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STUDY OF LAMINAR FORCED CONVECTION FOR DIFFERENT CONCENTRATIONS OF THE WATER-LITHIUM BROMIDE MIXTURE

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Abstract. *The present work aims to analyze the forced convection laminar of the water-lithium bromide mixture in the thermal entry region of a circular duct. Different concentrations of the water-lithium bromide mixture were used to verify the development of the thermal field. With the aid of the engineering equation solver (EES) it was possible to obtain the thermophysical properties of the studied fluid. The energy equation was solved by the generalized integral transformation technique (GITT), which combines the reliability and precision of the analytical techniques at a competitive computational cost, without neglecting the versatility of numerical methods. The results presented in the form of tables and graphs allow to analyze, for pre-defined mass flow rates, the influence that different concentrations of the fluid exert on the development of the temperature field.*

Keywords*: mixture water-lithium bromide, different concentrations, GITT, temperature field.*

1. INTRODUCTION

In the mid-1950s, the water-lithium bromide system (H_2O-LiBr) was created as a working liquid for industrial applications. Subsequently, other systems for the absorption of a single standard were created and launched as na industrial standard for high performance refrigeration systems (Srikhirin et al., 2001).

The literature has a wide range of studies on the water-drink of the lobe as a refrigerant and absorbent compound, for which it was submitted over two decades. Detergent, refrigerant-absorbent products that best meet the requirements in absorption refrigeration refrigerators, the water-lithium bromide blend are absolute in all requirements.

Filtration systems using a mixture of H_2O-LiBr are limited to an application where the temperature of the refrigerator is above the freezing temperature of water (273K), as well as the temperature of the mixture can not be less than 278K, not to occur the refrigerant freezing and compromising the system (Gordon and Ng, 2000).

The studies carried out by Herold, Radermacher and Klein (1996) in refrigeration systems by a mixture of H_2O-LiBr have become an outline of the system of these systems, when evaluated under the hypothesis that the processes operate a transient regime.

From the twenty-first century, the emergence of global multiple-stream systems in the different processes of utilization of refrigeration systems, which led to several studies, used a water-lithium bromide mixture (Varani, 2001., Younggy Shin et al., 2009, Cézar, 2012, Caldas et al., 2015).

Nowadays, with the new generations of computers, a problem solution, heat transfer by absorption communication systems, being used the numerical methods have been frequently analyzed. These techniques allow a resolution of the complete model of your text, including its multivariate feature with no time and no space. The present work aims at the transfer of heat by laminar forced convection of the water-lithium bromide mixture, using the formalism of the Generalized Integral Transformation (GITT) technique, where the obtained results were obtained through an analytical-numerical method. To Consider the analytical techniques at competitive computational cost, without neglecting the versatility of numerical methods.

The heat transfer in forced convection laminates in the inlet region of the circular duct grid has been extensively studied for various contour conditions (Brown, 1960; Shah 1975, Shih and Tsou 1978, Nagasue 1981, Gottifredi et al. 1983). Johnston, 1994; Lima et al., 2018). A very comprehensive series of literature can be found in the works of (Santos et al., 2001; Kakaç et al, 2014 and Shah and London, 2014).

The problem to be studied, in our work, it is a completely made space of the water-lithium bromide, in the region of thermal input of a circular tube that presents, concurrently, boundary conditions that are a symmetry of the profile and the constant temperature on the wall, as shown in Fig. 1.

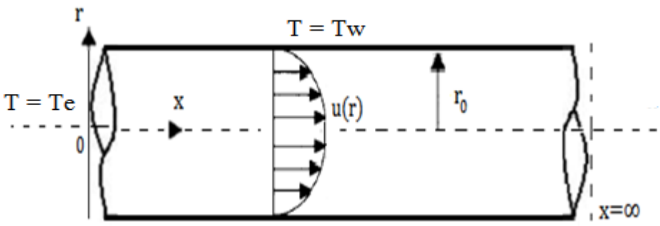

Figure 1. Illustration of the physical problem

2. MATHEMATICAL MODELING

For the mathematical modeling, the following considerations were adopted:

- Laminar flow, in steady state;
- The thermophysical properties of the fluid are considered constant;
- Incompressible fluid;
- The speed profile is fully developed at the thermal inlet;
- Impermeability and non-slip on walls;
- Neglecting the axial diffusion of the fluid;
- The effects of viscous dissipation not be considered.

