

ENCIT-2018-0817 EQUILIBRIUM AND PERFECT GAS AIR BEHAVIOR AT THE STAGNATION REGION

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Abstract. Undergraduate Students from UFRN's Mechanical Engineering are developing procedures to calculate the Earth's atmospheric thermodynamic properties and the conditions right after the normal shockwave considering air as calorically perfect gas and air in chemical and thermodynamic equilibrium. The Earth's atmosphere may be defined by several layers and closed relationships can be found for each of these layers. Thermodynamic properties (pressure, temperature and density) are presented for Earth's atmosphere geometric altitude up to 86 km. Thermodynamic properties (pressure, temperature and density) ratios right after normal shockwave can be obtained based on theoretical analysis using one-dimensional normal shockwave compressible flow. For calculating airflow conditions behind normal shockwaves considering two gas models for the air: calorically perfect and chemical and thermodynamic equilibrium are presented. For calorically and/or thermally perfect gas, the normal shock relationships can be easily obtained as closed form of the thermodynamic property (pressure, density and temperature) ratios and Mach number across the normal shockwave. For chemical and thermodynamic equilibrium, the high temperature effects in the air such as molecular vibration and dissociation and atomic ionization are taken into account by using the mathematical correlations for the air properties. By yielding pressure, temperature and Mach number of the freestream airflow, it is possible to obtain the results for the airflow conditions behind a normal shock wave.

Keywords: standard atmosphere, normal shock, calorically perfect gas, chemical equilibrium air, scramjet.

1. INTRODUCTION

In order to collaborate with the Instituto Nacional de Ciência e Tecnologia em Propulsão Hipersônica Aspirada (INCT PRO-HYPER), created by Instituto de Estudos Avançados (IEAv/DCTA) and approved by CNPq, The Universidade Federal do Rio Grande do Norte (UFRN) started to involve in hypersonic airbreathing propulsion research area.

INCT PRO-HYPER aims to carry out scientific research and technological development for the solution of problems related to the design of an aerospace vehicle that uses hypersonic airbreathing propulsion system based on supersonic combustion (scramjet). The proposal involves designs on aerothermodynamics, structural and thermal, integration with accelerator vehicle, material definitions, stability and trajectory studies, definition of embedded system, definition of monitoring parameters of atmospheric flight, navigation and control system studies, telemetry, post-flight analysis of telemetry data, covering the complete envelope of atmospheric flight from the launch up to hypersonic speeds.

Brazilian researchers, from Instituto de Estudos Avançados (IEAv) are designed two prototypes: 14-X waverider (Fig. 1) and 14-X S (Fig. 2), to demonstrate at 30 km geometric altitude the supersonic combustion during atmosphere flight at hypersonic velocity, corresponding to Mach number 10 (approximately 3000 m/s) and 7 (approximately 2100 m/s), respectively.

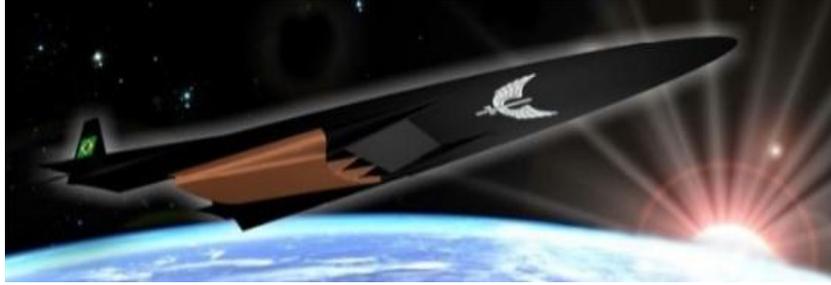


Figure 1. Demonstrator scramjet 14-X waverider.

