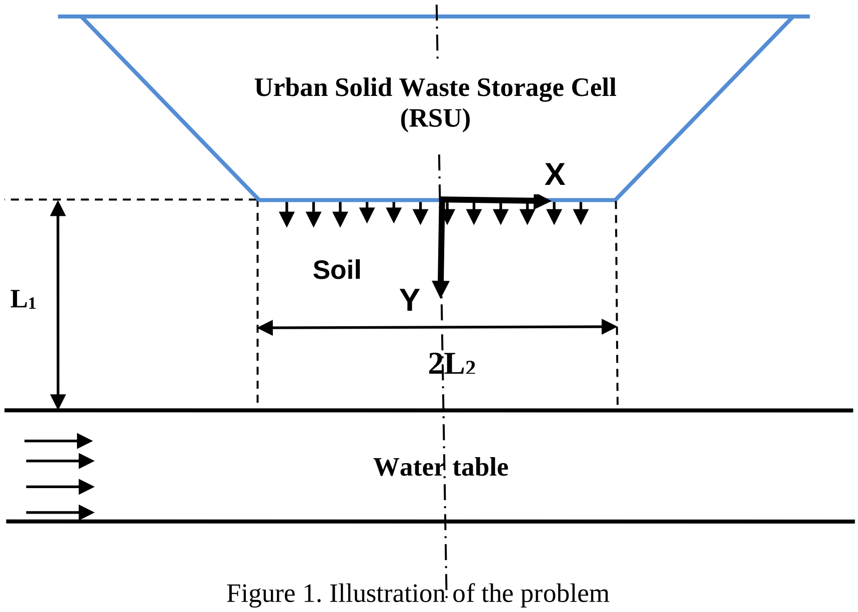

Figure 1. Illustration of the problem

2. MATHEMATICAL MODELING

For the mathematical modeling of the proposed physical problem, the following hypotheses are used:

- The soil is considered a homogeneous saturated porous medium;
- The pollutant flow through the soil is transient and two-dimensional;
- The launch of the pollutant is continuous;
- The physical properties of the pollutant medium and the soil are considered constant;
- The advective effect of the pollutant in the x-direction is negligible when compared to the diffusive term;
- The mass flux at y is much larger than at x;
- There is no migration of contaminants through the contours of the x-axis;
- The diffusive process in the soil is negligible when compared to diffusion in the pollutant;
- It is considered the uniform profile of velocity for the flow of the pollutant.

Mass transfer equation

$$R\frac{\partial C(x,y,t)}{\partial t} = D\left(\frac{\partial^2 C(x,y,t)}{\partial x^2} + \frac{\partial^2 C(x,y,t)}{\partial y^2}\right) - V_y \frac{\partial C(x,y,t)}{\partial y} \;;\; 0 \le x \le L_2,\; 0 \le y \le L_1 \text{ e } t \ge 0 \quad (1)$$

Where R represents the soil retardation factor, D refers to the dispersion coefficient and Vy represents the rate of soil pollutant propagation.

Contour Conditions

$$\frac{\partial C(x,y,t)}{\partial x} = 0,\; x = 0,\; 0 \le y \le L_1 \text{ e } t > 0 \quad (2)$$

$$\frac{\partial C(x,y,t)}{\partial x} = 0,\; x = L_2,\; 0 \le y \le L_1 \text{ e } t > 0 \quad (3)$$

$$C(x,y,t)=C_0,\ y=0,\ 0\le x\le L_2 \text{ e } t>0 \tag{4}$$

$$-D\frac{\partial C(x,y,t)}{\partial y}=h_m\left[C(x,y,t)-C_\infty\right]\ ,y=L_1,\ 0\le x\le L_2 \text{ e } t>0 \tag{5}$$

Initial condition

$$C(x,y,t)=C_{initial}(y)\ ,\ t=0,\ 0\le y\le L_1 \text{ e } 0\le x\le L_2 \tag{6}$$