Energy equation

$$\rho C_P u(r)\frac{\partial T(x,r)}{\partial x} = \kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T(x,r)}{\partial r}\right)\right]; \quad 0 < r < r_0 \; and \; x > 0 \tag{1}$$

Boundary conditions

$$\frac{\partial T(x,r)}{\partial r} = 0 \quad ; \quad r = 0 \; and \; x > 0 \tag{2}$$

$$T(x,r) = T_w \quad ; \quad r = r_0 \; and \; x > 0 \tag{3}$$

Inlet condition

$$T(x,r) = T_e \quad ; \quad 0 \le r \le r_0 \; and \; x = 0 \tag{4}$$

For the analysis of the problem were defined the following dimensionless parameters:

$$\xi = \frac{x}{r_0 \,\mathrm{Re}_h .\mathrm{Pr}} \quad ; \quad R = \frac{r}{r_0} \quad ; \quad U(R) = \frac{u(r)}{u_m} = 2\left(1 - R^2\right) \quad ; \quad Bi = \frac{h.r_0}{\kappa} \tag{5a-d}$$

$$\Theta(\xi,R)=\frac{T(x,r)-T_w}{T_e-T_w} \quad ; \quad \mathrm{Re}_h=\frac{D_h.u_m}{\upsilon} \quad ; \quad \mathrm{Pr}=\frac{\nu}{\alpha} \quad ; \quad P_e=\mathrm{Re}_h.\mathrm{Pr}=\frac{D_h u_m}{\alpha} \qquad \text{(5e-h)}$$

Applying the dimensionless parameters in equations (1), (2), (3) and (4), is found the energy equation, the boundary conditions and the inlet condition in the dimensionless form:

Dimensionless energy equation

$$U(R)\frac{\partial\Theta(\xi,R)}{\partial\xi}=\frac{2}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Theta(\xi,R)}{\partial R}\right); \quad 0<R<1 \ \ and \ \ \xi>0 \qquad (6)$$

Dimensionless boundary conditions

$$\frac{\partial\Theta(\xi,R)}{\partial R}=0 \quad ; \quad R=0 \ \ and \ \ \xi>0 \qquad (7)$$

$$\Theta(\xi,R)=0 \quad ; \quad R=1 \ \ and \ \ \xi>0 \qquad (8)$$

Dimensionless inlet condition

$$\Theta(\xi,R)=1 \ ; \quad 0\le R\le 1 \ \ and \ \ \xi=0 \qquad (9)$$

3. APPLICATION OF THE GENERALIZED INTEGRAL TRANSFORMING TECHNIQUE

3.1 Auxiliary problem of eigenvalue in the radial direction

The auxiliary problem, in the radial direction, for determining the temperature field is written as follows:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Psi_i(\mu_i,R)}{\partial R}\right)+\mu_i^2.U(R).\Psi_i(\mu_i,R)=0 \quad ; \qquad 0<R<1 \qquad (10)$$

$$\frac{\partial\Psi_i(\mu_i,R)}{\partial R}=0 \quad ; \quad R=0 \ \ and \ \ \mu_i>0 \qquad (11)$$

$$\Psi_i(\mu_i,R)=0 \quad ; \quad R=1 \ \ and \ \ \mu_i>0 \qquad (12)$$

The sign-count method developed by (Mikhailov e Vulchanov, 1983) is used to determine the eigenvalues (μ_i), the eigenfunctions, $\Psi_i(\mu_i,R)$, and the norms (N_i).

