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SPECTRAL AND FINITE DIFFERENCE SIMULATIONS FOR TEMPERATURE DISTRIBUTION IN A POROUS FIN

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Abstract. *This work analyzes the heat distribution in a rectangular profile porous fin accounting for the effects of radiation and convection heat transfer. Two different numerical methods for spatial discretization are compared, namely the Second Order Finite Difference and the Gauss-Lobatto Spectral Method. The analysis consists of comparing results obtained by these two methods. Results show that both methods show good agreement. Besides, the results concord with those in the literature. CPU times and convergence rates were compared, behaving as expected.*

Keywords: *porous fin, finite difference, spectral method*

1. INTRODUCTION

Fins are widely used to dissipate heat. They should be compact, have preferentially low cost and small weight and offer high heat transfer rates. One possibility to enhance these heat transfer rates is to employ porous materials of high thermal conductivity. Heat transfer in fins occurs by conduction, convection and radiation and the maximum fin efficiency may be attained by increasing the heat transfer rates.

Porous media present an increasing importance in technological advancements, giving rise to many important studies on the subject. For instance, Bejan and Nield (2006) highligh balance equations in various conditions and Kaviany (1991) focus on a widely used method for dealing with flows through porous media: the local volume-averaging methods. An alternative approach would be a mixture theory one, specifically developed to deal with multiphase phenomena. Examples are the packed-bed heat exchanger (Martins-Costa, 1996) or the flow in a channel with two distinct regions, a porous matrix region (mixture) and a pure fluid region, as it can be observed in Martins-Costa and Saldanha da Gama (1994).

Porous media are largely used to enhance heat and mass transfer rate, since, according to Lauriat and Ghafir (2000), Nusselt number in the presence of porous media can reach values circa 50% above the ones predicted for laminar flows in channels without porous materials. Also, the convective heat transfer coefficient increases when a porous matrix is included in a channel, since the porous matrix usually presents a much larger thermal conductivity than a fluid. On the other hand, the presence of a porous matrix significantly enhances the pressure drop. However, Siddique et al. (2010) mention that partially fillings of porous media could be used to minimize pressure drop.

This work considers the model used in the work of Gorla and Bakier (2011): a rectangular profile porous fin, though which the fluid flows. The momentum equation is simplified by Darcy's model and the energy equation assumes local thermal equilibrium, temperature variation only along the fin length and neglects surface radiant exchange. Three classical fins boundary conditions are considered: very long fin, insulated tip and convective tip. The heat distribution along a porous fin considering the effect of some important parameters are analyzed using two different numerical methods for spatial discretization: Second Order Finite Difference and Gauss-Lobatto Spectral Method and their performance is compared. The results show good agreement between both methods and with the results obtained by Gorla and Bakier (2011). So, this work may be seen an increment upon the work of Gorla and Bakier (2011).

2. METHODOLOGY

The rectangular porous fin with constant cross section is modeled as a homogeneous and isotropic porous matrix saturated by a fluid both with constant physical properties, so that the momentum equation is assumed to satisfy Darcy´s law. Supposing in local thermal equilibrium between fluid and porous medium, neglecting surface radiant exchange, and supposing temperature variation solely along the fin length, the scaled energy balance equation considering steady state condition may be expressed as (Gorla and Bakier, 2011):

$$\frac{\partial^2 \theta}{\partial X^2} - S_H \theta^2 - G[(\theta + C_T)^4 - C_T^4] = 0 \tag{1}$$

where θ represents the dimensionless temperature, X the dimensionless axial coordinate S_H is a porous parameter (accounting for permeability and buoyancy effects), G the radiation parameter (indicating the effect of the fin surface emissivity) and C_T is the temperature ratio (a dimensionless ratio of the ambient temperature to the difference of the base and ambient temperatures).

Applying a second order central finite difference method to Eq. (1) gives rise to:

$$\frac{\theta_{i-1} - 2\theta_i + \theta_{i+1}}{\Delta h^2} - S_H \theta^2 - G[(\theta + C_T)^4 - C_T^4] = 0 \tag{2}$$

in which i is the current position along the fin and h is the step size. Furthermore, as the resulting equation is non-linear, a convergence criterium must be established as well as an iterative relaxation method as follows, where n is the current iteration:

$$\theta_{i,n+1} = \frac{1}{3}\left[(\theta_{i-1,n} + \theta_{i,n} + \theta_{i+1,n}) - S_H \Delta h^2 \theta_n^2 - G\Delta h^2 (\theta_n + C_T)^4 + G\Delta h^2 C_T^4\right] \tag{3}$$

