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Stochastic model for multiphased flow induced pipe vibrations

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Abstract: This work is interested in investigating uncertainty propagation in the coupled fluid-structure dynamics of a pipe conveying fluid. Specifically, a two phased flow is considered, and parametric uncertainties are taken into account using a probabilistic approach. The Euler-Bernoulli beam is employed to model the pipe structure, with simply supported boundary conditions. The inner fluids are modeled using the plug flow approximation, and the extended Hamilton's principle is applied to obtain the partial differential equation of the system under analysis. The system is discretized by finite element method, and the Runge-Kutta scheme is used to integrate the equations. Monte Carlo simulations are obtained to approximate the statistics of the variables of interest. The uncertain parameters considered in the work are: the value of the flow velocity and the fluid volumetric fraction. Fluid velocity acts in the pipe like a compression load reducing system stiffness, which might cause fatigue failure, and buckling.

Keywords: pipe vibrations, two phased flow, nonlinear beam, stochastic model

INTRODUCTION

In oil and gas facilities process piping commonly have a multiphased flow composed by petroleum and natural gas. Normally in the industry, the piping plays a fundamental role connecting equipment and conveying fluids. According to Telles (2001), the costs involving the piping are significant, corresponding to approximately 45% of total assembly costs, 20% of total processing unit design and 25% of total industrial installation. Studying piping vibration is important to avoid problems like fatigue cracking, leakage, damage on the supports and equipment nozzles. It is also important to provide operational security and operational continuity of the facilities.

Uncertainties in the modeling should be taken into account, such that robust predictions can be performed. Ritto et al. (2014) proposed a probabilistic model for the fluid-structure interaction considering modeling errors, and analyzed the stability and reliability of the stochastic system.

This work proposes a stochastic model to propagate uncertainties through a pipe conveying an internally two phased flow. The uncertain parameters are the value of the flow velocity and the fluid volumetric fraction. Finally, the likelihood of failure, proposed by Energy (1999) guideline, is analyzed.

DETERMINISTIC AND STOCHASTIC MODELS

A pipe conveying an internally two phased flow is modeled as done by Monette and Pettigrew (2004). A simply supported beam represents the pipe, which is modeled with the Euler-Bernoulli beam assumptions. The internal flow is modeled considering a plug flow model Paidoussis (1998). The equation of the system is obtained applying the extended Hamilton's principle Meirovitch (1986), as shown in Equation 1.

$$\int_{t_1}^{t_2} (\delta \mathscr{T}_{f1} + \delta \mathscr{T}_{f2} + \delta \mathscr{T}_p - \delta \mathscr{V}_p - \delta \mathscr{V}_g + \overline{\delta W}_{NC}) dt = 0 \,, \tag{1}$$

where:"$\mathscr{T}_{f1}$" and "$\mathscr{T}_{f2}$" are the kinetic energies for the first and second fluids, "$\mathscr{T}_p$" is the kinetic energy for structure, "$\mathscr{V}_p$" is the elastic potential energy of the pipe,"$\mathscr{V}_g$" is the gravitational potential energy, and "W_{NC}" is the work of nonconservative forces.

The linear Euler-Bernoulli beam model has the potential elastic energy function considering given by Equation 2.

$$\delta \mathscr{V}_p = EI \int_0^L u_2'' \delta u_2'' dx \,. \tag{2}$$

The nonlinear extensible Euler-Bernoulli beam, according Paidoussis (1998) yields the variational of potential elastic energy as shown in Equation 3.

$$\delta\mathscr{V}_p = \int_0^L \{-EA(u_1'' + u_2'u_2'') - EI(u_2'u_2'''' + u_2''u_2''')\}\delta u_1 dx + \int_0^L \{-EA(u_1''u_2' + u_1'u_2'' + \frac{3}{2}u_2'^2u_2'') + \\ +EI(u_2'''' - 3u_1'''u_2'' - 4u_1''u_2''' - 2u_1'u_2'''' - u_1''''u_2' - 2u_2'^2u_2'''' - 8u_2'u_2''u_2''' - 2u_2''^3)\}\delta u_2 dx. \tag{3}$$

And the fluid kinetic energy approximation is given by Equation 4.

$$\mathscr{T}_f = \frac{\alpha m_{f1}}{2}\int_0^L \{(\dot{u}_1 + \mathbf{V}_{f1}(1+u_1'))^2 + (\dot{u}_2 + \mathbf{V}_{f1}u_2')^2\}dx + \frac{(1-\alpha)m_{f2}}{2}\int_0^L \{(\dot{u}_1 + \mathbf{V}_{f2}(1+u_1'))^2 + (\dot{u}_2 + \mathbf{V}_{f2}u_2')^2\}dx, \tag{4}$$

where V_{f1} and V_{f2} are the fluids speeds, m_{f1} and m_{f2} are fluids mass multiplied by pipe cross section area, α is the volumetric fraction of fluid one, and L is the pipe length. The over dot represent the time differential and the prime is the space differential.

