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A STUDY OF THE LAMINAR THERMAL BOUNDARY LAYER IN ROUND DUCTS USING GITT

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Abstract: *Thermal boundary layer corresponds to a variation on the temperature field that occurs in a narrow region close to the wall of a rigid body when this is exposed to a fluid flow at a different temperature. Studying this phenomenon is important to the various industrial application, such as designing cooling systems for electronic components, capturing solar energy, geothermal reservoirs, and enhanced oil recovery. The present work aims to perform a characterization of the laminar thermal boundary layer considering a Newtonian fluid flow inside a round tube. In order to do that, it was developed a mathematical model using GITT (Generalized Integral Transform Technique), the obtained results allow to verify the temperature development inside the tube and therefore to show the characteristics of the boundary layer including its thickness in certain points.*

Keywords: *thermal boundary layer, GITT, round tube.*

1. INTRODUCTION

According to Dewitt et al. (2008), the concept of the boundary layer is crucial to the understanding of convective heat and mass transfers between a surface and a fluid flowing in contact with the surface. Dewitt (2008) concludes that these boundary layers can be classified as velocity, thermal and concentration. The temperature field variation in a rigid body exposed to a fluid flow at a different temperature is going to happen in a narrow region close to the wall, exactly like it occurs with a velocity field. This region, by analogy to the dynamic process, is generally called thermal boundary layer (Pantaleão, 1990)

The boundary layer study (in this case, the thermal one) is important to various industrial applications, such as designing cooling systems for electronic components, capturing solar energy, geothermal reservoirs, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, among others applications (Makinde, 2012). Thermal boundary layer characterization is a limiting factor in various applications because if the temperature distribution in the thermal boundary layer is known, it is possible to calculate directly the heat transfer of a solid surface (Kulkarni, 2011).

The boundary layer can be laminar or turbulent. Convective heat transfer coefficients are generally higher for turbulent flow than for laminar flow. There can be the two types of boundary layers in the same flow, and it is necessary to consider both in the determination of the mean convective heat transfer coefficient. The laminar zone length can be either ignored or taken into consideration according to the flow geometry and speed. (Silva, 2016).

Even with all important applications presented, the best-known methods to characterize the thermal boundary layer, whether in ducts or plates, are usually invasive, in other words, there’s contact between the sensor and the liquid, and it interferes directly with its flow (Carey, 1978; Slagen, 2013; Puttmaker, 2013; Bhattacharyya, 2016; Bellec, 2016), generally the sensors utilized for this are thermocouples and hot wire anemometer.

Theoretical characterizations of thermal boundary layer were carried out by Azeman (2012); Makinde (2012) and Shishkina (2015) using mathematical methods like Runge-Kutta-Fehlberg, integral energy equation, DNS (Direct Numerical Simulation), and analyzed the variations in the Prandtl or Nusselt number for theoretical analysis of thermal boundary layer.

Also, for the theoretical analysis of the thermal boundary layer, a numerical method that has been used, presenting satisfactory results, is the GITT (Generalized Integral Transform Technique), (Ghiaasiaan, 2011); it shows a theoretical solution in which it is ignored the heat transfer in the axial direction and it applies Hagen-Poiseuille in the thermal boundary layer problem, while for the present work it will be considered the axial direction and the GITT will be utilized for the mathematical modeling.

The physical problem consists in analyzing the Newtonian fluid flow. According to Ghiaasiaan (2011), the fluids that follow the proportionality equation, in other words, when there's a linear relationship between the applied shear stress value and the strain rate resulting from it, they are called Newtonian fluids, including the water and gases in general. Figure 1 presents this kind of problem considering a round duct.

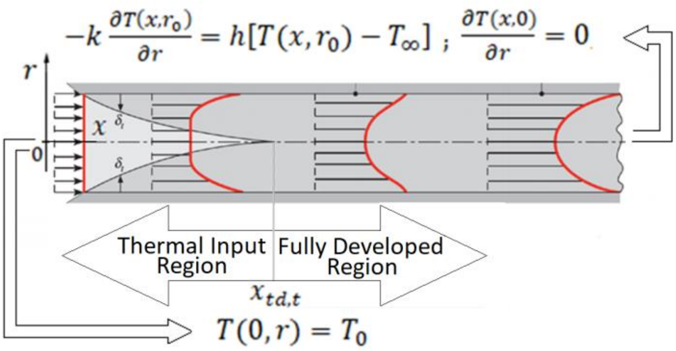

Figure 1: Physical problem illustration, Çengel, 2012.

