

## ENDOREVERSIBLE THERMODYNAMIC OPTIMIZATION OF A RANKINE CYCLE FOR NUCLEAR PROPULSION

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**Abstract.** Besides the final efficiency, the most crucial factor of an energy conversion system for propulsion purposes is the total mass or size of the system. Considering this aspect, the present study aims the development of a design-based model that predicts the thermodynamic performance of a Rankine cycle applied in a Pressurized Water Reactor (PWR) nuclear plant. The endoreversible model considers the thermal conductance of the main components in order to predict the energy conversion performance, allowing its use as preliminary tool for the heat exchanger optimization. The centrifugal-flow turbine and pump characterization were achieved using isentropic efficiencies. Moreover, an optimization analysis based on the final area/size of heat exchangers is performed. Based on the proposed endoreversible model, efficiency or size optimization can be achieved with a proper reallocation of thermal conductances related to the steam generator and condenser.

**Keywords:** Optimization, Energy conversion, Nuclear Propulsion, Rankine Cycle.

### 1. INTRODUCTION

When compared to ground-based power systems, nuclear power systems must present a set of novel aspects, such as low power level and lightweight. For propulsion purposes, lighter power systems for the same power output promote more available mass for the payload. Furthermore, an adequate optimization of mass and size is considered a critical factor that can constrain the feasibility of nuclear power systems for naval and space propulsion (Ribeiro *et al.*, 2015). Therefore, special attention should be focused on these aspects, since they perform high influence on the viability of use of such power systems.

Most early endoreversible models for energy conversion and refrigerating systems apply simple modeling and commonly assume a pre-defined system capacity, system power input or heat exchanger thermal conductance (Bejan, 1989; Bejan, 1993). These studies demonstrated via analytical equations that power plants and refrigeration systems could share a common optimization principle, mapping thermodynamic irreversibilities of such thermal cycles.

Considering these aforementioned characteristics, this study proposes a methodology and a design-based thermodynamic model that can be used as a design tool for performance prediction and heat exchanger optimization of a Rankine cycle applied to a Pressurized Water Reactor (PWR) nuclear plant. A sensitivity analysis of the sizing of heat exchangers on the final efficiency will be discussed in detail. Furthermore, an optimization approach (Negrão and Hermes, 2011) is applied in this paper, allowing the match between nuclear power system size and energy conversion efficiency. The same optimization methodology was carried out in the work of Ribeiro *et al.* (2015), for a closed regenerative Brayton cycle used to space power systems. In this study, water was used as working fluid.

The Fig. 1 shows schematically a representation of a typical PWR plant. The primary cycle has the purpose of transferring the generated energy from the nuclear fission in the reactor core to a steam generator through a high-pressure coolant, and the secondary cycle receives that energy in the form of heat to generate superheated steam, later used by a turbine to generate electricity through a generator coupled to the craft.

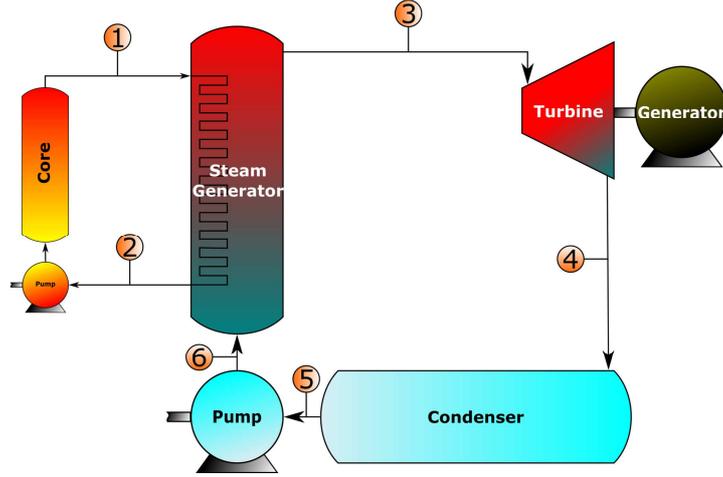


Figure 1. Schematic representation of a simple PWR.