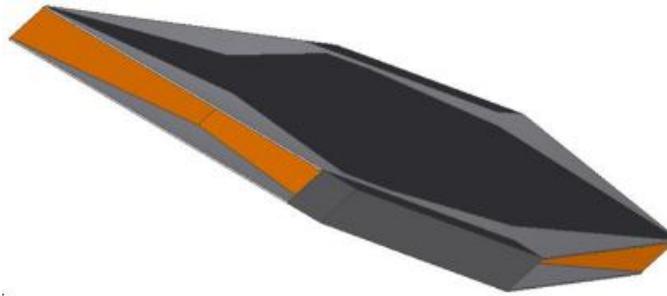


Figure 2. Demonstrator scramjet 14-X S.

2. METHODOLOGY

2.1 Earth's atmosphere

In general, researchers use the U.S. Standard Atmosphere (1976) to determine the atmospheric thermodynamic properties. For the present purpose air is composed of a mixture of nitrogen and oxygen, and its behavior as perfect gas.

Standard atmospheres have been providing from many sources, and the most useful standard atmosphere is provided by the work developed by three U.S. Agencies: National Oceanic and Atmospheric Administration, National Aeronautics and Space Administration and United States Air Force (U.S. Standard Atmosphere, 1976).

One may be recognized that the Earth's atmosphere is not a static medium. U.S. Standard Atmosphere (1976) mentions, there is a continuously changing in pressure, temperature, chemical constituents, particulate presence and electrical characteristics, primarily due to changes in solar radiation. Finally, U.S. Standard Atmosphere (1976) presents the governing equations to obtain the Earth's thermodynamic properties up to 1000 km geometric altitude.

It's important point out the aerospace vehicle integrated to hypersonic airbreathing propulsion based on scramjet technology will fly in a dense Earth's atmosphere up to 70 km geometric altitude, where there is air enough to be captured by the scramjet inlet.

The static temperature is function of molecular temperature and the ratio of average molecular weight at desired level to of average molecular weight at sea level the mixture of perfect gas

$$T_M = \frac{M_0}{M} T \quad (1)$$

where: up to 86 km one may consider the $M = M_0$, and the molecular temperature is given by

$$T_M = T_{M,b} + L_{M,b} (H - H_b) \quad (2)$$

where the geopotential altitude (here the geopotential of any point with respect to mean sea level, assumed zero potential), is given by

$$H = \frac{r_0}{r_0 + Z} \frac{Z}{g_o} \quad (3)$$

where Z is the geometric altitude, $r_0 = 6356,766$ km is the Earth's radius, and $\frac{Z}{g_o} = 1 \text{ m}^2/\text{m}$.

There are two static pressure relations used for computing pressure p at altitude from the sea level up to 86 km, within which the argument of the computation is geopotential. One should be applied when $L_{M,b}$ is zero (isothermal atmosphere layer) and the other is for $L_{M,b}$ is different of zero.

$$p = p_b e^{\frac{-g_o M_o (H-H_b)}{R^* T_{M,b}}} \quad (4)$$

$$p = p_b \left[\frac{T_{m,b}}{T_{M,b} + L_{M,b} (H - H_b)} \right]^{\frac{g_o M_o}{R^* L_{M,b}}} \quad (5)$$

In these equations $g_o' = 9,806650$ [m/s²], $M_o = 28.9644$ [kg/kmol], and $R^* = 8.314,32$ [N m/(kmol K)] are each defined single-valued constants, while $L_{M,b}$ and H_b are each defined multi-valued constants in accordance with the value of b as indicated in Table 1. The quantity $T_{M,b}$ is a multi-valued constant dependent on $L_{M,b}$ and H_b . The reference-level value for p_b for $b = 0$ is the defined sea-level value, $P_o = 101325.0$ N/m². Values of p_b for $b = 1$ through $b = 6$ are obtained from the application of the appropriate member of the pair (Tab. 1).

Table 1. The defined reference levels and gradients of the linearly segmented temperature-height profile from the surface to 86 geometric kilometers.