3.2 Integral transformation of the temperature field

The pair transformed integral, defined for this problem is given by:

$$\overline{\Theta}_i(\xi)=\frac{1}{N_i^{1/2}}\int_0^1 RU(R)\Psi_i(\mu_i,R)\Theta(\xi,R)dR, \qquad Transform \qquad (13)$$

$$\Theta(\xi,R)=\sum_{i=1}^{\infty}\frac{\Psi_i(\mu_i,R)\overline{\Theta}_i(\xi)}{N_i^{1/2}} \qquad \textit{Inverse} \tag{14}$$

Applying integral operators in equation (6), with the aid of the auxiliary problem and the transformed-inverse pair, it is possible to transform this partial differential equation into a system of ordinary differential equations given by:

$$\frac{1}{2}\frac{d\overline{\Theta}_i(\xi)}{d\xi}=-\mu_i^2.\overline{\Theta}_i(\xi) \tag{15}$$

This system has a classical analytical solution, given by:

$$\overline{\Theta}_i(\xi)=\overline{\Theta}_i(0).e^{-2\mu_i^2\xi} \tag{16}$$

Where:

$$\overline{\Theta}_i(0)=\frac{1}{N_i^{1/2}}\int_0^1 RU(R).\Psi_i(\mu_i,R)dR=\bar{f}_i \tag{17}$$

3.3 Temperature field solution

Using the inverse formula it is possible find the general solution of the temperature field for the proposed physical problem. The temperature field for the thermal input region takes the form:

$$\Theta(\xi,R)=\sum_{i=1}^{\infty}\frac{\Psi_i(\mu_i,R)\bar{f}_i.e^{-2\mu_i^2\xi}}{N_i^{1/2}} \tag{18}$$

4. RESULTS

The mathematical model was implemented using computer code written in FORTRAN programming language to obtain the results. For the analysis of the convergence of results, 300 eigenvalues and 300 corresponding eigenfunctions were used in the auxiliary problem. For the analysis performed in the present work is considered a tube with an internal radius of $4x10^{-3}$ m and that the water-lithium bromide mixtures have an initial temperature of 333 K and are cooled to 303 K.

The table 1 contains the values of the thermophysical properties of the water-lithium bromide mixture provided by the software (EES). In the temperature range evaluated in the present work, from 303 K to 333K, a small variation in the values of the parameters was noticed, so that are considered them constant from an average value.

Table 1. Mean values of thermophysical properties for different fractions of water-lithium bromide in the temperature range 303 K to 333K.

Lithium Bromide Concentration [%]	k[W/m.K]	ρ[kg/m^3]	Cp[10^3 J/kg.K]	α[10^{-4} m^2/s]
50	0.4663	1524.0	2.172	1.40846
52	0.4584	1557.1	2.117	1.39048
54	0.4502	1591.4	2.062	1.37195
56	0.4420	1627.3	2.007	1.35358
58	0.4336	1664.7	1.951	1.33503
60	0.4250	1703.6	1.893	1.31777

The graphs represented in topics 4.1 and 4.2 analyze the development of the thermal field for different concentrations of the water-lithium bromide mixture, for mass flow rates of 0.05 kg/s and 0.1 kg/s and volumetric flow rates of 4.10^{-5} m³/s and 4.10^{-5} m³/s. The evolution in the temperature field, for each concentration analyzed, can be observed through the development in the longitudinal axis. It can be seen that as the concentration of lithium bromide increases the thermal development length shows a small decrease. It can also be verified that, for a given concentration, the thermal development length increases proportionally with the mass flow rate and the volumetric flow rate.

4.1 Development of thermal field X mass flow

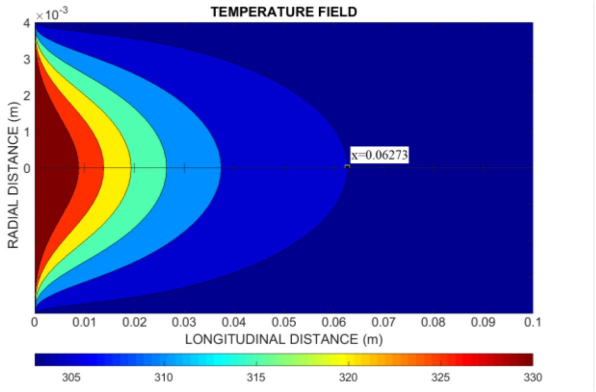

Figure 2. Evolution of the temperature field for the concentration of 50% lithium bromide and mass flow of 0.05Kg/s.