2.1 Adimensionalization of the problem

For the analysis of the problem, the following dimensionless parameters were defined, given by equations (7a-i), with the objective of solving not only a particular problem, but a class of problems that are defined by the same proposed model

$$X=\frac{x}{L_2}\quad;\quad Y=\frac{y}{L_1}\quad;\quad \tau=\frac{Dt}{L_1^2}\quad;\quad V^*=\frac{V_y}{V_{av}}=1\quad;\quad Bi=\frac{h_m.L_1}{D} \tag{7a-e}$$

$$C^*(X,Y,\tau)=\frac{C(x,y,t)-C_\infty}{C_0-C_\infty}\quad;\quad C_0^{\ *}(Y)=\frac{C_{initial}(y)-C_\infty}{C_0-C_\infty}\quad;\quad Pe=\frac{V_{av}.L_1}{D}\quad;\quad L^*=\left(\frac{L_1}{L_2}\right)^2 \tag{7f-i}$$

Where C_∞ represents the concentration of contaminant at the soil / water table interface, V_{av} represents the mean flow velocity of the contaminant, Bi represents the Biot number and Pe represents the Peclet number.

Applying the dimensionless parameters in equations (1), (2), (3), (4), (5) and (6), we find the main equation, the boundary conditions and the initial condition in the dimensionless form:

Dimensionless principal equation

$$R\frac{\partial C^*(X,Y,\tau)}{\partial \tau}=L^*\frac{\partial^2 C^*(X,Y,\tau)}{\partial X^2}+\frac{\partial^2 C^*(X,Y,\tau)}{\partial Y^2}-Pe\frac{\partial C^*(X,Y,\tau)}{\partial Y};\ 0\le X\le 1\ ;0\le Y\le 1 \text{ e } \tau\ge 0 \tag{8}$$

Dimensionless contour conditions

$$\frac{\partial C^*(X,Y,\tau)}{\partial X}=0\ ;\quad X=0\ ;\ \tau>0 \text{ e } 0\le Y\le 1 \tag{9}$$

$$\frac{\partial C^*(X,Y,\tau)}{\partial X}=0\ ;\quad X=1\ ;\ \tau>0 \text{ e } 0\le Y\le 1 \tag{10}$$

$$C^*(X,Y,\tau)=1;\quad Y=0\ ;\ \tau>0 \text{ e } 0\le X\le 1 \tag{11}$$

$$\frac{\partial C^*(X,Y,\tau)}{\partial Y}+BiC^*(X,Y,\tau)=0;\ \ Y=1\ ;\ \tau>0 \text{ e } 0\le X\le 1 \tag{12}$$

Initial condition dimensionless

$$C^*(X,Y,\tau)=C_0^{\ *}(Y)\quad;\quad \tau=0\ ;\ 0\le X\le 1 \text{ e } 0\le Y\le 1 \tag{13}$$

3. APPLICATION OF THE GENERALIZED INTEGRAL TRANSFORMATION TECHNIQUE

The generalized integral transform technique, described by (Cotta, 1993), is used to solve the proposed physical problem. Using the ideas provided by this technique, we aim to find an analytical solution for the case where The Bi = ∞.

Since the original problem has a non-homogeneous boundary condition in the Y direction, given by eq. (11), a mathematical filter will be used for GITT to be applied properly, as well as to improve computational performance. The proposed filter for the concentration field will be of the form:

$$C^*(X,Y,\tau) = C^{**}(X,Y,\tau) + C_F(Y) \qquad (14)$$

The mathematical filter introduced must be such that:

$$\begin{cases} \dfrac{d^2 C_F(Y)}{dY^2} - Pe\dfrac{dC_F(Y)}{dY} = 0 \\ C_F(Y) = 1 \qquad ; \quad Y = 0 \\ \dfrac{dC_F(Y)}{dY} + BiC_F(Y) = 0 \qquad ; \quad Y = 1 \end{cases} \qquad (15a\text{-}c)$$

Therefore, for Bi = ∞, the filter solution takes the form:

$$C_F(Y) = \frac{1}{1-e^{Pe}}\left[e^{PeY} - e^{Pe}\right] \qquad (16)$$

GITT will be applied to $C^{**}(X,Y,\tau)$, where it will be possible to obtain its solution. Given the same, we will use equations (16) and (14) to find the general solution of the proposed physical problem. Following the GITT methodology, appropriate auxiliary problems must be defined as well as the development of a transformed-inverse pair.