## 2. THERMODYNAMIC MODELING

### 2.1 Governing Equations

For this endoreversible model, the heat exchanger thermal conductance  $UA$  is applied to estimate the boiling and condensing pressures of the PWR secondary circuit. Thus, the reactor core, steam generator and condenser were modeled based on the heat exchange effectiveness. By a simple energy balance along the heat exchanger (Holman, 2009), it is possible to associate the heat transfer rate extracted by each heat exchanger as follows

$$\dot{q}_{UA} = UA \cdot \Delta T_{ml} \quad (1)$$

where the  $\dot{q}$  is the heat transfer rate and the  $\Delta T_{ml}$  is the logarithmic mean temperature difference. The term  $\Delta T_{ml}$  is represented as

$$\Delta T_{ml} = \frac{(T_{outlet_a} - T_{inlet_b}) - (T_{inlet_a} - T_{outlet_b})}{\ln \frac{T_{outlet_a} - T_{inlet_b}}{T_{inlet_a} - T_{outlet_b}}} \quad (2)$$

where  $T_{inlet_a}$  and  $T_{outlet_a}$  are the inlet and outlet temperature of hot side fluid, whereas  $T_{inlet_b}$  and  $T_{outlet_b}$  are the inlet and outlet temperature of cold side fluid. For this study, it is assumed that the heat exchanger thermal conductance is function of heat transfer available area and consequently, heat exchanger volume and mass. Therefore, changes on the heat exchanger thermal conductance can be inferred as changes on the heat exchanger mass.

In order to overcome the complexities involved in the modeling of the working fluid charge, it was adopted a fixed super-heating value at the output of the steam generator (Point 3, in Fig. 1) and sub-cooling at the condenser outlet (Point 5). This procedure not only eliminates possible convergence problems, but also simplifies the mathematical modeling (Hermes *et al.*, 2009; Ribeiro and Barbosa, 2016). Thus, we can rewrite the  $T_3$  as function of super-heating level  $\Delta T_{sh}$  as follows

$$T_3 = T_{sat}(P_{sg}) + \Delta T_{sh} \quad (3)$$

and for sub-cooling:

$$T_5 = T_{sat}(P_{con}) + \Delta T_{sc} \quad (4)$$

where  $T_{sat}(P_{sg})$  and  $T_{sat}(P_{con})$  are the saturated temperature at the steam generator and condenser, respectively. The term  $\Delta T_{sh}$  is the super-heating temperature level and  $\Delta T_{sc}$  is the sub-cooling temperature level. Neglecting of the kinetic and potential energy, the energy balance considering the heat exchanger as the control volume yields

$$\dot{q}_f = \dot{m} \cdot (h_{outlet} - h_{inlet}) \quad (5)$$

where the  $h_{inlet}$  and  $h_{outlet}$  are the inlet and outlet enthalpy of fluid, respectively. The term  $\dot{m}$  denotes the mass flow rate of the secondary circuit. The total cycle efficiency is expressed as

$$\eta_{tot} = \frac{\dot{W}_{tur} - \dot{W}_{pum}}{\dot{q}_{core}} \quad (6)$$

where the  $\dot{W}_{tur}$  is the work generated by the turbine,  $\dot{W}_{pum}$  is the mechanical work necessary for the pump operation and  $\dot{q}_{core}$  denotes the heat supplied by the nuclear reactor core. The total heat exchange conductance (roughly, heat exchange mass) of the PWR secondary circuit is the sum of the steam generator and condenser thermal conductance, as follows

$$UA_{tot} = UA_{sg} + UA_{con} \quad (7)$$

The turbine modeling is based on the energy balance where it is assumed an adiabatic expansion, followed by an isentropic efficiency which characterizes internal thermodynamic irreversibilities (Moran and Shapiro, 2013). Thus, the ideal (isentropic) work  $\dot{W}_i$  and the actual turbine work  $\dot{W}$  are represented as follows