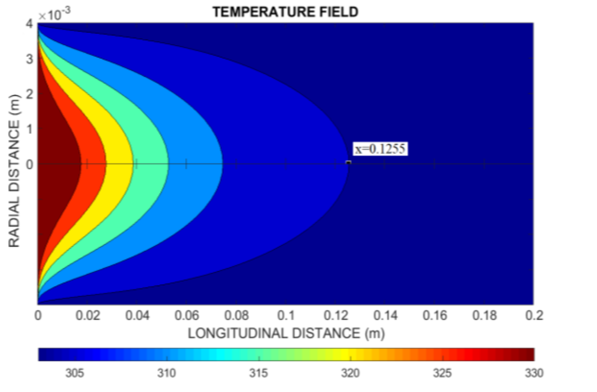

Figure 3. Evolution of the temperature field for the concentration of 50% lithium bromide and mass flow of 0.1Kg/s.

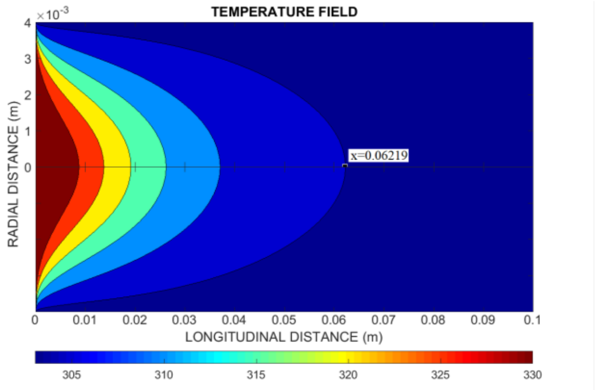

Figure 4. Evolution of the temperature field for the concentration of 52% lithium bromide and mass flow of 0.05Kg/s.

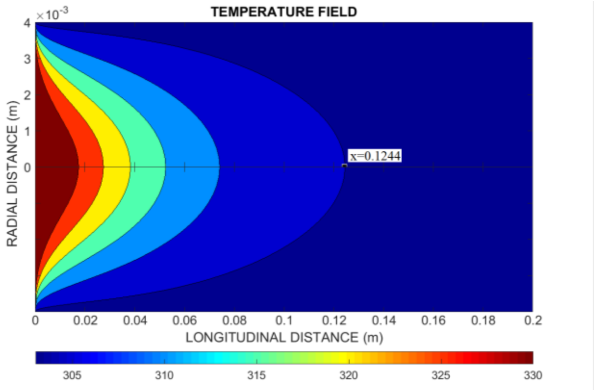

Figure 5. Evolution of the temperature field for the concentration of 52% lithium bromide and mass flow of 0.1Kg/s.

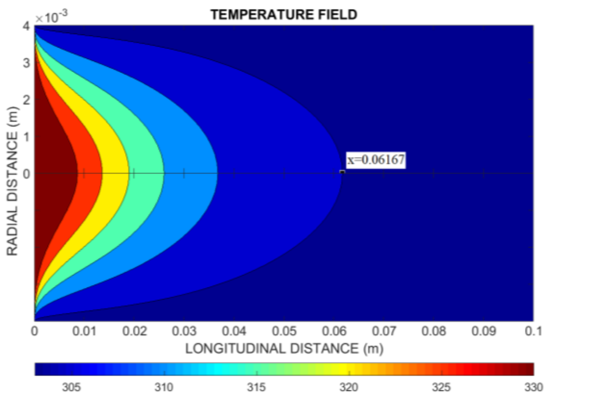

Figure 6. Evolution of the temperature field for the concentration of 54% lithium bromide and mass flow of 0.05Kg/s.

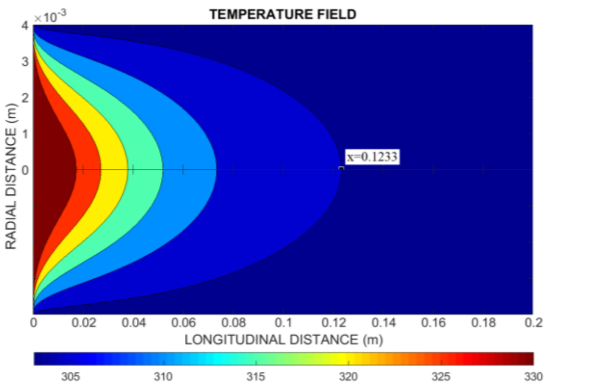

Figure 7. Evolution of the temperature field for the concentration of 54% lithium bromide and mass flow of 0.1Kg/s.