Spectral methods can be used methods for numerically solving many types of differential equations, since they may be easily applied to both finite and infinite domains. Other interesting features is their convergence speed (they present exponential convergence rates) and their high accuracy level. The Gauss-Lobatto-Chebyshev collocation point (when the collocation points are related to the structure of the classical orthogonal polynomials) are most commonly used in Chebyshev spectral methods, because this set of points also includes the boundary points (which makes it possible to easily incorporate the boundary conditions in the collocation approach). In this work, the spectral method used is based on a combination of Gauss-Lobatto-Chebyshev collocation point method along with the methodology explored in Ma *et al.* (2016). Therefore, the first procedure is to apply the Gauss-Lobatto-Chebyshev derivation to the dimensionless temperature distribution equation seen in Eq. (1):

$$\sum_{j=0}^{N} d_{ij}^2 \theta_j - S_H \theta_i^2 - G[(\theta_i + C_T)^4 - C_T^4] = 0 \tag{4}$$

where N is the total number of points and d_{ij} is an element of the Gauss-Lobatto-Chebyshev derivative matrix. Further, by applying an iterative relaxation method similar to the one previously used for the finite difference method Eq. (4) becomes:

$$\theta_{i,n+1} = \frac{\theta_{i,n}}{2} + \frac{S_H \theta_{i,n}^2 + G(\theta_{i,n} + C_T)^4 - G\,C_T^4 - \sum\limits_{j=0, j\neq i}^{N} d_{ij}^2 \theta_{j,n}}{2 d_{ii}^2} \tag{5}$$

Equation (5), however, needs to be remapped to the correct domain, this can be done by using Eq. (6):

$$\hat{d}_{i,j}^2(\eta) = \left(\frac{dx}{d\eta}\right)^2 d_{i,j}^2(x) + \frac{d^2 x}{d\eta^2} d_{i,j}^1 \tag{6}$$

In Eq. (6) $\hat{d}_{i,j}$ represents the element of the Gauss-Lobatto-Chebyshev derivative matrix remapped to the problem's domain, x represents the distance variable along the fin. Moreover, η is given by Eq. (7), where A and B are constants:

$$\eta = Ax + B \tag{7}$$

Equation (5), thus becomes Eq. (8) for both the insulated tip and convective tip cases, as they possess the same domain:

$$\theta_{\mathrm{i},n+1} = \frac{\theta_{\mathrm{i},n}}{2} + \frac{S_H \theta_{\mathrm{i},n}^2 + G(\theta_{\mathrm{i},n} + C_T)^4 - G\,\mathrm{C}_T^4 - 4 \sum_{j=0,\, j\neq i}^{N} d_{ij}^2 \theta_{j,n}}{8 d_{ii}^2} \tag{8}$$

Whereas, for the long fin case that possesses a different domain, Eq. (5) remapped gives rise to

$$\theta_{\mathrm{i},n+1} = \frac{\theta_{\mathrm{i},n}}{2} + \frac{S_H \theta_{\mathrm{i},n}^2 + G(\theta_{\mathrm{i},n} + C_T)^4 - G\,\mathrm{C}_T^4 - \left(4/9\right) \sum_{j=0,\, j\neq i}^{N} d_{ij}^2 \theta_{j,n}}{\left(8/9\right) d_{ii}^2} \tag{9}$$

The d_{ij} term is defined as:

$$d_{\mathrm{ij}} = \begin{cases} d_{00} = -d_{NN} = \left(2N^2 + 1\right)/6 \\ d_{jj} = \dfrac{x_j}{2\left(x_j^2 - 1\right)}, 1 \leq j \leq N-1 \\ \left(-1\right)^{i-j} \dfrac{c_i / c_j}{x_i - x_j}, i \neq j \end{cases} \tag{10}$$

Where c_j and x_i are given by Eq. (11) and Eq. (12) respectively:

$$c_j = \begin{cases} 2, \mathrm{j} = 0; \mathrm{j} = \mathrm{N} \\ 1, 1 \leq \mathrm{j} \leq \mathrm{N} - 1 \end{cases} \tag{11}$$

$$x_{\mathrm{i}} = \cos\left(i\pi/N\right), 0 \leq i \leq N \tag{12}$$

3. RESULTS

Figures 1 to 3 present the dimensionless temperature distribution for the three usual fin cases, namely a very long fin an insulated tip and a finite length fin with tip subjected to convection boundary conditions described by Eqs. (7), (8) and (9), respectively. In all the considered cases, the results obtained in this work are depicted in black while the results by Gorla and Bakier (2011) are depicted in red. The results obtained by the second order finite difference and the Gauss-Lobatto spectral methods, in all cases, present a very low discrepancy (around 5%). Since this would prevent to see any distinction among the curves, the results obtained by the second order finite difference method (which requires less CPU time) are chosen to be depicted.