$$\dot{W}_i = \dot{m} \cdot (h_{outlet_i} - h_{inlet}) \quad (8)$$

and

$$\dot{W} = \dot{m} \cdot (h_{outlet} - h_{inlet}) \quad (9)$$

where the  $h_{inlet}$  and  $h_{outlet}$  are the fluid inlet and outlet enthalpy, respectively. The term  $h_{outlet_i}$  denotes the outlet enthalpy for an isentropic expansion. The turbine isentropic efficiency is characterized by the relationship between the actual work and the ideal (isentropic) work. Thus, the turbine efficiency is calculated as

$$\eta_{tur} = \frac{\dot{W}}{\dot{W}_i} = \frac{(h_{outlet} - h_{inlet})}{(h_{outlet_i} - h_{inlet})} \quad (10)$$

The work related to the pumping process is analogous turbine work, and due to this reason, its energy balance and the isentropic efficiency  $\eta_{pum}$  are similar to eq. (8) to (10). Considering the pump as a positive displacement device, the mass flow rate supplied by each pump is given as

$$\dot{m} = V \cdot \rho \cdot \omega \cdot \eta_v \quad (11)$$

where the  $V$  is volume,  $\rho$  is density,  $\omega$  is the speed and the  $\eta_v$  is the volumetric efficiency. The input data displayed in Table 1 were used as the baseline data for the present model. The heat exchangers thermal conductance and turbo machinery parameters were extracted from the model, as a manner to maintain 48MW of thermal power ( $\dot{q}_{core}$ ) and temperature and pressure values commonly found in PWR power plants.

Table 1. Input parameters

Components	Parameters	Values	Unit
Core	$T_{core}$	348.6	°C
	$P_{core}$	15396	kPa
	$UA_{core}$	1643.43	kW/°C
Pump 1	$\eta_{pum,1}$	70	%
	$V$	$17.273 \cdot 10^{-3}$	$m^3$
	$\omega$	1200	rpm
Pump 2	$\eta_{pum,2}$	70	%
	$\eta_v$	70	%
	$V$	$938 \cdot 10^{-6}$	$m^3$
	$\omega$	1800	rpm
Steam Generator	$UA_{sg}$	397.56	kW/°C
	$P_{sg}$	6296	kPa
	$\Delta T_{sh}$	1.22	°C
Turbine	$\eta_{tur}$	90	%

Condenser	$P_{con}$	58.9	kPa
.	$UA_{con}$	688.75	kW/°C
.	$\Delta T_{sc}$	16.91	°C

## 2.2 Numerical Procedure

The software EES was the chosen code used in this study. EES is a general equation-solving program that can numerically solve thousands of coupled non-linear algebraic and differential equations. The program can also be used to solve differential and integral equations, perform optimization techniques, provide measurement uncertainty analysis, and perform linear and non-linear regression. A major feature of EES is its complete and high-accuracy thermodynamic and transport properties database.

Fig. 2 displays the model flowchart implemented in EES. The parameters presented in Table 1 are defined as data inputs in b.1 (block 1), as well as guessed values of temperature and pressure along the energy conversion cycle. In b.3, the mass flow rate at the primary circuit is computed as function of the thermodynamic state at the pump inlet. In b.4, the nuclear core outlet temperature is obtained, then, in c.b.1 (conditional block), the energy balance of the nuclear core is verified, relating the core thermal conductance to the thermodynamic states at the core inlet and outlet. When a difference between  $\dot{q}_{UA}$  and  $\dot{q}_f$  is found, the thermodynamic state is updated and the procedure is repeated. The same procedure is applied to the steam generator and condenser, as shown in Fig. 2. When the energy balances  $\dot{q}_{UA}$  and  $\dot{q}_f$  of all heat exchangers are equal, it is assumed that thermodynamic state along the energy conversion cycle were achieved.