3.1 Auxiliary problem of eigenvalue in the X direction

The auxiliary problem, in the X direction, for determining the concentration field is written as follows:

$$\frac{\partial^2 \phi_i(\sigma_i, X)}{\partial X^2} + \frac{\sigma_i^2}{L^*}\phi_i(\sigma_i, X) = 0 \quad ; \quad 0 \le X \le 1 \qquad (17)$$

$$\frac{\partial \phi_i(\sigma_i, X)}{\partial X} = 0 \quad ; \quad X = 0 \ \text{ e } \ \sigma_i > 0 \qquad (18)$$

$$\frac{\partial \phi_i(\sigma_i, X)}{\partial X} = 0 \quad ; \quad X = 1 \ \text{ e } \ \sigma_i > 0 \qquad (19)$$

The eigenvalues (σ_i), the autofunctions, $\Phi_i(\sigma_i,X)$, and the norms (N_i), for the above-mentioned eigenvalue problem, can be developed analytically. After mathematical manipulations, we obtain that:

$$\sigma_i = (i-1)\pi\sqrt{L^*} \qquad ; \qquad i = 1,2,3,... \qquad (20)$$

$$\phi_i(\sigma_i, X) = \cos\left(\frac{\sigma_i}{\sqrt{L^*}} X\right) \tag{21}$$

$$N_i = \int_0^1 \frac{1}{L^*} \phi_i^2(\sigma_i, X) dX \tag{22}$$

Since the first eigenvalue is zero, the norm for the first eigenvalue is given by: $N_1 = \frac{1}{L^*}$. For the other eigenvalues, the norm is: $N_i = \frac{1}{2L^*} \quad ; \quad i = 2,3,4,...$

3.2 Auxiliary problem of eigenvalue in the Y direction

The auxiliary problem in the Y direction is defined as:

$$\frac{\partial}{\partial Y}\left(e^{-Pe.Y} . \frac{\partial \Psi_j(\mu_j, Y)}{\partial Y}\right) + \mu_j^2 e^{-Pe.Y} \Psi_j(\mu_j, Y) = 0, \quad 0 \le Y \le 1 \tag{23}$$

$$\Psi_j(\mu_j, Y) = 0 \quad ; \quad Y = 0 \text{ e } \mu_j > 0 \tag{24}$$

$$\frac{\partial \Psi_j(\mu_j, Y)}{\partial Y} + Bi.\Psi_j(\mu_j, Y) = 0 \quad ; \quad Y = 1 \text{ e } \mu_j > 0 \tag{25}$$

The proposed auxiliary problem was solved using the Integral Transformation Method (Cotta, 1993), for solving problems of eigenvalues (μj), of the eigenfunctions φj (μj, Y) and of norms (Kj) with structure in the typical problem of Sturm-Liouuville.

3.3 Integral transformation of the concentration field

Following the methodology of GITT's use, the transformed-inverse pair was defined with the purpose of reducing the original problem, which is a partial differential equation, in an infinite and coupled system of ordinary differential equations. In a second moment, the inverse formula can be used to obtain the solution of the original problem. The pair transformed integral, defined for this problem is given by:

$$\overline{C}_i(Y, \tau) = \frac{1}{N_i^{1/2}} \int_0^1 \frac{1}{L^*} \phi_i(\sigma_i, X) C^{**}(X, Y, \tau) dX, \qquad \textit{Transformada} \quad 1 \tag{26}$$

$$\overline{C}_{ij}(\tau) = \frac{1}{K_j^{1/2}} \int_0^1 e^{-Pe.Y} . \Psi_j(\mu_j, Y) \overline{C}_i(Y, \tau) dY, \qquad \textit{Transformada} \quad 2 \tag{27}$$

$$C^{**}(X, Y, \tau) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\phi_i(\sigma_i, X) \Psi_j(\mu_j, Y) \overline{C}_{ij}(\tau)}{N_i^{1/2} K_j^{1/2}} \qquad \textit{Inversa} \tag{28}$$