Layer	geopotential Altitude	molecular scale temperature gradient	molecular temperature	Form of function relating T to H	geopotential pressure
Subscripto b	H_b (km)	$L_{M,b}$ (K/km)	$T_{M,b}$ (K)		p_b (Pa)
0	0	-6.5	288.15	Linear	101325.00
1	11	0.0	216.65	Isotérmico	22632.04
2	20	1.0	216.65	Linear	5474.875
3	32	2.8	228.65	Linear	868.0153
4	47	0.0	270.65	Isotérmico	110.9057
5	51	-2.8	270.65	Linear	66.93847
6	71	-2.0	214.65	Linear	3.956387
7	84.852		186.87		0.373377

2.2 Governing conservation for normal shockwave

In the analytical theoretical analysis, the subscripts 1 and 2 are used to identify the upstream and the downstream conditions, across the normal shockwave (Anderson, 1990). The mass, momentum and energy conservation laws in one-dimensional steady state, non-viscous, no heat conduction compressible flow, applied to fixed normal shock wave (Fig. 3), are given by:

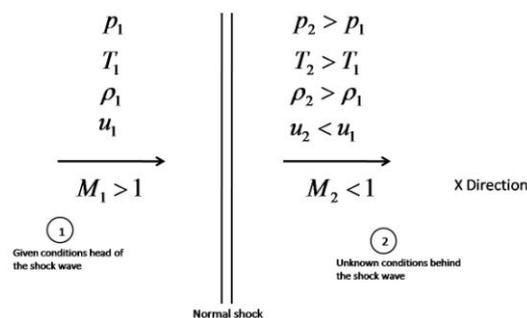


Figure 3: Normal shock diagram (Anderson, 1990).

$$\rho_1 u_1 = \rho_2 u_2 \quad (6)$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad (7)$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad (8)$$

where: ρ , p , u , and h are density, pressure, velocity and enthalpy of the gas across the normal shock wave, respectively.

2.2.1 Normal shock wave relationships for perfect gas behavior

The normal shockwave relationships can be easily obtained as closed form of the thermodynamic property (static pressure, static density, static temperature, ...) ratios and Mach number across the normal shock given by:

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{(\gamma+1)} [M_1^2 - 1] \quad (9)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{[(\gamma-1)M_1^2 + 2]} \quad (10)$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \frac{\rho_1}{\rho_2} = \left\{ 1 + \frac{2\gamma}{(\gamma+1)} [M_1^2 - 1] \right\} \frac{[(\gamma-1)M_1^2 + 2]}{(\gamma+1)M_1^2} \quad (11)$$

$$M_2^2 = \frac{M_1^2 + \frac{2}{(\gamma-1)}}{\frac{2\gamma}{(\gamma-1)} M_1^2 - 1} \quad (12)$$

Note, the flow across the normal shock wave promote an increase of pressure, density, temperature, and a decrease of Mach number for Mach number lower than 1.

2.2.2 Normal shock wave relationships for air in chemical and thermodynamic equilibrium

Tannehill and Muggge (1974) present the polynomial curve fittings to estimate the thermodynamic properties of the air in chemical equilibrium conditions, given by

$$h = \frac{p}{\rho} \left[\frac{\tilde{\gamma}}{\tilde{\gamma} - 1} \right] \quad (13)$$

where, the specific enthalpy as function of pressure and density. The specific heat ratio $\tilde{\gamma}$ considering high temperature effects may be calculated by

$$\tilde{\gamma} = c_1 + c_2 Y + c_3 Z + c_4 Y Z + \frac{c_5 + c_6 + c_7 Z + c_8 Y Z}{1 + \exp[c_9 (X + c_{10} Y + c_{11})]} \quad (14)$$

where, X, Y, and Z are given by

$$X = \log_{10} \left(\frac{p}{1.013 \times 10^5} \right) \quad (15)$$

$$Y = \log_{10} \left(\frac{\rho}{1.292} \right) \quad (16)$$

$$Z = X - Y \quad (17)$$

and the coefficients c_1 to c_{11} are tabulated and may be found in Tannehill and Mugge (1974).