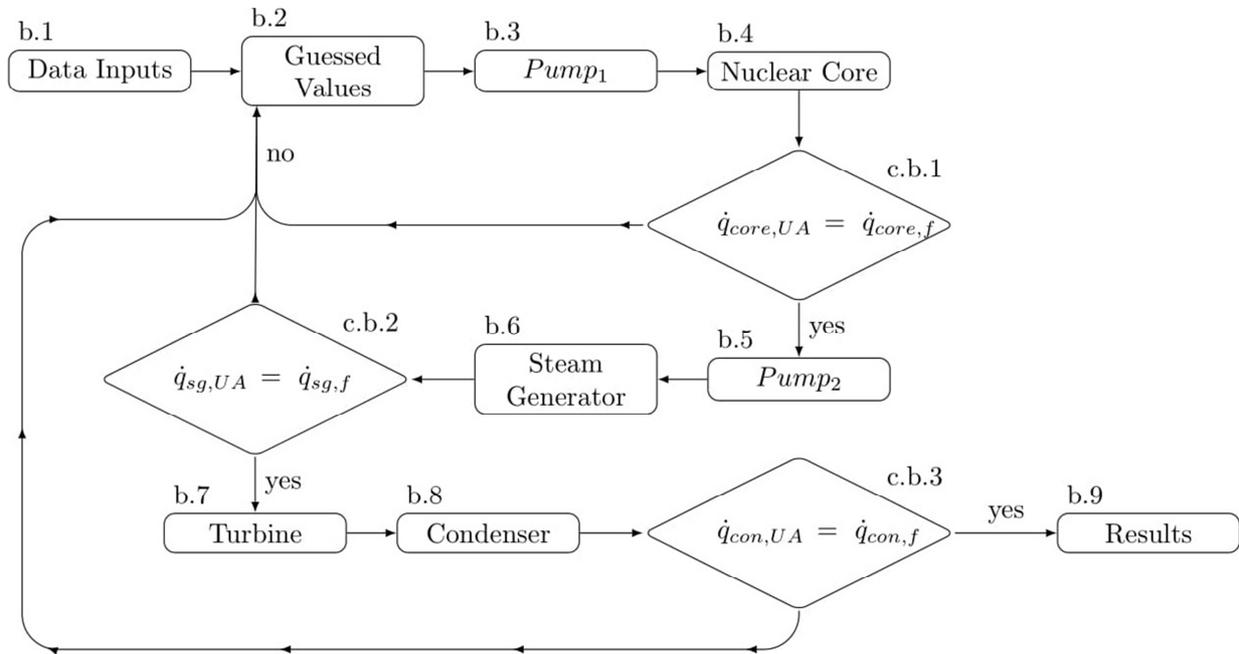


Figure 2. Flowchart of the endoreversible modeling.

## 3. RESULTS

Fig. 3 displays conversion cycle efficiency  $\eta_{tot}$  as a function of the overall thermal conductance of the core  $UA_{core}$ . As can be seen, the cycle efficiency  $\eta_{tot}$  increases with the increasing  $UA_{core}$ . Higher values of nuclear core thermal conductance promoted higher temperatures of the hot side of the secondary circuit. Therefore, the thermodynamic irreversibilities related to the finite difference of temperature are reduced, and due to this reason the cycle efficiency  $\eta_{tot}$  is increased when the core thermal conductance  $UA_{core}$  is increased.

Fig. 4 displays the conversion cycle efficiency  $\eta_{tot}$  as a function of the steam generator thermal conductance  $UA_{sg}$ . The optimization of cycle efficiency can be achieved varying the  $UA_{sg}$ , for a fixed total thermal conductance  $UA_{tot}$ . In this analysis, by varying  $UA_{sg}$  (and consequently  $UA_{con}$ ), a maximum cycle efficiency  $\eta_{tot}$  is obtained (marked in Fig. 4 as  $\times$ ). Therefore, for a fixed total heat exchange area (and as a result, mass), a maximum cycle efficiency is obtained with a proper thermal conductance distribution between the steam generator and the condenser. This result shows the importance of a conciliation between total heat exchanger mass and desired system efficiency during the design phase, for nuclear reactors for propulsion purposes.

