17th Brazilian Congress of Thermal Sciences and Engineering
November 25th-28th, 2018, Águas de Lindóia, SP, Brazil

ENCIT2018-0066
COMPRESSIBILITY EFFECTS ON THE CENTRIFUGAL INSTABILITY OF LAMINAR SEPARATION BUBBLES

Elmer Mateus Gennaro
Bruna Dias Pires de Souza
São Paulo State University (UNESP), Campus of São João da Boa Vista, Brazil
elmer.gennaro@unesp.br

Daniel Rodríguez
Graduate Program in Mechanical Engineering (PGMEC), Universidade Federal Fluminense, Brazil
danielrodriguez@id.uff.br

Abstract. *Separation bubbles have an intrinsic instability mechanism, which results in the appearance of spanwise-periodic three-dimensional structures. In incompressible flow, the instability mechanism responsible for three-dimensionality was found to become active under certain conditions and require of continuous external excitation to dominate the physics. We investigate if the compressibility could alter the qualitative picture. The model configuration for laminar separation bubble was a flat-plate boundary layer subjected to an adverse pressure gradient. The bubble considered is of the kind that occurs in the leading edge of a thin airfoil when it is at a high angle of attack. Direct numerical simulations were carried out to obtain two-dimensional base flows, and we achieved steady two-dimensional flows with peak reversed up to* 16%. *We find that the instability mechanism responsible for three-dimensionality has also become active in the compressible flow.*

Keywords: *Laminar separation bubble, Compressible flow, Flow instability*

1. INTRODUCTION

The instability and transition characteristics of laminar separation bubbles have been addressed by many experimental and numerical works considering the simple set-up of a flat-plate boundary layer subjected to an external flow deceleration, equivalent to an adverse pressure gradient Rist and Maucher (1994, 2002); Diwan and Ramesh (2012); Marxen *et al.* (2013); Embacher and Fasel (2014). Environmental disturbances are presented naturally in wind tunnel experiments, while controlled disturbances are commonly introduced at the inlet of the computational domain, that feed on the separated-flow instability trigger the laminar-turbulent transition process. Thus, the details of the transition process depend completely on the kind of external perturbations that are imposed upstream of the separation. In fact, it has been demonstrated that many different instability mechanisms, linear and nonlinear, may play a role in the amplification of environment or externally imposed perturbations. However, this classic description of the transition process on laminar separation bubbles is based on what is called "the amplifier behavior of laminar separation bubbles" which assumes that external perturbations are present and consequently dominate the physics (Dallmann and Schewe, 1987; Gaster, 2004).

Much less work has been done considering the potential of laminar separation bubbles of presenting self-destabilizing mechanisms, that could initiate transition in scenarios in which external disturbances are very weak or even totally absent. These intrinsic or self-excited instability mechanisms acting on nominally two-dimensional and steady separation flow would be responsible for the break-down of the two main symmetries existing in the set-up, namely the invariance with respect to time and with respect to the spanwise direction.

Considering incompressible flow, Rodríguez *et al.* (2013) studied the competence of two self-excited instability mechanisms in order to ascertain which of the two possible mechanisms should be expected to dominate. The first mechanism corresponded to a global oscillator that turns the steady two-dimensional bubble into an unsteady one, ultimately leading to vortex shedding; the second one was a three-dimensional global eigenmode, associated to a centrifugal instability, that was conjectured by Dallmann (1988) and then confirmed by Rodríguez *et al.* (2013) which showed the dominance of the three-dimensional instability, concluding that, in the absence of external forcing, incompressible laminar separation bubbles should be expected to become three-dimensional and give rise to spanwise periodic flow topologies as described by Rodríguez and Theofilis (2010).

The implications of this discovery were addressed by Rodríguez and Gennaro (2015) and Gennaro *et al.* (2015). Provided that unforced bubbles in a perfectly two-dimensional and time-independent set-up become three-dimensional as a result of a self-excited global flow instability, secondary instabilities could appear resulting from the steady distortion of the separation bubble that give rise to self-sustained oscillations. This possibility was confirmed in Rodríguez and Gennaro (2015) for separation bubbles characterized by a reversed flow velocity lower than that expected for the global oscillator to appear as a primary instability, explaining the process through which an unforced bubble becomes unsteady and three-dimensional and initiates the laminar-turbulent transition.

The aim of this work is to extend the previous analyses to compressible subsonic separation bubbles, and to investigate if the compressibility could alter the qualitative picture. A flat-plate boundary layer subjected to an adverse pressure gradient was considered again as the model configuration for laminar separation bubble. A laminar separation bubble develops if a laminar boundary layer is subjected to a sufficiently strong adverse pressure gradient (Gaster and Grant, 197). The bubble considered here is of the kind that occurs in the leading edge of a thin airfoil when it is at high angle of attack. For this, direct numerical simulations were carried out to obtain two-dimensional base flows.