Temperature as function of pressure and density may be calculated by

$$\log\left(\frac{T}{T_0}\right) = d_1 + d_2Y + d_3Z + d_4YZ + d_5Z^2 + \frac{d_6 + d_7Y + d_8Z + d_9YZ + d_{10}Z^2}{1 + \exp[d_{11}(Z + d_{12})]} \quad (18)$$

where, $T_0 = 151.78[\text{K}]$ and X, Y, and Z are given by

$$Y = \log\left(\frac{\rho}{1.225}\right) \quad (19)$$

$$X = \log\left(\frac{p}{1.0134 \times 10^5}\right) \quad (20)$$

$$Z = X - Y \quad (21)$$

and the coefficients d_1 to d_{13} are tabulated and may be found in Tannehill and Mugge (1974).

3. RESULTS AND COMMENTARIES

First is necessary to define the thermodynamic atmospheric air properties, which any aerospace vehicle (subsonic aircraft to hypersonic rocket engines) will perform atmospheric flight. As mention early, the US Standard Atmosphere 1976 presents the equations needed to obtain the thermodynamic atmospheric air properties.

In the present paper the Figs. 4 to 6 show the temperature, pressure and density, respectively.

As one can observe the temperature profile is function of altitude, and it is always lower than the sea level temperature (Fig. 4). The pressure (Fig. 5) and density (Fig. 6) are exponential function of geometric altitude.

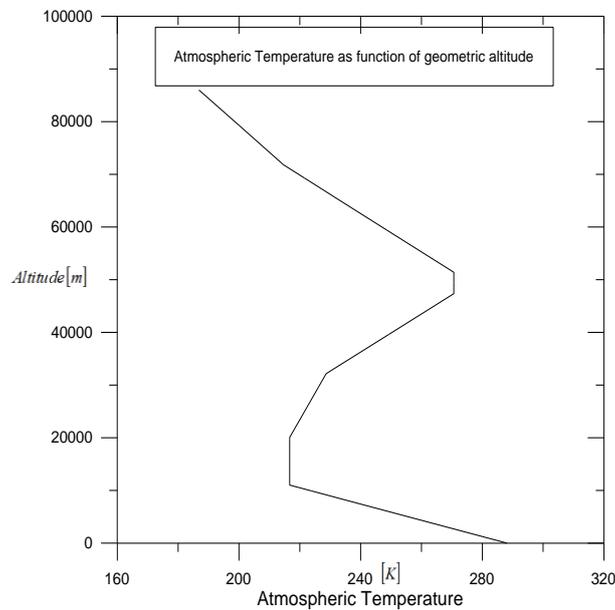


Figure 4. Static temperature as function of geometric altitude.