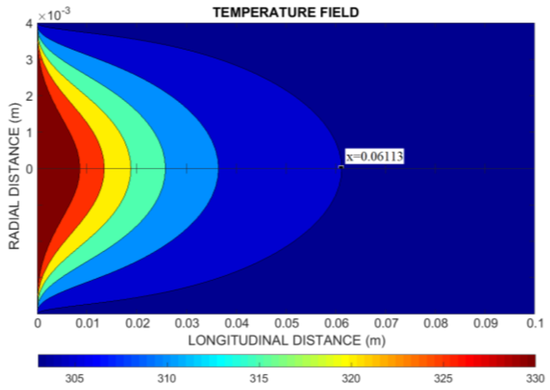

Figure 8. Evolution of the temperature field for the concentration of 56% lithium bromide and mass flow of 0.05Kg/s.

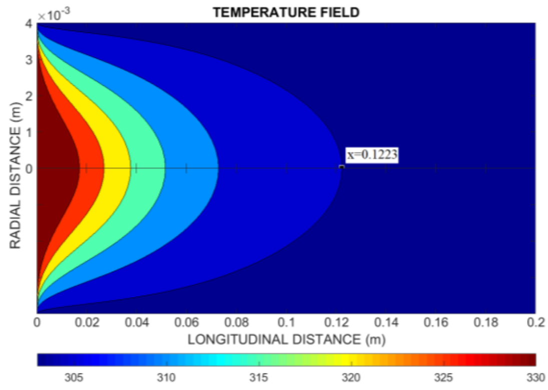

Figure 9. Evolution of the temperature field for the concentration of 56% lithium bromide and mass flow of 0.1Kg/s.

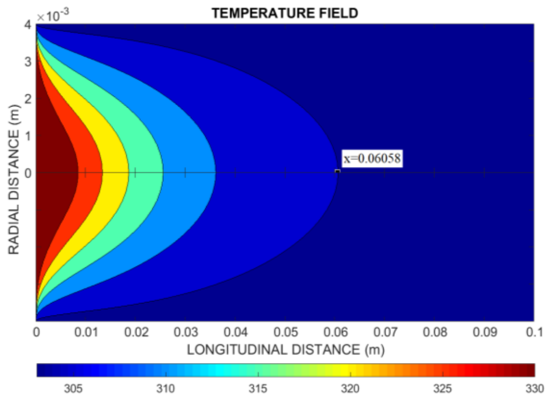

Figure 10. Evolution of the temperature field for the concentration of 58% lithium bromide and mass flow of 0.05Kg/s.

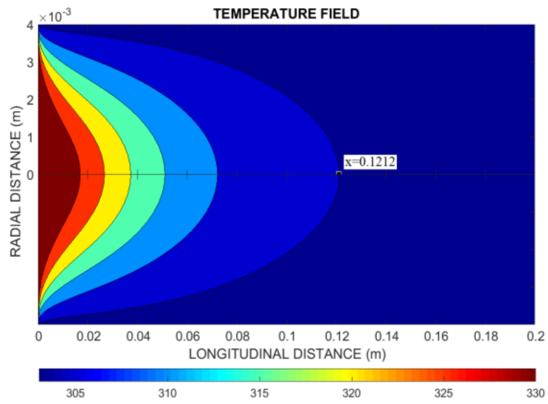

Figure 11. Evolution of the temperature field for the concentration of 58% lithium bromide and mass flow of 0.1Kg/s.