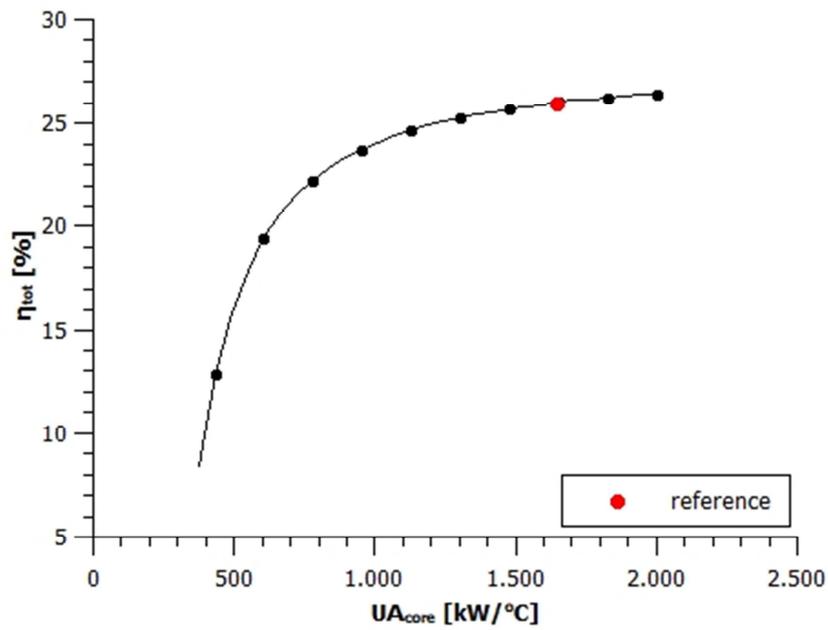


Figure 3. Power plant efficiency as a function of the nuclear core thermal conductance.

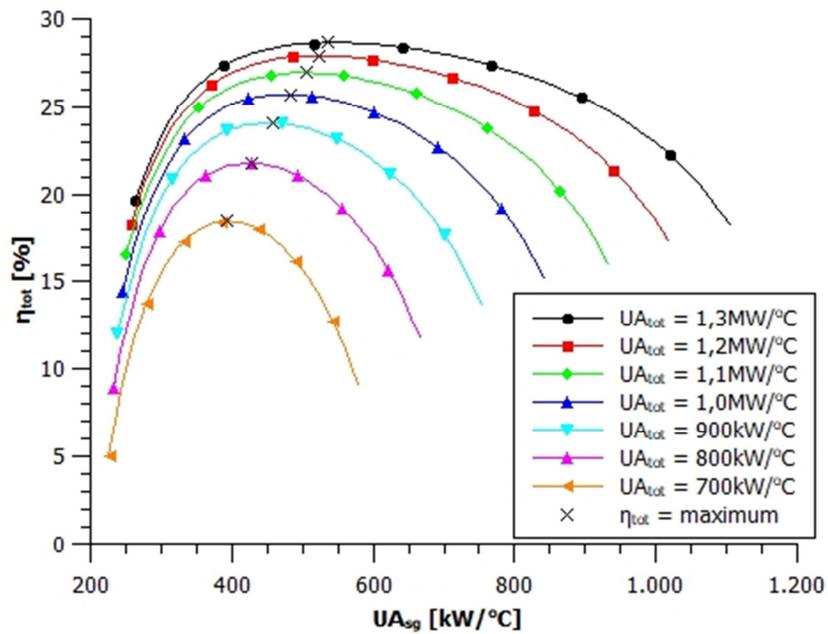


Figure 4. Power plant efficiency as a function of the steam generator thermal conductance, for a fixed total thermal conductance.

The influence of the  $UA_{sg}$  and  $UA_{con}$  and in the efficiency  $\eta_{tot}$ , for different number of secondary cycles is shown in Fig. 5 and 6, respectively. As displayed in the results, the influence of  $UA_{sg}$  and  $UA_{con}$  is greater in the  $\eta_{tot}$  for higher numbers of secondary cycles. As expected, a nuclear power plant with two secondary circuits, with a specific  $UA_{sg}$  or  $UA_{con}$  each, would yield approximately the same  $\eta_{tot}$  of a power plant with a single secondary circuit with a steam generator thermal conductance two times higher. However, as can be seen in Fig. 5 and 6, working with more than a single secondary cycle is not thermodynamically justifiable, due to a marginal increase of total cycle efficiency  $\eta_{tot}$ .