2. MATHEMATICAL MODELING

Considering the flow regime is laminar and steady, and the fluid is incompressible with constant thermophysical properties, the energy equation, the inlet and contour conditions for a cylindrical coordinate system can be written as:

Energy Equation:

$$\rho c_p u(r)\frac{\partial T(x,r)}{\partial x} = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(x,r)}{\partial r}\right)\right]; \quad 0 < r < r_0, \quad x > 0 \tag{1a}$$

Contour conditions:

$$\frac{\partial T(x,r)}{\partial r} = 0, \quad r = 0, \ x > 0 \tag{1b}$$

$$k\frac{\partial T}{\partial r} + h(T(x,r) - T_\infty) = 0, \ r = r_0, \ x > 0 \tag{1c}$$

Inlet condition:

$$T(x,r) = T_0, \quad x = 0 \tag{1d}$$

Where ρ, c_p e k represent, respectively to the fluid, specific mass, specific heat at constant pressure and thermal conductivity. The velocity profile of the fully developed laminar flow is represented by u(r). In the present work, the velocity profile is fully developed in the thermal inlet (Kakaç, 1995 e Lipkis, 1954), viscous dissipation effects are not considered; no-slip and impermeable walls, body forces will be ignored; no internal energy generation and the axial diffusion of the fluid throughout the flow will also be ignored.

2.1 Non-dimensionalization of the problem

Non-dimensional parameters and groups proposed by Ghiaasiaan (2011) will be utilized with the objective of solving a family of problems defined by the proposed model:

$$R=\frac{r}{r_0};\qquad \xi=\frac{2x}{D_h\,\mathrm{Re}_d\,\mathrm{Pr}};\qquad \theta(\xi,R)=\frac{T(x,r)-T_\infty}{T_0-T_\infty};\qquad U(R)=\frac{u(r)}{u_m} \tag{2a-d}$$

Where T_∞ e T_0 represent, respectively, ambient temperature and fluid inlet temperature Bi, Re_d, and Pr are nondimensional, called Biot, Reynolds number and Prandtl number, respectively, whose definitions are given by

$$Bi=\frac{hr_0}{k};\qquad \mathrm{Re}_D=\frac{u_m D_h}{\nu};\qquad \mathrm{Pr}=\frac{\nu}{\alpha} \tag{2e-g}$$

With u_m representing the average flow velocity, r_0 is the round tube radius, $D_h=2r_0$ is the tube hydraulic diameter, ν, kinematic viscosity of the fluid and $\alpha = k/\rho c_p$ thermal diffusivity of the fluid. Even though the work does not objectify the hydrodynamic boundary layer analysis, it is known that if Prandtl number is higher than 1, the thermal boundary layer occurs before the hydrodynamic boundary layer, if it is lower than 1, it occurs after it. Reynolds number (Re_d) is a non-dimensional number used in fluid mechanics to calculate the flow regime in a certain fluid over a surface. The fundamental meaning of the Reynolds number is that it allows evaluating the type of flow (flux stability) and it can indicate if it flows in a laminar or turbulent way (Abramowits, 1953 e Brown, 1960). When the water flows in a cylindrical tube, the numbers 2000 and 2400 are adopted as limits. Thus, for amounts less than 2000, the flux will be laminar, for amounts higher than 2400 the flux will be turbulent, and if the number is between those limits the flux will be transient (Silva, 2016)

Biot number (Bi) is a non-dimensional parameter that provides a ratio between convective heat transfer coefficient in the surface of the rigid body and specific conductance of the rigid body, the ratio between the resistances inside of the rigid body and on its surface. For $Bi\to\infty$ there is the first type of contour condition (temperature specified on the wall), for $Bi\to 0$ there is the second type (flux specified on the wall) and $0<Bi<\infty$ there is the third type (it is considered heat transfer between fluid and environment). Biot is considered infinite for the results presented in this work.

Using the parameters and non-dimensional groups defined earlier, and after some algebraic manipulations, the energy equation, contour and inlet conditions will take the following non-dimensional forms:

Non-dimensional energy equation:

$$U(R)\frac{\partial\theta(\xi,R)}{\partial\xi}=\frac{2}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(\xi,R)}{\partial R}\right];\,0<R<1,\xi>0 \tag{3a}$$

Non-dimensional contour conditions:

$$\left.\frac{\partial\theta(\xi,R)}{\partial R}\right|_{R=0}=0\,,R=0\,;\xi>0 \tag{3b}$$

$$\frac{\partial\theta(\xi,R)}{\partial R}+Bi\,\theta(\xi,R)=0\qquad R=1\,,\quad \xi>0 \tag{3c}$$

Non-dimensional inlet conditions:

$$\theta=1\,;\xi=0,\quad 0\le R\le 1 \tag{3d}$$

Non-dimensional velocity profile:

$$U(R)=\;2[1-R^2] \tag{4}$$

2.2. Methodology – Generalized Integral Technique to solve the problem

The chosen auxiliary problem to determine the temperature field was:

$$\frac{d}{dR}\left[R\,\frac{d\psi_i(\mu_i,R)}{dR}\right] + \left[\mu_i^2 R\, U(R)\psi_i(\mu_i,R)\right] = 0,\ \ 0 < R < 1 \tag{5a}$$

$$\frac{\partial\psi_i(\mu_i,R)}{\partial R} = 0,\ \ R = 0,\ \ \mu_i > 0 \tag{5b}$$

$$\frac{\partial\psi_i(\mu_i,R)}{\partial R} + Bi\psi_i(\mu_i,R) = 0,\ \ R = 1,\ \ \mu_i > 0 \tag{5c}$$

In order to approach heat and mass diffusion problems analytically, usually, the answer to a correspondent eigenvalue problem is necessary, defined as the auxiliary problem, (Cotta, 1993). Mikhailov (1984), developed the signal counting method; this method allows calculating the eigenvalues, the eigenfunctions and the standards without loss of information in the application of the methodology.