$$\theta\left(0\right) = 1 \qquad \theta\left(\infty\right) = 0 \tag{7}$$

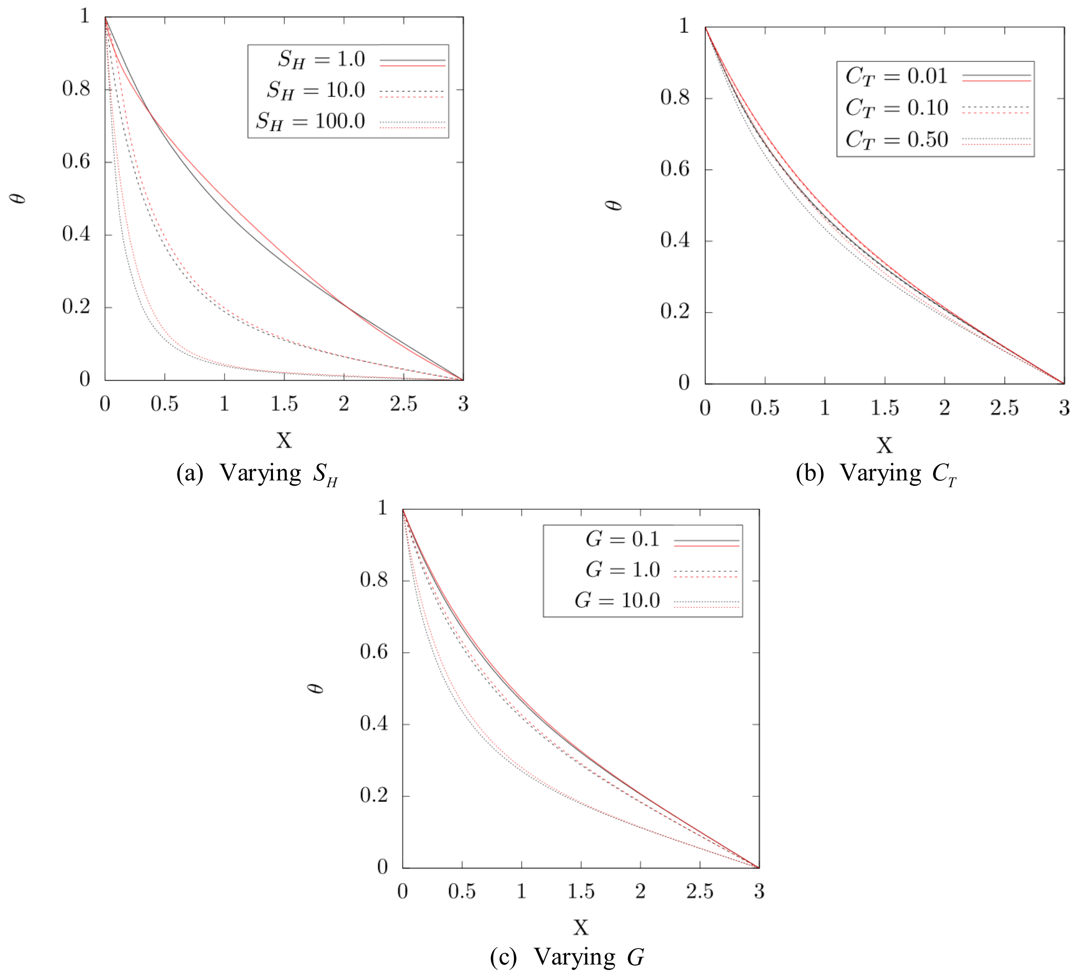


Figure 1. Dimensionless temperature distribution along the axial distance for very long fin boundary condition

In Fig. 1(a) a porous parameter that accounts for permeability and buoyancy effects S_H varies while the parameters $G = 0.1$ and $C_T = 0.01$ are kept constant. Clearly as S_H increases θ decreases along the porous fin length. Also, the results obtained in this work show good agreement with those obtained by Gorla and Bakier (2011) using a 4th order Runge-Kutta method. In Fig. 1(b) the temperature ratio C_T, varies while $S_H = 1$ and $G = 0.1$ are kept constant. It is very clear the small influence of the temperature ratio on the dimensionless temperature θ along the porous fin. Also, any difference is barely visible between these work´s results and those by Gorla and Bakier (2011). Finally, in Fig. 1(c) the radiation parameter G varies while $S_H = 1$ and $C_T = 0.01$. As in the previous cases, these work´s results are in good agreement with those by Gorla and Bakier (2011) and, as G increases, θ decreases along the porous fin length, although the radiation parameter variation is less sensible than the one caused by S_H variation.