Substituting Eq. (14) into Eq. (8), the resulting equation can be treated analytically by integral operators. With the aid of the auxiliary problems and the transformed-inverse pair, this partial differential equation can be transformed into a system of ordinary differential equations given by:

$$R\frac{\partial \overline{C}_{ij}(\tau)}{\partial \tau}=-\left(\mu_j^{\,2}+\sigma_i^2\right)\overline{C}_{ij}(\tau) \tag{29}$$

whose general solution is classical, given by:

$$\overline{C}_{ij}(\tau)=\overline{C}_{ij}(0).e^{-\frac{\left(\mu_j^{\,2}+\sigma_i^2\right)}{R}\tau} \tag{30}$$

such that: $\overline{C}_{ij}(0)=\bar{f}_i.\bar{g}_j$, where:

$$\bar{f}_i=\frac{1}{N_i^{1/2}}\int_0^1\frac{1}{L^*}\phi_i(\sigma_i,X)dX \rightarrow \bar{f}_1=\frac{1}{\sqrt{L^*}} \quad e \quad \bar{f}_i=0\ ;\ i=2,3,4,... \tag{31}$$

$$\bar{g}_j=\frac{1}{K_j^{1/2}}\int_0^1\left[C_0^{\,*}(Y)-C_F(Y)\right]e^{-Pe.Y}.\Psi_j\left(\mu_j,Y\right)dY \tag{32}$$

Combining equations (28), (16) and (14), one can find the general solution of the proposed physical problem, which concerns the calculation of $C^*(X,Y,\tau)$. Proceeding as described, it is concluded that:

$$C^*(X,Y,\tau)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{\phi_i(\sigma_i,X)\Psi_j\left(\mu_j,Y\right)\bar{f}_i.\bar{g}_j.e^{-\frac{\left(\mu_j^{\,2}+\sigma_i^2\right)}{R}\tau}}{N_i^{1/2}K_j^{1/2}}+\frac{1}{1-e^{Pe}}\left[e^{PeY}-e^{Pe}\right] \tag{33}$$

4. RESULTS

In the figures that follow The solution obtained in the present work was implemented through in MATHEMATICS software 9.0 to obtain the results. In the auxiliary problems 100 eigenvalues and 100 corresponding eigenfunctions were used to analyze the convergence of the results. In all cases presented it is considered $C_0^{\,*}(Y)=0$ and L^* = 1. In the figures that follow we can observe the effect of the Peclet on the process of soil contamination.

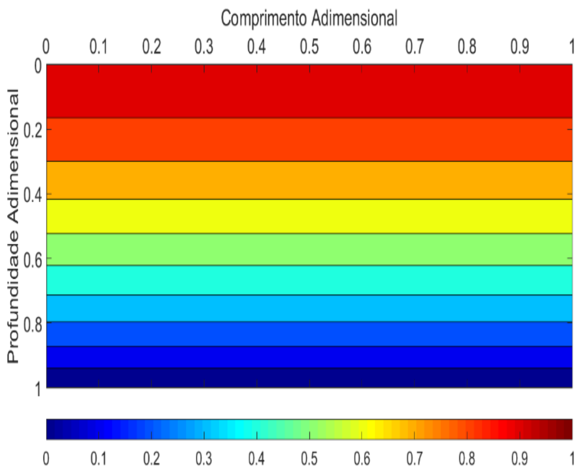

Figure 2. Distribution of the dimensionless concentration of the pollutant for Pe = 2, R = 1 e τ = 0,20