2.3. Determining the Inverse-Transform pair of formulas

Inverse-transform pair defined for the problem studied:

$$\overline{\theta}_i(\xi) = \frac{1}{N_i^{1/2}}\int_0^1 R\,U(R)\psi_i(\mu_i,R)\theta(\xi,R)dR\ , \qquad \text{Transform} \tag{6a}$$

$$\theta(\xi,R) = \sum_{i=1}^{\infty}\frac{1}{N_i^{1/2}}\psi_i(\mu_i,R)\,\overline{\theta}_i(\xi)\ , \qquad \text{Inverse} \tag{6b}$$

Applying the transform definition, given by equation (6a), and using the auxiliary problem defined by (5a), the non-dimensional energy equation (3a) can be transformed into the following system of ordinary differential equations.

$$\frac{1}{2}\frac{d\overline{\theta}_i(\xi)}{d\xi} + \mu_i^2\,\overline{\theta}_i(\xi) = 0 \tag{7}$$

This system has classical analytical solution given by (8):

$$\overline{\theta}_i(\xi) = \overline{f}_i e^{-2\mu_i^2\xi} \tag{8}$$

Such that

$$\overline{f}_i = \frac{B_i\psi_i(\mu_i,1)}{N_i^{1/2}\mu_i^2}$$

The temperature field for the thermal inlet region takes on the form:

$$\theta(\xi,R) = \sum_{i=0}^{\infty}\frac{\psi_i(\mu_i,R).\bar{f}_i.e^{-2\mu_i^2\xi}}{N_i^{1/2}} \tag{9}$$

3. IMPLEMENTATION OF THE MATHEMATICAL MODEL – PRELIMINARY RESULTS

The system current mathematical model was simulated using *FORTRAN* programming language, achieving a numeric result. Temperature considered on the wall was 293K and various cases were conducted at inlet temperatures varying from 296K to 298K. The established flow rates for the flow of the fluid in simulation were 8.3 X 10^{-6} m^3/s and 16,67 X 10^{-6} m^3/s, the results that were obtained are presented in figures 2, 3, 4, and 5 from a to c.

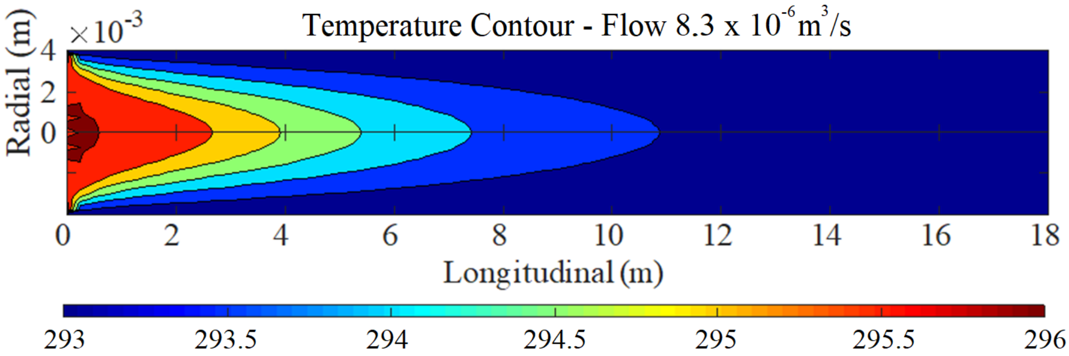

Figure 2a. Temperature Contour- Flow rate 8.3 X 10^{-6} m^3/s – Temperature of 296K

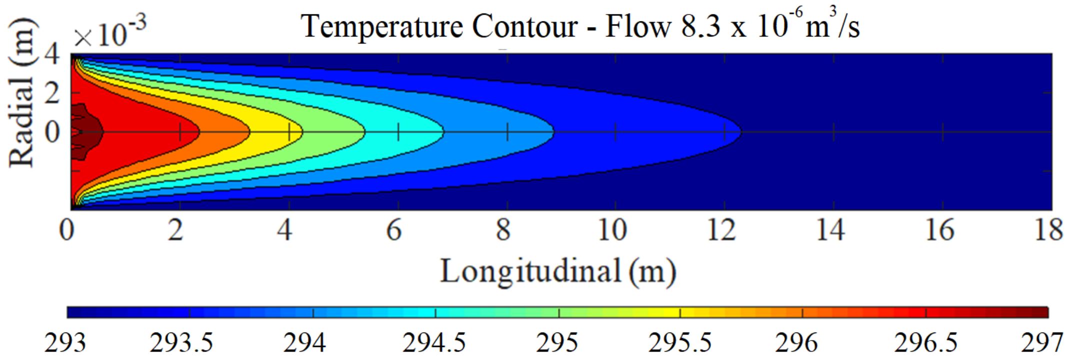

Figure 2b. Temperature Contour – Flow rate 8.3 X 10^{-6} m^3/s – Temperature of 297K