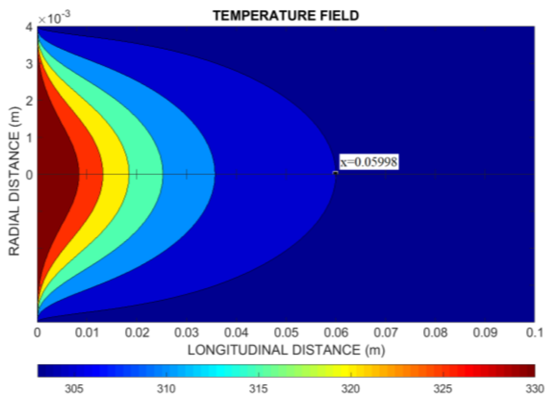

Figure 12. Evolution of the temperature field for the concentration of 60% lithium bromide and mass flow of 0.05Kg/s.

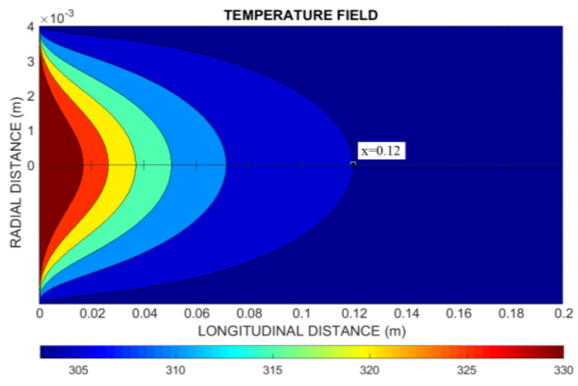

Figure 13. Evolution of the temperature field for the concentration of 60% lithium bromide and mass flow of 0.1Kg/s.

4.2 Development of the temperature field X volumetric flow

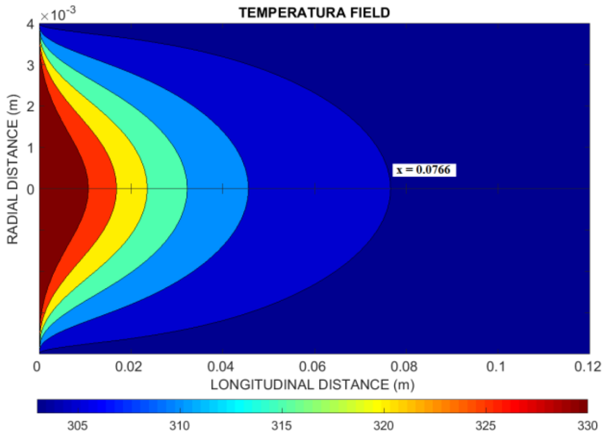

Figure 14. Evolution of the temperature field for the concentration of 50% of lithium bromide and volumetric flow of 4.10^{-5} m³/s.

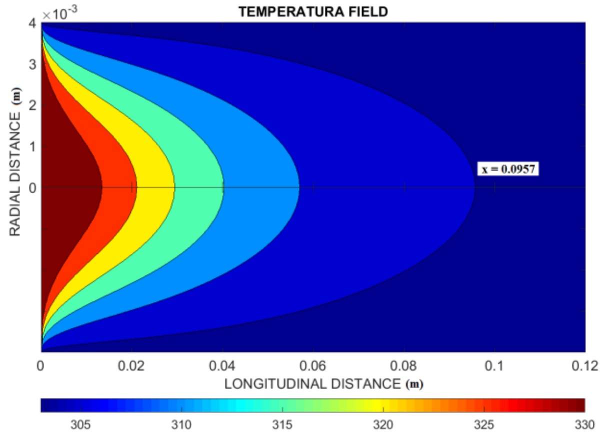

Figure 15. Evolution of the temperature field for the concentration of 50% of lithium bromide and volumetric flow of 5.10^{-5} m³/s.