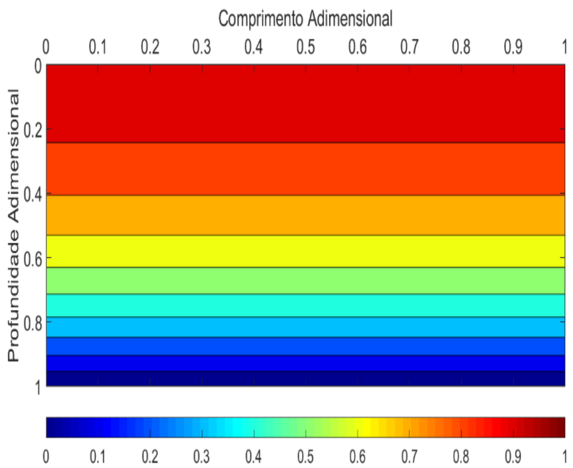

Figure 3. Distribution of the dimensionless concentration of the pollutant for Pe = 2, R = 1 e τ = 0,50

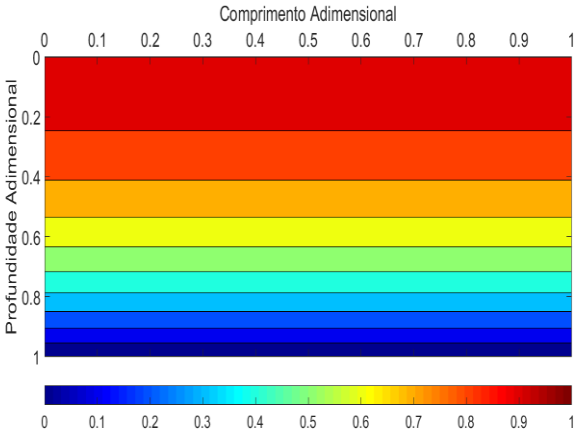

Figure 4. Distribution of the dimensionless concentration of the pollutant for Pe = 2, R = 1 e τ = 0,75

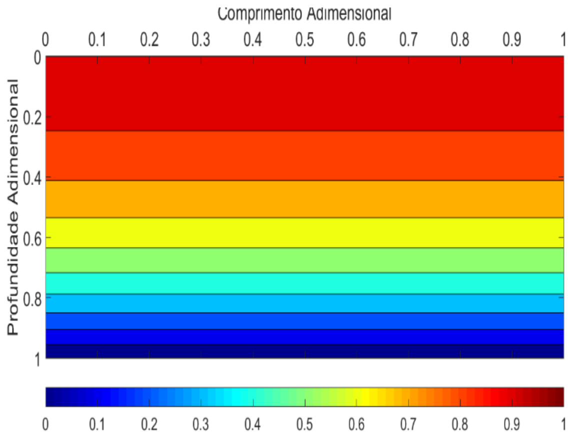

Figure 5. Distribution of the dimensionless concentration of the pollutant for Pe = 2, R = 1 e τ = 0,90

In Figs. (2-5) the concentration distribution was evaluated for different dimensionless times e Pe = 2 e R=1, where it is verified that as the dimensionless time increases the contamination coming from the RSU cell tends to reach greater dimensionless depths.

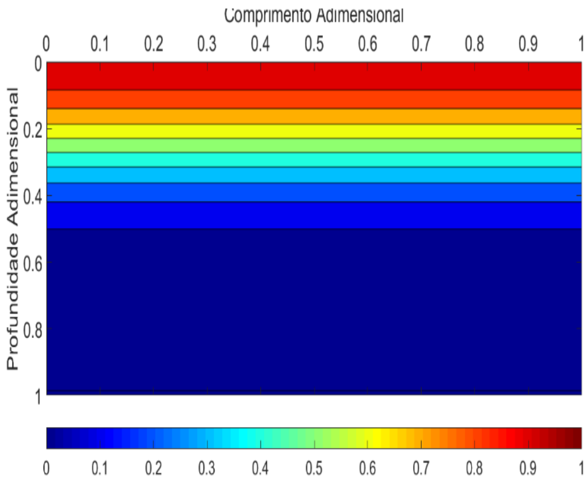

Figura 6. Distribution of the dimensionless concentration of the pollutant for Pe = 10, R = 1 e τ = 0,02