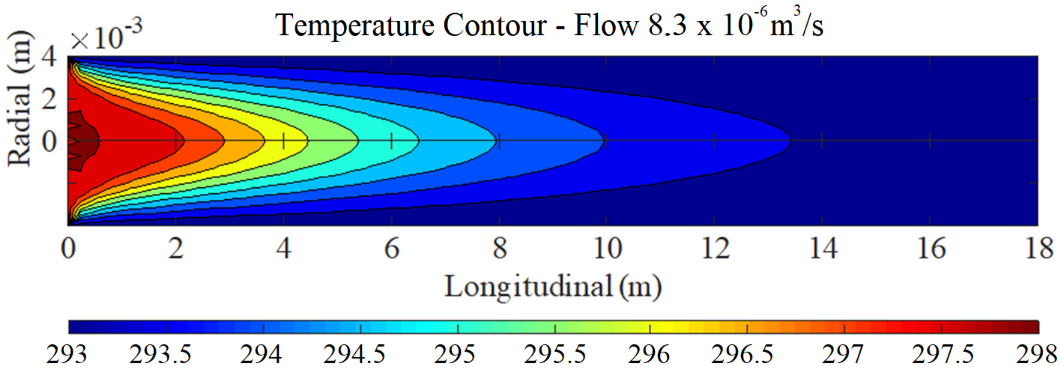

Figure 2c. Temperature Contour – Flow rate 8.3 X 10^{-6} m^3/s - Temperature of 298K

It is possible to observe in the longitudinal axis in figures 2a, 2b, and 2c that the thermal boundary layer is fully developed between 10 and 14 meters lengthwise in the round tube, results were better detailed on the charts from figures 3a, 3b, and 3c. According to Dewitt et al. (2008), far from the tube surface, the temperature profile is uniform $T(y) = T\infty$, however, as fluid particles get in touch with the tube surface, they reach a thermal balance and temperature gradients are developed. The region of the fluid which these temperature gradients exist is called the thermal boundary layer and its thickness δt is typically defined as the value of y for which the ratio:

$$\theta = \frac{T - T_s}{T_\infty - T_s} = 0.9 \tag{10}$$

A margin of 10% is defined in this work in order to verify the full heat development of the analyzed fluid. The proposed method allows, therefore, verify the thermal boundary layer thickness for a fluid that flows in a tube in laminar regime. Considering the equation (10) for inlet temperatures from 296K to 298K and considering 293K on the wall of the tube, the simulations were carried out as shown in the charts from 3a to 3c. Calculations performed reveal that the value of T which represents the full development of the thermal boundary layer is 293.5 K. Analyzing the temperature field evolution in each meter in the longitudinal direction, it is verified in figures 2a, 2b, and 2c the correspondents points to the height throughout the radial axis of the tube, thus, it was obtained the graphs which detail the thermal boundary layer thickness of the tube.

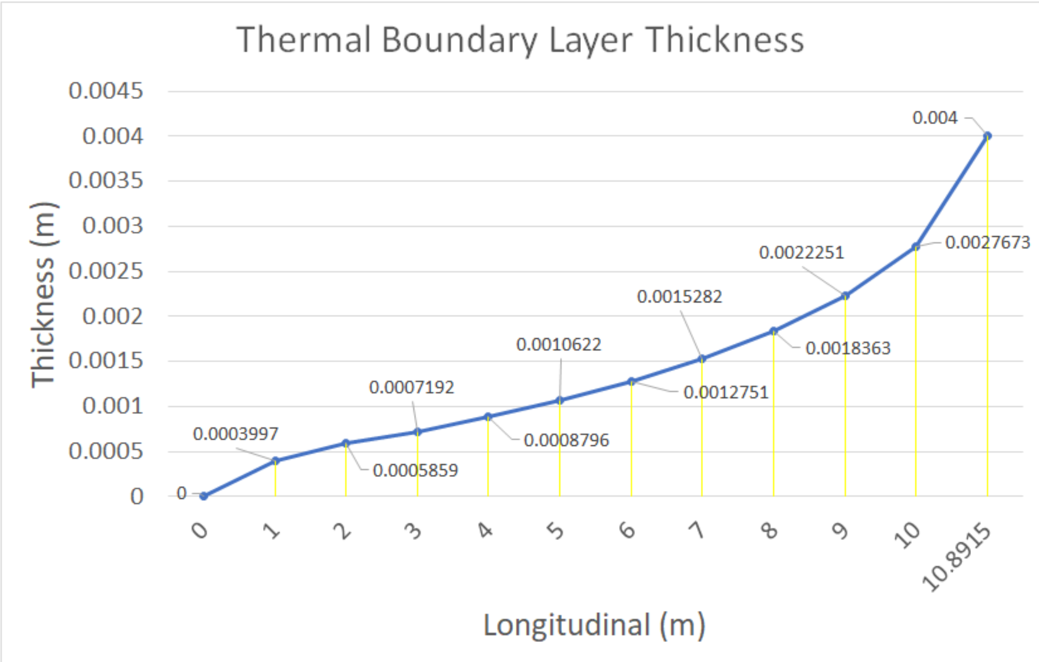

Figure 3a. Thermal boundary layer thickness versus longitudinal position for an inlet temperature of 296K