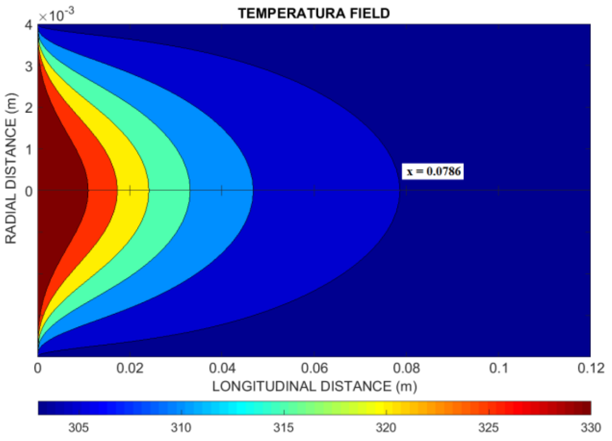

Figure 16. Evolution of the temperature field for the concentration of 54% of lithium bromide and volumetric flow of 4.10^{-5} m³/s.

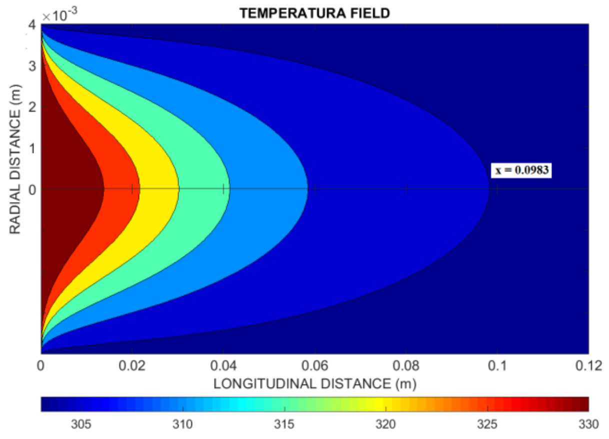

Figure 17. Evolution of the temperature field for the concentration of 54% of lithium bromide and volumetric flow of 5.10^{-5} m³/s.

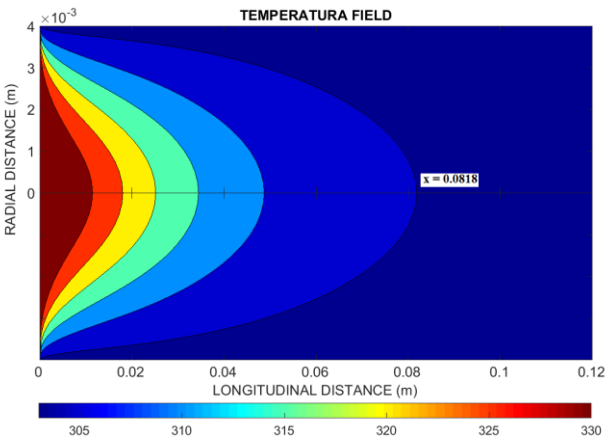

Figure 18. Evolution of the temperature field for the concentration of 60% of lithium bromide and volumetric flow of 4.10^{-5} m³/s.

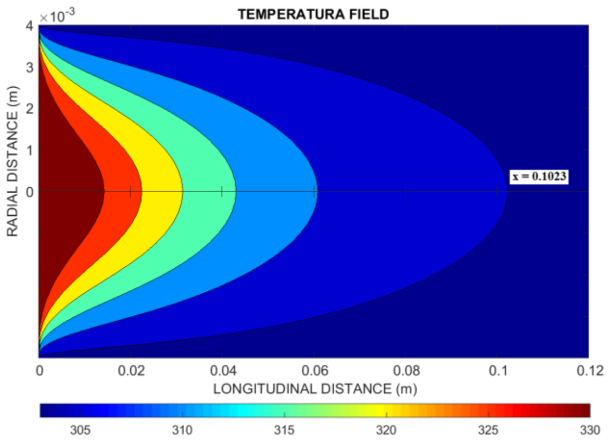

Figure 19. Evolution of the temperature field for the concentration of 60% of lithium bromide and volumetric flow of 5.10^{-5} m³/s.

5. CONCLUSIONS

The results obtained reiterate how effective and appropriate GITT is in the solution of heat and mass transfer problems, as can be found in (Mikhailov and Özisik, 1984, Cotta, 1993 and 1998). In this way, the objectives were satisfactorily achieved, showing the analysis of the laminar forced convection of the water-lithium bromide mixture and analyzing the influence of the concentrations, the mass and volumetric flow rates on the development of the thermal field. The thermal input length is shown to be faster established for higher concentration values and lower mass flow values.

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