2. Methodology

In the present paper, the base flow depends solely on two of the three spatial directions, and the flow quantities are then decomposed according to

$$\mathbf{q}(x,y,z,t) = \bar{\mathbf{q}}(x,y) + \varepsilon\mathbf{q}'(x,y,z,t), \tag{1}$$

where the time-independent *base flow* is prescribed as

$$\bar{\mathbf{q}} = (\bar{u}, \bar{v}, \bar{w}, \bar{T}, \bar{p})^T. \tag{2}$$

In this case, the linear operators are homogeneous in the $z-$direction and modal disturbances take the form

$$\mathbf{q}'(x,y,z,t) \sim \hat{\mathbf{q}}(x,y)\mathrm{e}^{\mathrm{i}(\beta z - \omega t)}, \tag{3}$$

where β is a number related to a wavelength the frequency (or wavelength) $L_z = 2\pi/\beta$.

The introduction of β is equivalent to performing the Fourier transform of the equations in the $z-$direction. The linear disturbance equations of two-dimensional eigenfunction problem are obtained by substituting the decomposition (1) into the governing equations, taking $\varepsilon \ll 1$, linearizing about $\bar{\mathbf{q}}$ and neglecting terms at $O(\varepsilon^2)$. β is taken to be a real wavenumber parameter describing an eingenmode in the z-direction. The complex ω and the eigenfunctions $\hat{\mathbf{q}}$ are sought. The real part of the eigenvalue is related with the frequency of the global eigenmode while the imaginary part is the temporal growth/damping rate: $\omega_r > 0$ indicates exponential growth of the instability mode in the time, and $\omega_i < 0$ indicates decay of perturbation in time. Thus, we can be written as the complex non-symmetric generalized eigenvalue problem in general form

$$\mathcal{L}(\bar{\mathbf{q}}, \beta, Re, Ma)\hat{\mathbf{q}} = \omega\mathcal{R}\hat{\mathbf{q}}. \tag{4}$$

where the coefficients of the operators are described in Gennaro (2012).

The two-dimensional eigenmode problem is solved numerically using a high-order finite-difference scheme and the algorithm and numerical methods described in Gennaro *et al.* (2011, 2013a). The eigenvalue problem is solved using an in-house implementation of the shift-and-invert Arnoldi algorithm Rodríguez and Gennaro (2017). Matrix operators are formed and operated on in sparse format. The most demanding task in the Arnoldi algorithm, namely the LU decomposition of the matrix, is performed using the open-source library MUMPS Amestoy *et al.* (2001).

3. Base Flow Calculations

The bubbles were generated by two-dimensional direct numerical simulation of a boundary-layer flow on a flat plate. The inflow velocity condition is a uniform flow and the bubble is generated by imposing a suction-blowing profile in the upper boundary, analogous to that produced by the pressure gradient around the nose pressure peak in medium-thickness airfoils. At the wall, a no-slip boundary condition is imposed. An isothermal wall condition is used for the temperature and the density is calculated through the compatibility condition. A zero-vorticity gradient is imposed at the far-field boundary, which together with the suction-blowing profile imposed to the wall-normal velocity produces a deceleration of the free-stream velocity.

The simulations were performed using a structured algorithm developed by Gennaro *et al.* (2013b); Bergamo *et al.* (2015) for solving the compressible Navier-Stokes equations. The algorithm is implemented in Fortran language and parallelized with the MPI protocol using domain decomposition strategies. The compressible Navier-Stokes equations are spatially discretized using a compact finite differences scheme with spectral like resolution of 6th-order accuracy, based on Lele (1992). The temporal integration was performed by the Runge-Kutta's method of fourth order. To prevent aliasing-related problems the flow variables were filtered in the last pseudo time-step of the Runge-Kutta scheme by a 10th-order low pass filter Visbal and Gaitonde (2002).

In the present paper, the laminar separation bubbles were obtained after a computational convergence study of mesh and domain with numeric residual of the $\sim 10^{-8}$ order and only results from converged solutions are presented.

Table 1. The critical values.

Mach	β_c	Re_c	u_rev/U_∞
0.3	1.2013	1291	8.29%
0.4	1.3713	1693	7.74%
0.5	1.5273	2033	7.25%
0.7	1.7763	2343	6.10%

4. Results

The stability of the compressible laminar separation bubble flows described in section 3is studied in the scope of the global stability analysis described in section 2. The linear stability eigenspectra was found to be dominated by a discrete eigenmode for all the combinations of Reynolds and Mach number considered.