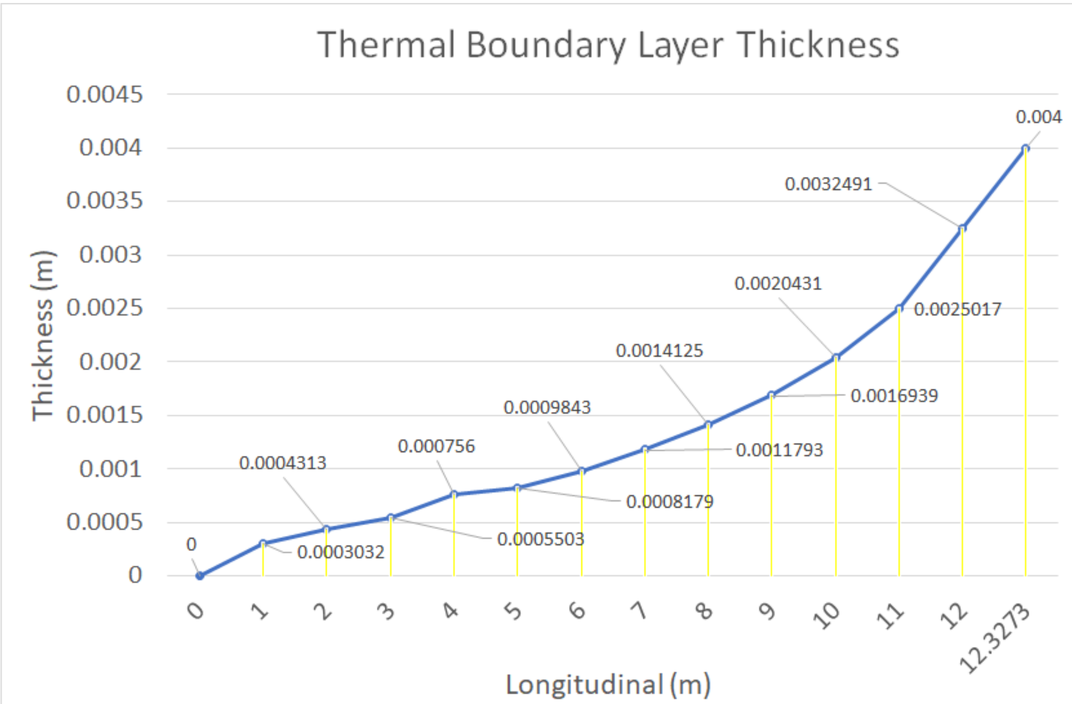

Figure 3b. Thermal boundary layer thickness versus longitudinal position for an inlet temperature of 297K

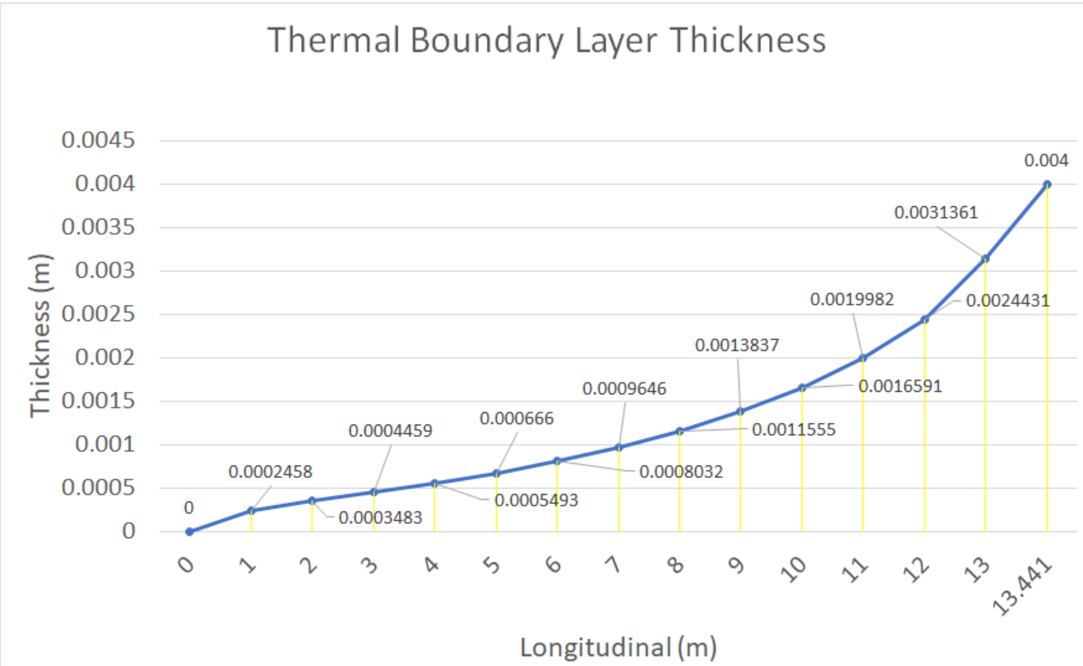

Figure 3c. Thermal boundary layer thickness versus longitudinal position for an inlet temperature of 298K

Doubling the flow rate value, going from 8.3 to 16.67 X 10^{-6} m^3/s, temperature contours were plotted as shown in the figures 4 a-c. It is possible to verify that for the model and adopted conditions the behavior of the thermal boundary layer presents direct linear variation - i. e., doubling the flow rate ended up doubling the longitudinal temperature profile. Similar to the figures 2 from a to c, for the figures 4 a-c it was considered the temperature of 293K on the wall and inlet temperatures from 296K to 298K.