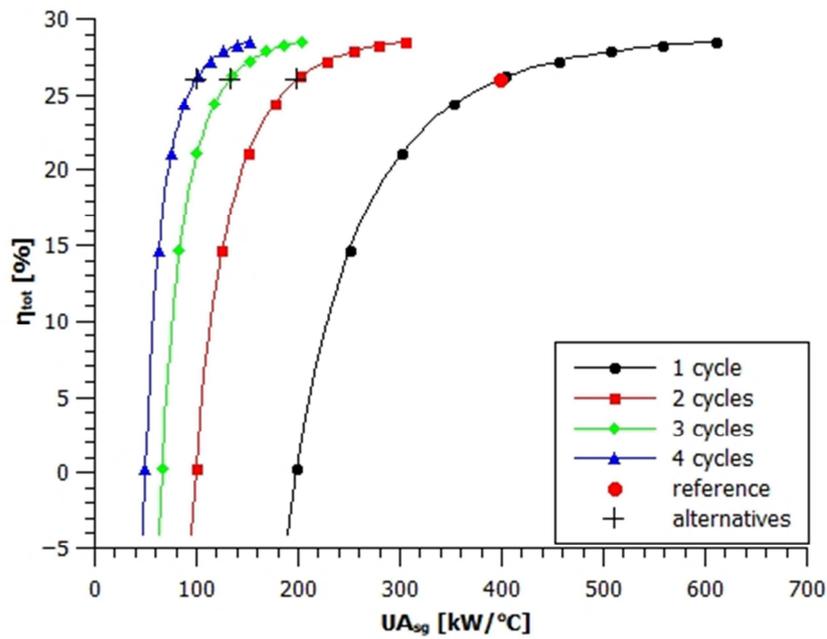


Figure 5. Power plant efficiency as a function of the steam generator thermal conductance, for different number of secondary cycles.

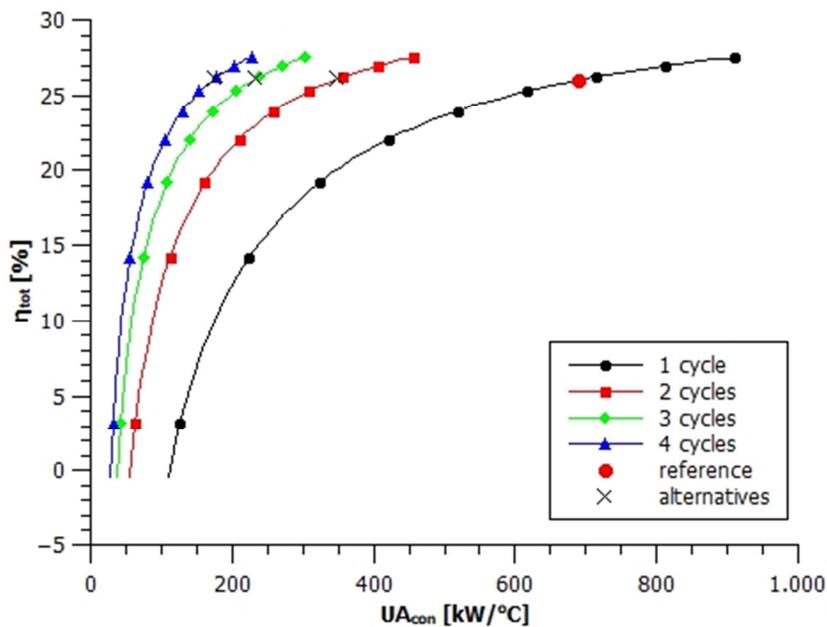


Figure 6. Power plant efficiency curve as a function of the condenser thermal conductance, for different number of secondary cycles.

Fig. 7 shows the influence of total cycle efficiency as a function of the turbine isentropic efficiency  $\eta_i$ . The red dot is the result related to the baseline case, where a turbine sized with a 90% of isentropic efficiency  $\eta_{tur}$  yields a total cycle efficiency of 26.02%. By keeping fixed the condenser and steam generator thermal conductances, we conclude that the cycle efficiency  $\eta_{tot}$  decreases almost linearly with the turbine isentropic efficiency  $\eta_{tur}$ . Therefore, this cycle component present a great influence on the final performance of this Rankine cycle.