The system is discretized by means of the finite element method. Considering a pipe element with two nodes (node A and node B), where x_A and x_B are the horizontal nodal displacements, y_A and y_B are the vertical nodal displacements, and θ_A and θ_B are the rotational angles for nodes "A" and "B". Equation 5 shows the polynomial approximation for a pipe element with horizontal u_1 and vertical u_2 displacements.

$$u_1^{(e)} = \begin{bmatrix} 1 - \frac{x}{L} \\ \frac{x}{L} \end{bmatrix}^T \begin{Bmatrix} x_A \\ x_B \end{Bmatrix}; \quad u_2^{(e)} = \begin{bmatrix} \frac{2x^3}{L^3} - \frac{3x^2}{L^2} + 1 \\ \frac{x^3}{L^2} - \frac{2x^2}{L} + x \\ -\frac{2x^3}{L^3} + \frac{3x^2}{L^2} \\ \frac{x^3}{L^2} - \frac{2x^2}{L} \end{bmatrix}^T \begin{Bmatrix} y_A \\ \theta_A \\ y_B \\ \theta_B \end{Bmatrix} \tag{5}$$

After assembling the finite element matrices and obtaining the system mass $[M]$, damping $[C]$, and stiffness $[K]$ matrices, the space state representation is constructed according Equation 6. Where space state variables are represented as: $\{z\} = \{u,\ \dot{u}\}$.

$$\{\dot{z}\} = \begin{bmatrix} \mathbb{0} & \mathbb{1} \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix}\{z\} + \begin{bmatrix} \mathbb{0} \\ [M]^{-1}\{F\} \end{bmatrix} = [A]\{z\} + [B]. \tag{6}$$

Now it is possible to compute the natural frequencies and mode shapes of the deterministic system, and also apply the Runge-Kutta integration scheme to analyze the response in the time domain.

Two variables are chosen as uncertain: the flow velocity V_f, and the volumetric fraction α. These are two important parameters, and we want to investigate how they impact the response of the system. The maximum entropy principle, Jaynes (2003), is applied to construct the probability density function for the two random variables. Since no information other than the support of the distribution is given, only Uniform random variables are considered in the analyses.

The stochastic system can be written using random variables according Equation 7. Where $\{\dot{Z}\}$ is the random response because $[\mathbf{A}]$ and $[\mathbf{B}]$ are random. As explained in a previous paragraph, the uncertainties come from two parameters, V_f and α. The statistics of the stochastic response is approximated using the Monte Carlo method.

$$\{\dot{Z}\} = [\mathbf{A}]\{Z\} + [\mathbf{B}], \tag{7}$$

NUMERICAL RESULTS

In this section some results are discussed. The parameters used in the simulations are shown in Table 1. Both random fluid volumetric fraction and random fluid velocities are taken into account. The x-axis of the next graphics shows the mean value of the fluid velocities, where $V_{f1} = V_{f2}$.

According to guideline Energy (1999), the Likelihood of Failure (LOF) parameter is calculated according Equation 8. Where ρ is the fluid density, V_f is the fluid velocity and F_v is the flow induced vibration factor and it depends on the pipe flexibility.

$$LOF = \frac{\rho\, V_f^2}{F_v}, \tag{8}$$

Table 1 – Pipe and fluids parameters values.

Parameter	Dimension
Pipe: Length / External Diameter / Thickness	15 m / 0.36 m (NPS14) / 0.0095 m (SCH40)
Pipe Density	7600 kg/m^3
Pipe Second Moments of Area	$1.5515 \cdot 10^{-4} m^4$
Pipe Young's Module	$210 \cdot 10^9$Pa
1st Fluid Density / 2nd Fluid Density	$650 kg/m^3$ / 1.5 kg/m^3
1st Fluid Speed and 2nd Fluid Speed	$UNIFORM \sim (3;10)\ m/s$
Volumetric Fraction "α"	$UNIFORM \sim (0.5;0.7)$

Figure 1 expresses the reduction of the first natural frequency while the mean flow speed increases. The mean value, the upper limit and lower limit for 95% of 200 samples are shown. The statistical envelope increases as the mean flow speed increases. When the mean flow speed is close to $200 m/s$, the first structural bifurcation, inducing to a compressive buckling failure, is observed. Considering the uncertainties, this value drops to $180 m/s$.