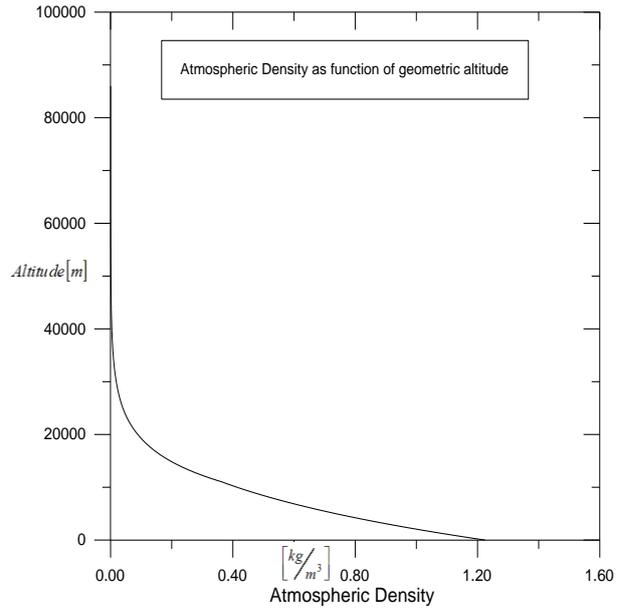
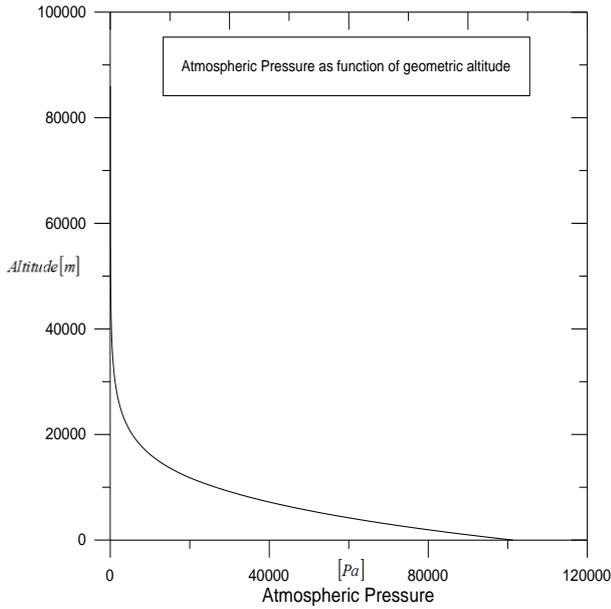


Figure 5. Static pressure as function of geometric altitude. Figure 6: Static density as function of geometric altitude.

Figs. 7 to 9 present, considering air as perfect gas, the thermodynamic property (pressure, temperature and density) ratios across the normal shockwave, respectively, and Fig. 10 presents the flow Mach number airflow after the normal shockwave.

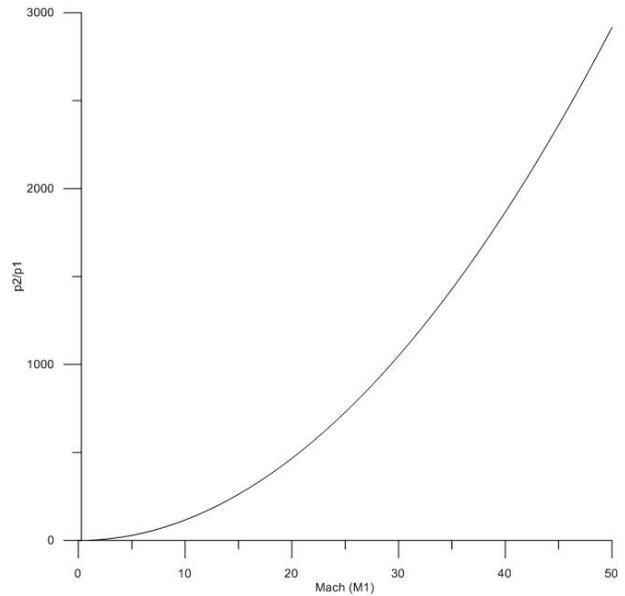
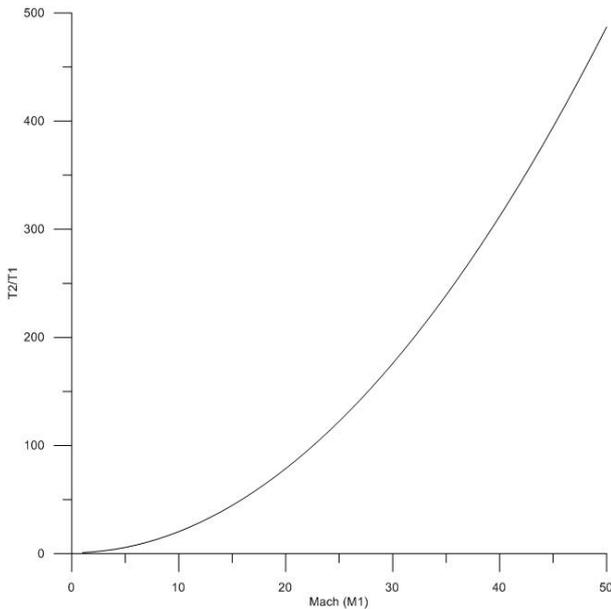


Figure 7. Temperature ratio as function of freestream Mach number.