This eigenmode shares the same characteristics of the three-dimensional instability that dominates incompressible separation bubbles (Theofilis *et al.*, 2000; Rodríguez and Theofilis, 2010; Rodríguez *et al.*, 2013).

Both the range of unstable wavenumbers and the maximum amplification is found to increase with higher Reynolds numbers, which can be due to two main reasons. On one hand, the higher the Reynolds number, the larger the recirculation region and the peak reversed flow, resulting in stronger bubbles, more prone to instability. On the other hand, for centrifugal instabilities like the present one, viscosity has a stabilizing effect which is reduced with increasing values of the Reynolds number.

The neutral curve corresponding to the centrifugal instability eigenmode is shown in Fig.1, as a function of the spanwise wavenumber and Reynolds number. The eigenmode is unstable for a bounded range of β values, and attains the maximum amplification for a finite spanwise wavenumber. Qualitatively identical behavior is found for the incompressible laminar separation bubble (Rodríguez *et al.*, 2013). The critical parameters The value of the reversed flow for which the eigenmode becomes unstable, i.e., the critical peaks recirculation are shown in the table 1 and in all cases ara below $u_rev = 10\%$.

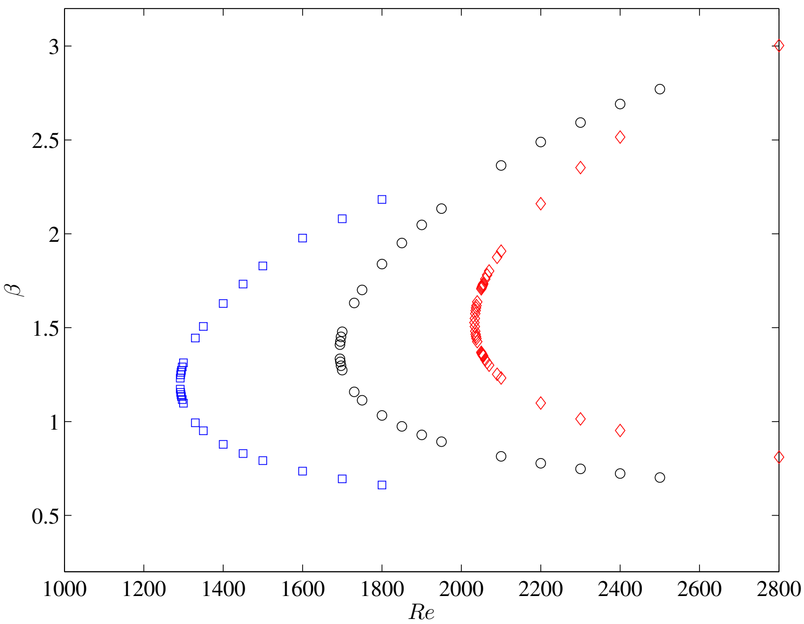

Figure 1. Neutral curve for the steady three-dimensional global mode, depending on the spanwise wavenumber β, the Reynolds number and Mach numbers 0.3 (squares), 0.4 (circles) and 0.5 (diamonds).

5. Conclusion

The combined influence of the Reynolds and Mach numbers on the linear instability of laminar separation bubbles in the absent of environmental or externally-introduced disturbances is studied here, thus extending the work done in Rodríguez *et al.* (2013) to compressible flow. A validated direct numerical simulation code is used to compute a family of model separation bubbles on a flat-plate boundary layer with different Reynolds and Mach numbers. Steady two-dimensional flows with peak reversed up to 16% were computed in this manner. A modal stability analysis was performed for these steady flows, which showed the dominance of a discrete three-dimensional eigenmode analogous to the one found for incompressible bubbles in previous researches.

6. ACKNOWLEDGEMENTS

The authors acknowledge funding from FAPESP (grants 15/26759-9, 17/01586-0), CNPq (grants 405144/2016-4, 305512/2016-1, 423846/2016-7) and FAPERJ (grants 200003, 223669).

7. REFERENCES

Amestoy, P.R., Duff, I.S., L'Excellent, J.Y. and Koster, J., 2001. "A fully asynchronous multifrontal solver using distributed dynamic scheduling". *SIAM J. Matrix Anal. Appl.*, Vol. 23, No. 1, pp. 15–41. ISSN 0895-4798.

Bergamo, L., Gennaro, E.M., Theofilis, V. and Medeiros, M., 2015. "Compressible modes in a square lid-driven cavity". *Aerospace Science and Technology*, Vol. 44, pp. 125 – 134.

Dallmann, U., 1988. "Three-dimensional vortex structures and vorticity topology". *Fluid Dynamics Resarch*, , No. 3, pp. 183–189.