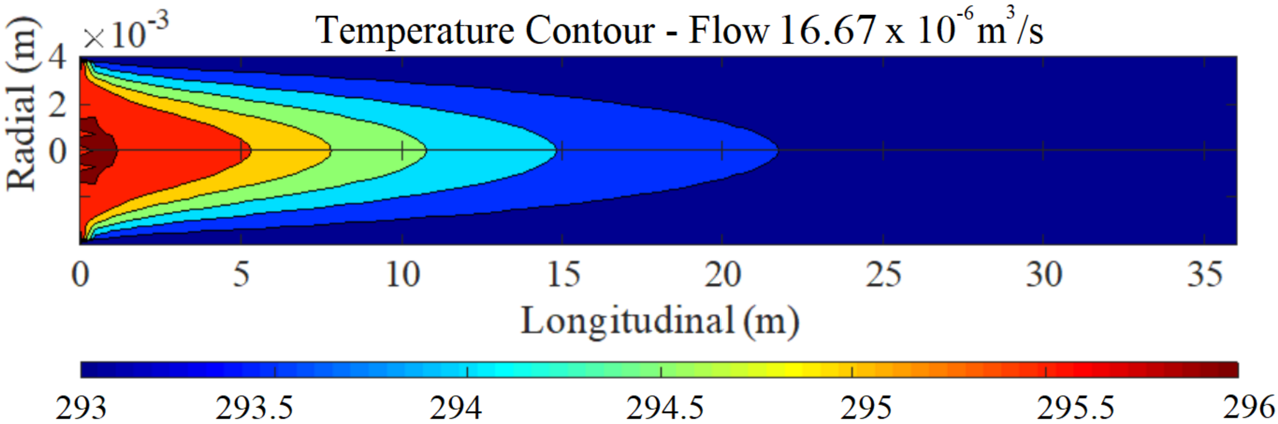

Figure 4a. Temperature Contour – Flow rate 16.67 10^{-6} m^3/s – Temperature of 296K

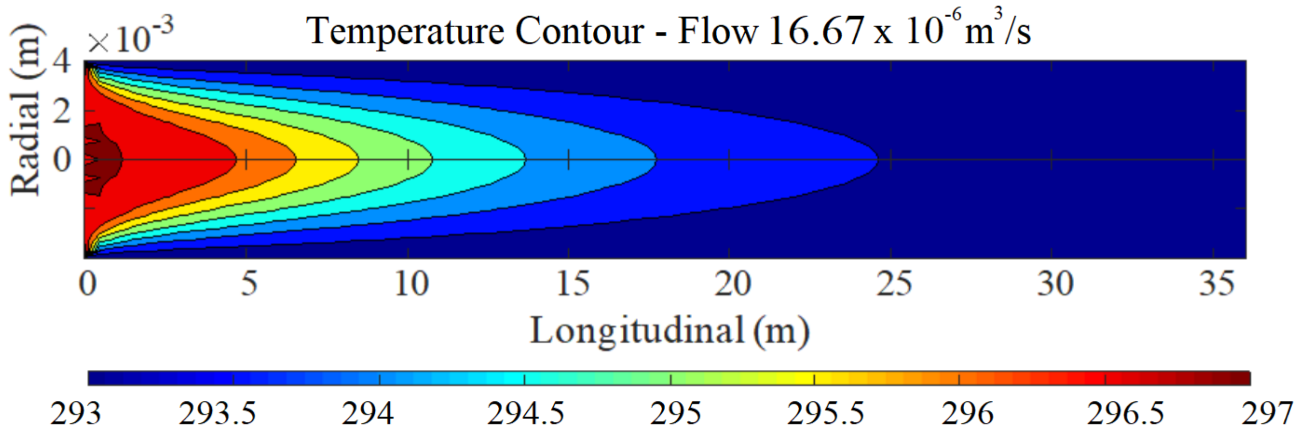

Figure 4b. Temperature Contour – Flow rate 16.67 10^{-6} m^3/s – Temperature of 297K