Figure 2 shows the pipe likelihood of failure (LOF), as proposed by Energy (1999). As expected, it increases as the mean flow speed increases. However the value of LOF is greater than 0.5 (critical value) for a mean flow speed of $1.8 m/s$. Considering the uncertainties, this value drops to $1.65 m/s$. The statistical envelope also increases its width as the mean flow speed increases. This limit value considering LOF is much lower than the critical buckling speed.

This is because LOF is a method to evaluate system natural frequency informing when it is lower enough to start a vibration issue. While the buckling fracture occurs when system stiffness does not work anymore leading to a natural frequency equal to zero.

Analyzes using Runge-Kutta time response and the nonlinear beam model are also used in this work. However, this content will be discussed in the full text.

CONCLUDING REMARKS

A stochastic model of a pipe conveying two-phased fluids was developed in this work. The system equation was obtained by Hamilton's principle and discretized by means of the finite element method. Pitchfork bifurcation (buckling) is observed when the flow speeds increases. The sensitivity of the system response to uncertainties in the flow velocities and in the fluid volumetric fraction parameters are computed.

It was verified that the pipe buckling happens when the mean flow velocities are over to $200 m/s$ while the LOF limit proposed by Energy (1999) happens for mean flow velocities close to $2 m/s$. Then it can be concluded that LOF quantitative assessment proposed by Energy (1999) is more conservative than the strategy of computing the first natural frequency of the system.

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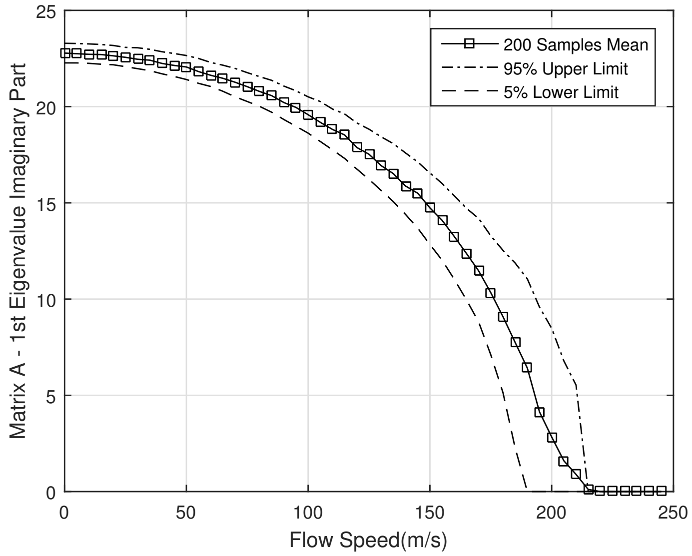

Figure 1 – Imaginary Part of Matrix A first Eigenvalue

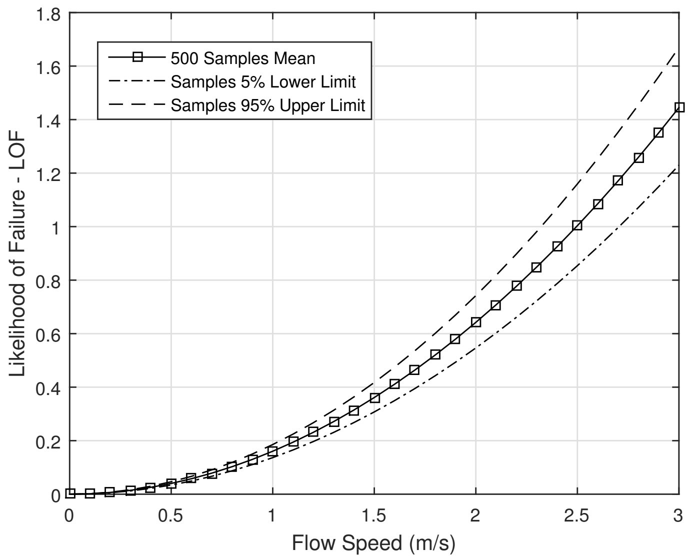

Figure 2 – Pipe Likelihood of Failure (LOF).