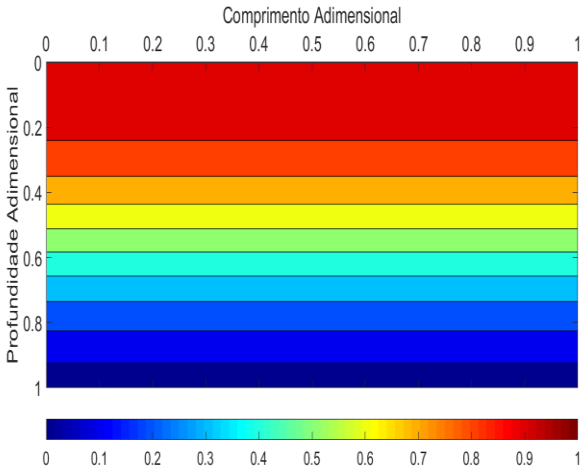

Figura 7. Distribution of the dimensionless concentration of the pollutant for Pe = 10, R = 1 e τ = 0,05

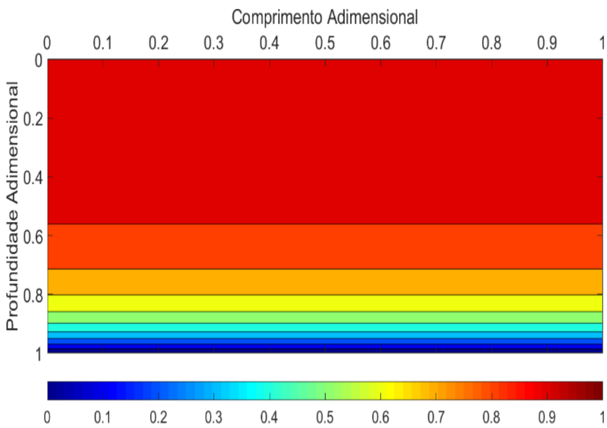

Figura 8. Distribution of the dimensionless concentration of the pollutant for Pe = 10, R = 1 e τ = 0,10

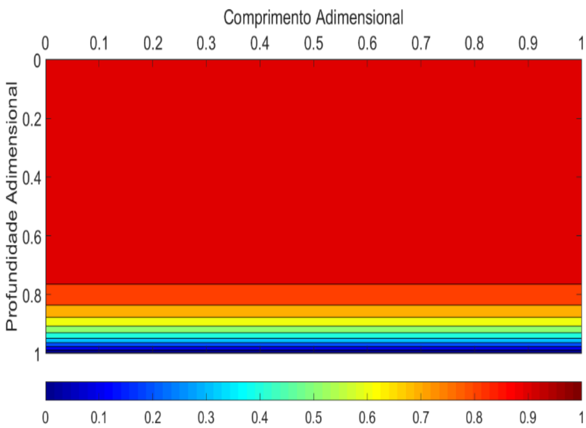

Figura 9. Distribution of the dimensionless concentration of the pollutant for Pe = 10, R = 1 e τ = 0,20

In Figs. (6-9) the concentration distribution was evaluated for different dimensionless times Pe = 10 e R=1. It was verified that the soil contamination occurs faster than the previous case, so that in the dimensionless time τ =0,20 virtually 80% of the soil is already contaminated.

5. CONCLUSIONS

It is concluded from the results analysis that the GITT application is effective in solving the proposed physical problem where it was possible to obtain an analytical solution for the concentration field. In this way, the objectives were achieved satisfactorily, showing the influence of the Peclet on the concentration field. This study shows is effective and important to contribute to the improvement and preservation of the environment.

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