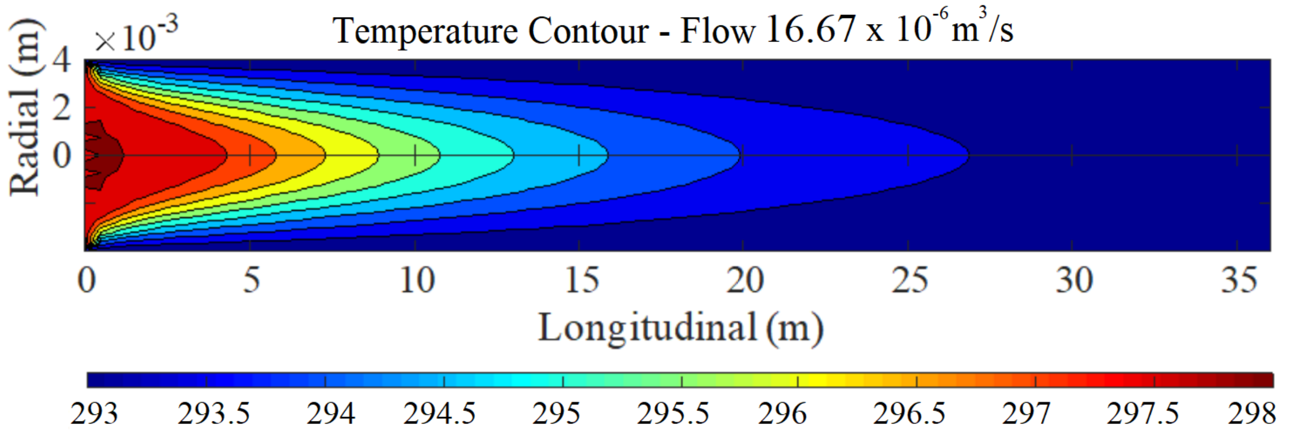

Figure 4c. Temperature Contour – Flow rate 16.67 10^{-6} m^3/s – Temperature of 298K

Despite the verification of linearity on the development of the thermal field, the corresponding amounts for the thermal boundary layer thickness to flow rate of 16.67 X 10^{-6} m^3/s showed on the figures 5 a-c do not present the same behavior, demonstrating that even though the temperature field changes linearly, the thermal boundary layer thickness do not display this property.

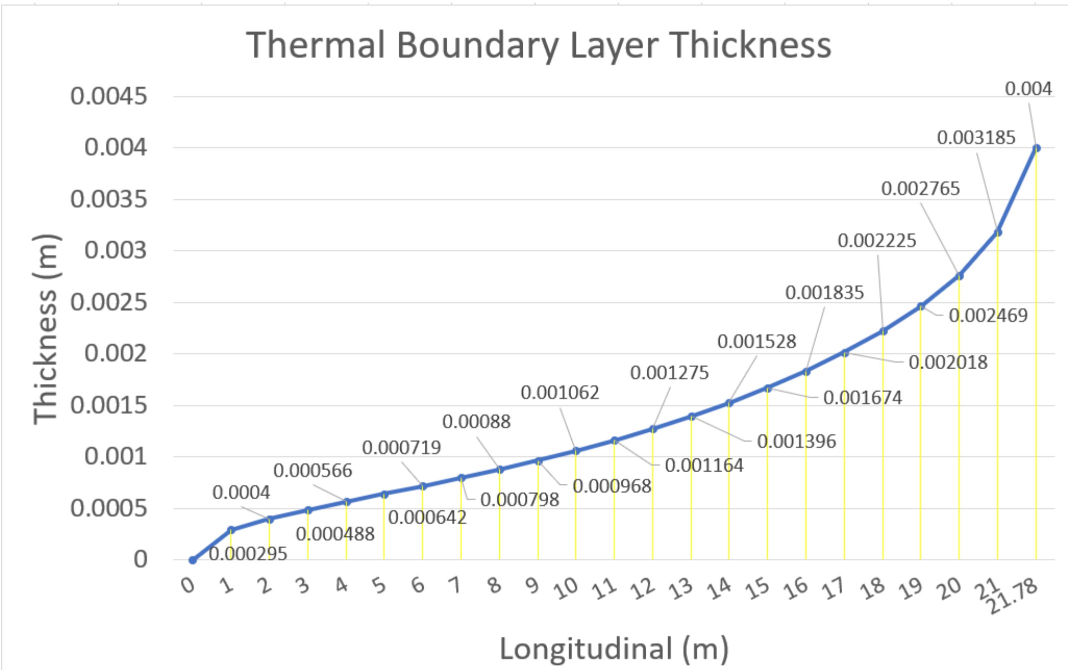

Figure 5a. Thermal boundary layer thickness versus longitudinal position for a inlet temperature of 296K