Furthermore, the impact of the isentropic efficiency of the secondary pump  $\eta_{pum,2}$  on the cycle efficiency  $\eta_{tot}$  is displayed in Fig. 8. As can be seen, the influence of  $\eta_{pum,2}$  on  $\eta_{tot}$  is marginal, especially when compared to the turbine isentropic efficiency  $\eta_{tur}$ . This conclusion is easily supported by the literature, where the pumping work is often

neglected from the cycle efficiency computation. For a pump efficiency  $\eta_{pum,2}$  of only 16%, an efficiency loss of only 1% would be found, when the baseline case (red dot) is used as reference.

The maximum points of power system efficiency  $\eta_{tot}$  seen in Fig. 4 were plotted as a function of the total heat exchanger thermal conductance  $UA_{tot}$ , in Fig. 9. It can be noticed that possible power systems sizing are positioned below the optimized curve, whereas any point above the optimized curve is not feasible according to the applied size restrictions. Therefore, an energy conversion system for nuclear propulsion should be designed as a manner to place its representative point as close as possible of the optimization boundary.

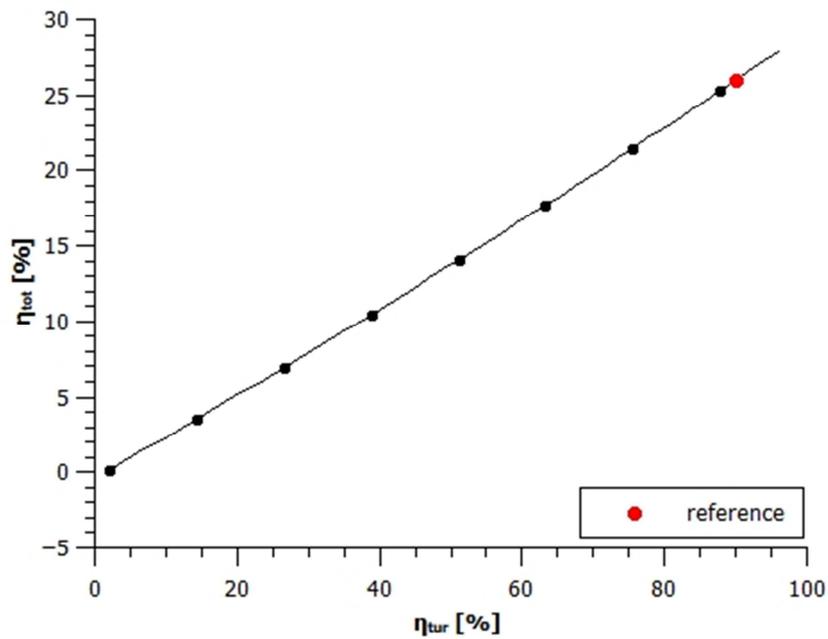


Figure 7. Power plant efficiency as a function of the turbine isentropic efficiency.