Figure 8. Pressure ratio as function of freestream Mach number.

The temperature (Fig. 7) and pressure (Fig. 8) are functions of freestream Mach number, and both increase (go to infinity) as freestream Mach number increases (go to infinity). However, the density (Fig. 9) approaches to values equal to 6 when freestream Mach number increases. Finally, the flow Mach number (Fig. 1) after the normal shockwave decreases to Mach number lower than 1 as freestream Mach number increases.

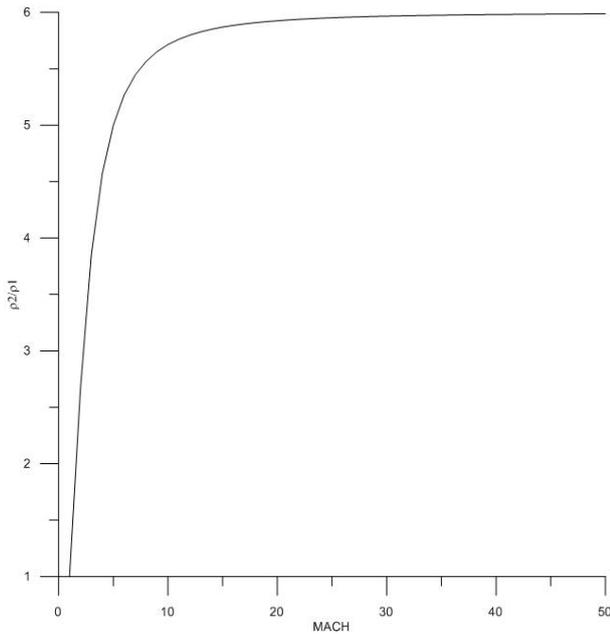


Figure 9. Density ratio as function of freestream Mach number.

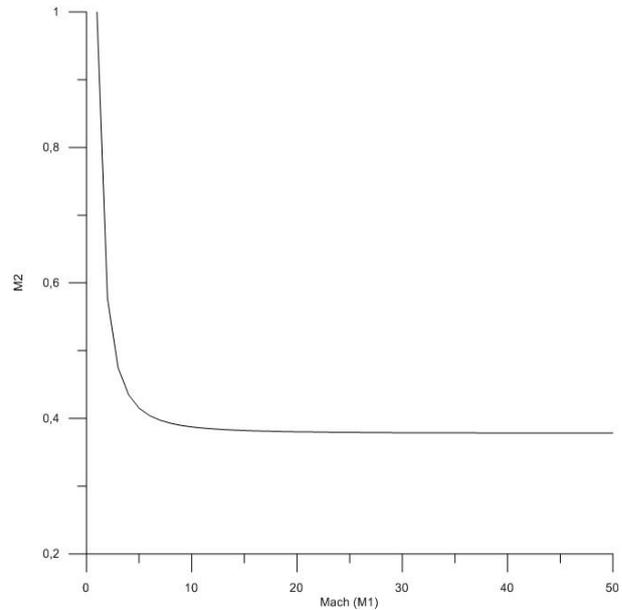


Figure 10. Mach number after the normal shockwave as function of freestream Mach number.

Considering the air in chemical and thermodynamic equilibrium behind the normal shockwave Figs. 11 to 13 present, as function of freestream Mach number, the thermodynamic properties (pressure, temperature and density), at 52 km of geometric altitude, across the normal shockwave, respectively, and Fig. 10 presents the flow Mach number airflow after the normal shockwave. The calorically perfect gas data has been verified with results found in the literature (for example: Anderson, 1990).