Dallmann, U. and Schewe, G., 1987. "On topological changes of separating flow structures at transition reynolds numbers". AIAA, Honolulu, HI, number Paper 87-1266 in 16th Fluid Dynamics, Plasma Dynamics and Lasers Conference.

Diwan, S. and Ramesh, O., 2012. "Relevance of local parallel theory to the linear stability of laminar separation bubbles". *J. Fluid Mech.*, Vol. 698, pp. 468–478.

Embacher, M. and Fasel, H.F., 2014. "Direct numerical simulations of laminar separation bubbles: investigation of absolute instability and active flow control of transition to turbulence". *Journal of Fluid Mechanics*, Vol. 747, pp. 141–185.

Gaster, M., 2004. "Laminar separation bubbles". In R. Govindarajan, ed., *Sixth IUTAM Symposium on Laminar-Turbulent Transition*. Springer, pp. 1–13.

Gaster, M. and Grant, I., 197. "An experimental investigation of the formation and development of a wave packet in a laminar boundary layer". *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, Vol. 347, pp. 253–269.

Gennaro, E.M., 2012. *Análise da estabilidade global de escoamentos compressíveis*. Ph.D. thesis, Universidade de São Paulo.

Gennaro, E.M., Rodríguez, D. and Souza, L.F., 2015. "Secondary instability analysis of 3d laminar separation bubbles based on cross-plane wkb approximation". In *Proceedings of 23rd ABCM International Congress of Mechanical Engineering*.

Gennaro, E.M., Rodríguez, D., Theofilis, V. and Medeiros, M.A.F., 2013a. "Sparse techniques in global flow instability with application to compressible leading-edge flow". *AIAA Journal*, Vol. 51, No. 9, pp. 2295 – 2303.

Gennaro, E.M., Rodríguez, D.A., Theofilis, V. and Medeiros, M.A.F., 2011. "Linear instability of orthogonal compressible leading-edge boundary layer flow". *AIAA-paper*, , No. 2011-3751.

Gennaro, E.M., Colaciti, A.K. and Medeiros, M.A., 2013b. "On the controversy regarding the effect of flow shear on the strouhal number of cylinder vortex shedding". *Aerospace Science and Technology*, Vol. 29, No. 1, pp. 313 – 320. ISSN 1270-9638. doi:https://doi.org/10.1016/j.ast.2013.04.002.

Lele, S., 1992. "Compact finite difference schemes with spectral-like resolution". *Journal of Computational Physics*, Vol. 103, No. 1, pp. 16–42.

Marxen, O., Lang, M. and Rist, U., 2013. "Vortex formation and vortex breakup in laminar separation bubbles". *J. Fluid Mech.*, Vol. 728, pp. 58–90.

Rist, U. and Maucher, U., 1994. "Direct numerical simulation of 2–d and 3–d instability waves in a laminar separation bubble". In B. Cantwell, ed., *AGARD-CP-551 Application of Direct and Large Eddy Simulation to Transition and Turbulence*. pp. 34–1 – 34–7.

Rist, U. and Maucher, U., 2002. "Investigations of time-growing instabilities in laminar separation bubbles". *European Journal of Mechanics - B/Fluids*, Vol. 21, No. 5, pp. 495 – 509. ISSN 0997-7546.

Rodríguez, D. and Gennaro, E.M., 2015. "On the secondary instability of forced and unforced laminar separation bubbles". *Procedia IUTAM*, Vol. 14, pp. 78 – 87.

Rodríguez, D. and Gennaro, E.M., 2017. "Three-dimensional flow stability analysis based on the matrix-forming approach made affordable". In *Lecture Notes in Computational Science and Engineering*, Springer.

Rodríguez, D., Gennaro, E.M. and Juniper, M.P., 2013. "The two classes of primary modal instability in laminar separation bubbles". *J. Fluid Mech.*, Vol. 734, p. R4.

Rodríguez, D. and Theofilis, V., 2010. "Structural changes of laminar separation bubbles induced by global linear instability". *J. Fluid Mech.*, Vol. 655, pp. 280 – 305.

Theofilis, V., Hein, S. and Dallmann, U., 2000. "On the origins of unsteadiness and three-dimensionality in a laminar separation bubble". *Phil. Trans. R. Soc. Lond.*, Vol. 358, No. 1777, pp. 3229 – 3246.

Visbal, M.R. and Gaitonde, D.V., 2002. "On the Use of Higher-Order Finite-Difference Schemes on Curvilinear and

Deforming Meshes". *Journal of Computational Physics*, Vol. 181, No. 1, pp. 155–185.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.