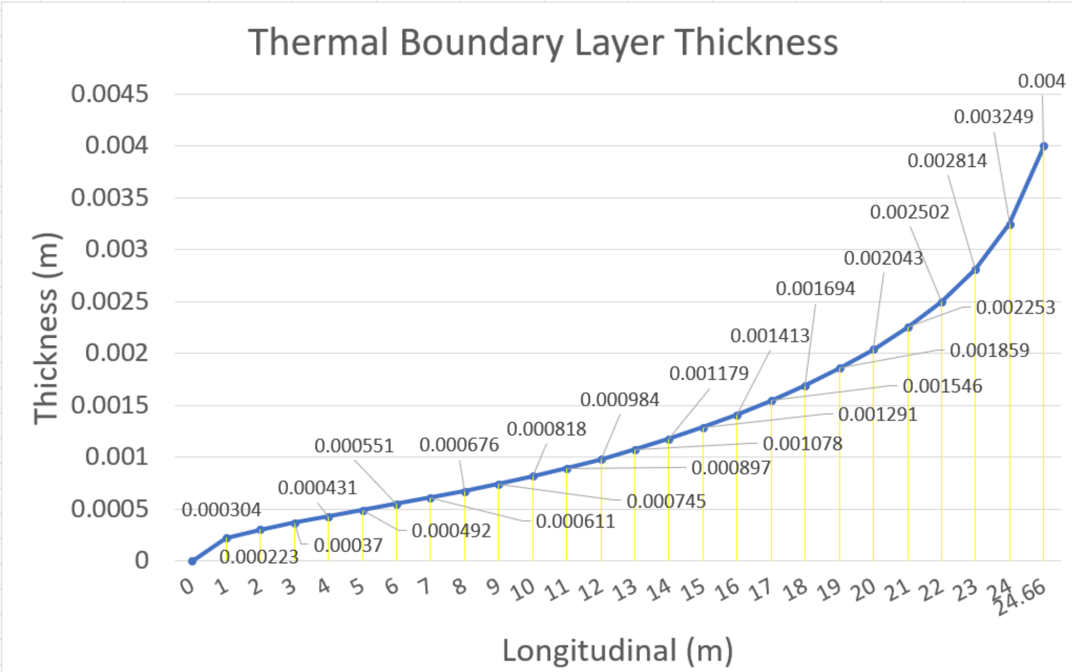

Figure 5b. Thermal boundary layer thickness versus longitudinal position for a inlet temperature of 297K

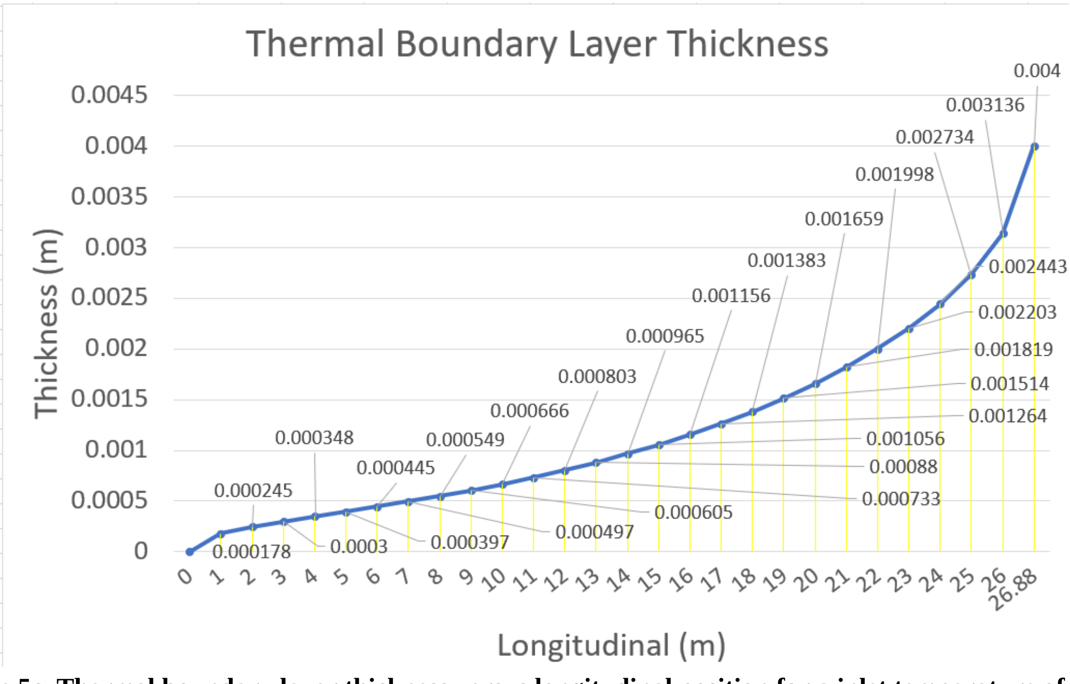

Figure 5c. Thermal boundary layer thickness versus longitudinal position for a inlet temperature of 298K

4. CONCLUSIONS

It is concluded with this piece of work that the model presented by GITT to characterize the laminar thermal boundary layer in round ducts provides a measuring possibility of the thermal boundary layer thickness at any place of a round tube with Newtonian fluid flowing laminarly. The development of the temperature field for a flow rate of 8.3 X 10^{-6} m^3/s and 16.67 X 10^{-6} m^3/s is verified with details, with fluid temperatures varying between 296K and 298K in the inlet end.

It is possible to note that there is a direct linear variation in the temperature field when the temperature in the tube wall is maintained constant and the temperature in the inlet end is between 296K and 298K, however, the thermal boundary layer thickness does not display direct linear variation, turning not possible to represent its values based on an equation which represents all models.

5. ACKNOWLEDGMENTS

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