The temperature after the normal shockwave was validated using the results presented by Anderson (2006). Table 2 presents atmospheric data used to validate the temperature after the normal shockwave, with velocities ranging from 330 m/s to 14030 m/s, at geometric altitude of 52000 m.

Table 2. Thermodynamic atmospheric properties at 52 km geometric altitude (U.S. Standard Atmosphere, 1976).

Altitude	Temperature	Pressure	Density	Sound speed
km	K	Pa	kg/m ³	m/s
52	269.03	62.21	0.00080562	288.81

Observe that, the temperature (Fig. 11) after normal shockwave, considering air as in thermodynamic equilibrium is lower than the temperature considering perfect gas, due to the activation of the vibrational energy of the molecules and eventual dissociation and even ionization, both consuming energy from the system and reducing its overall temperature.

Additionally, for velocities lower than 2000 m/s there is no differences between calorically perfect gas and chemical equilibrium models for atmospheric air. Therefore, for velocities higher than 2000 m/s the aerospace vehicle designers should consider the use of chemical equilibrium model.

Considering air in equilibrium the values of pressure (Fig. 12) and density (Fig. 13) are higher than the corresponding values considering air as calorically perfect gas. However, the influence of vibrational energy, dissociation and even ionization is very low in the pressure profile (Fig. 12). Additionally, Mach number after normal shockwave considering air in equilibrium is lower than air as perfect gas (Fig. 14).

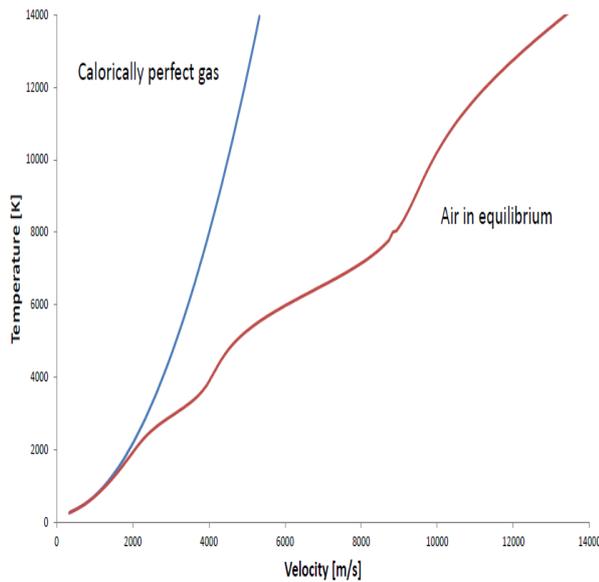


Figure 11. Temperature as function of freestream Mach number.

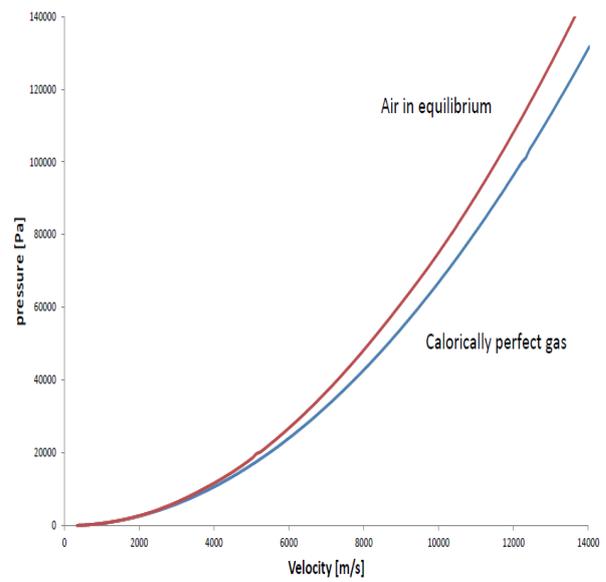


Figure 12. Pressure ratio as function of freestream Mach number.