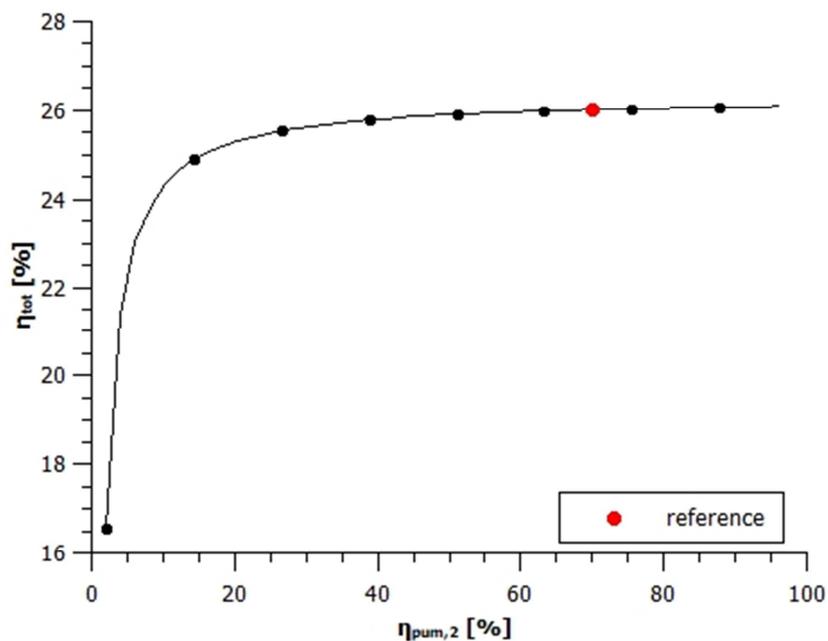


Figure 8. Power plant efficiency as a function of the secondary pump isentropic efficiency.

As an example, it is possible to keep the total cycle efficiency of 26.02% decreasing up to 6.6% of the total heat exchanger thermal conductance  $UA_{tot}$ , or we can maintain fixed the  $UA_{tot}$  and redistribute the  $UA_{sg}$  and the  $UA_{con}$  as a manner to perform efficiency optimization, which would imply a 1% efficiency increase.

The reference case considered in this study is shown in Fig. 9 as a red dot, and as expected, there is opportunity for a proper performance enhancement. As can be seen, a size optimization can be performed by changing the heat exchangers thermal conductance in order to achieve lower mass for the same power system efficiency. Likewise, heat exchanger thermal conductance can also be changed for efficiency optimization, where the best efficiency is obtained for a constant power system mass.

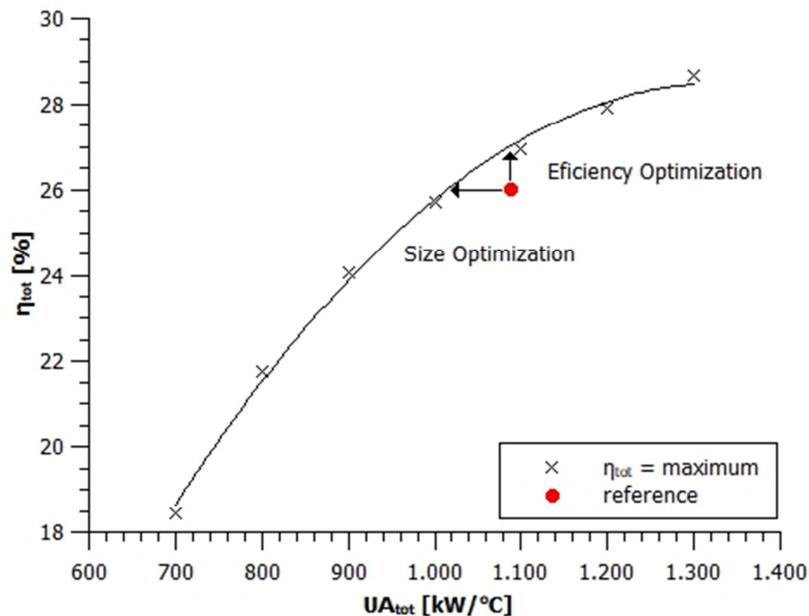


Figure 9. Performance curve as a function of the overall total secondary conductance.

#### 4. CONCLUSIONS

An endoreversible model was developed for the thermodynamic prediction of a Rankine cycle applied to a PWR nuclear power plant. The model considers overall thermal parameters for each component and is used during the project conception phase, since weight and size are key drivers that allow the use of nuclear energy for propulsion.

Finally, it has been concluded that for each total heat exchangers weight (represented by the total thermal conductance) there is maximum system efficiency. Thus, an efficiency optimization can be performed, changing each heat exchanger proportion on the final volume in order to accomplish the optimized efficiency. By the same token, the size optimization consists in achieve the lowest total heat exchanger volume keeping the same system efficiency, redistributing the volumes of the heat exchangers.

#### 5. ACKNOWLEDGEMENTS

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