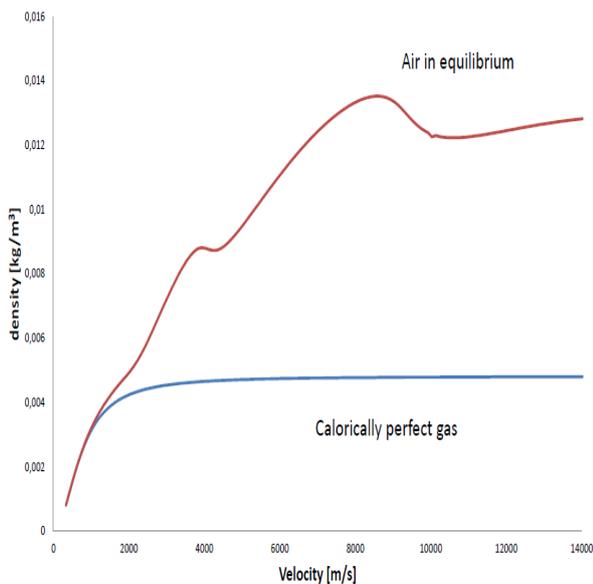


Figure 13. Density ratio as function of freestream Mach number.

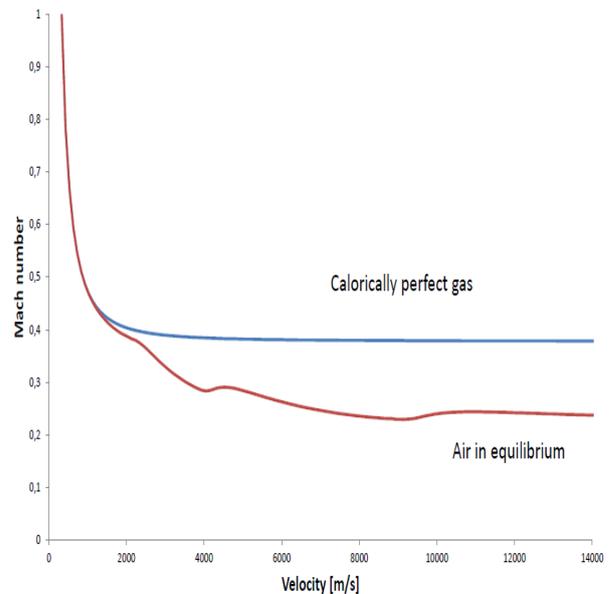


Figure 14. Mach number after the normal shockwave as function of freestream Mach number.

4. CONCLUSION AND OUTLOOK FOR FUTURE PROJECTS

Undergraduate Students from UFRN's Mechanical Engineering are developing procedures to calculate the Earth's atmospheric thermodynamic properties and the conditions right after the normal shockwave considering air as calorically perfect gas and air in chemical and thermodynamic equilibrium.

One may keep in mind that the Earth's thermodynamic properties presented in the present scientific paper can be used to up to 86 km altitude to design any aerospace vehicle (include one using airbreathing propulsion engine based on scramjet technology) flying through the Earth's atmosphere.

The Earth's thermodynamic atmosphere properties may be used to determine the derived quantities, such as the dimensionless numbers: Knudsen number $Kn = \frac{\lambda}{L}$, Mach number $M = \frac{V}{a}$ Reynolds number $Re = \frac{\rho V D}{\mu}$, relevant for the any analysis of the aerospace vehicle design.

The results from the air as perfect gas relationships and in chemical equilibrium, after normal shockwaves, show the influence of vibrational energy, and dissociation and ionization effects. Therefore, it is important the use of air in chemical equilibrium, in the design of any aerospace vehicle in hypersonic velocities higher than 2000 m/s.

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The authors declare that there is no conflict of interest regarding the publication of this scientific article.

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