$$\theta(0) = 1 \qquad \left.\frac{d\theta}{dX}\right|_{X=1} = 0 \tag{8}$$

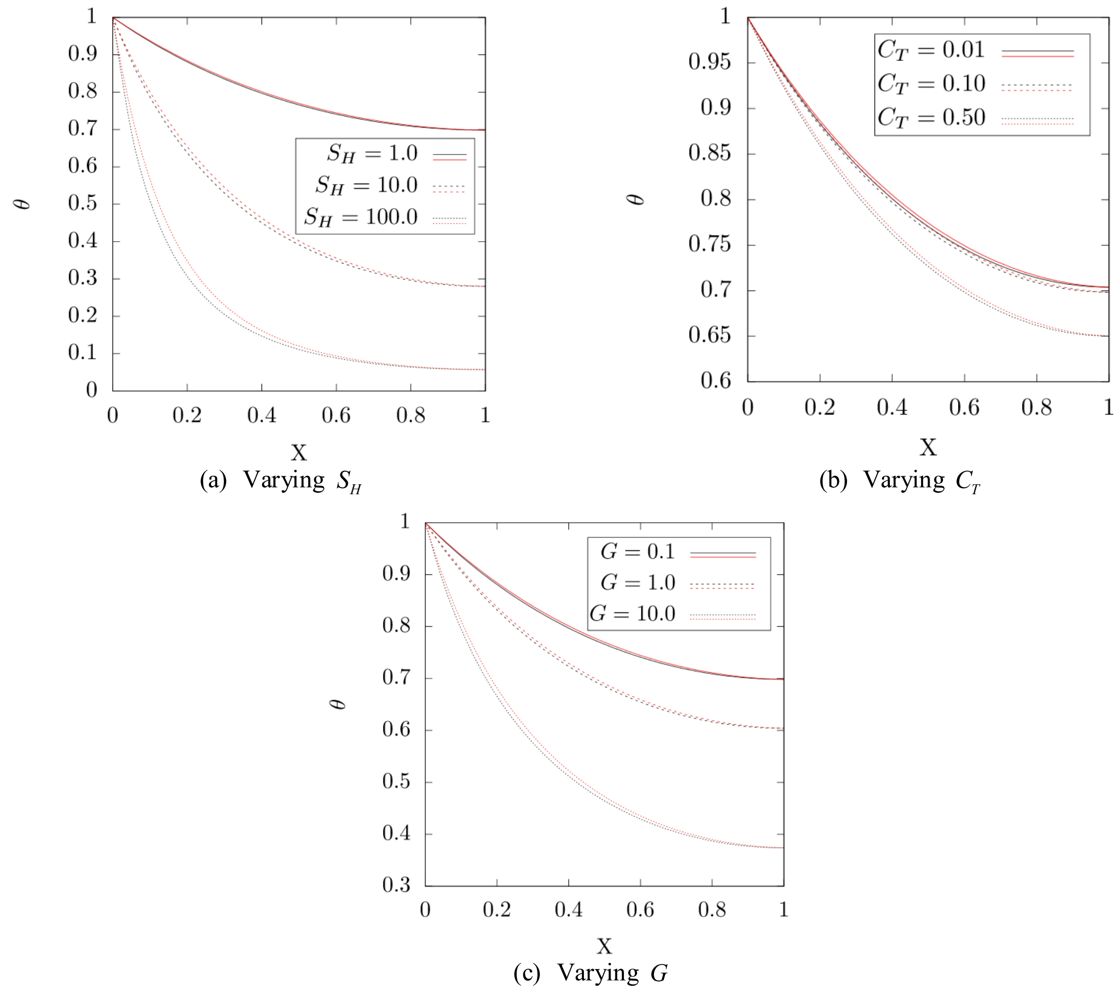
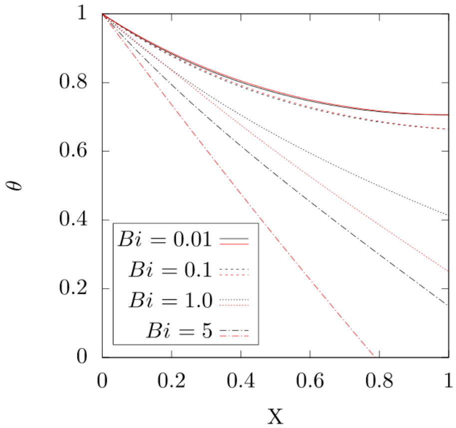


Figure 2. Dimensionless temperature distribution along the axial distance for the insulated tip boundary condition

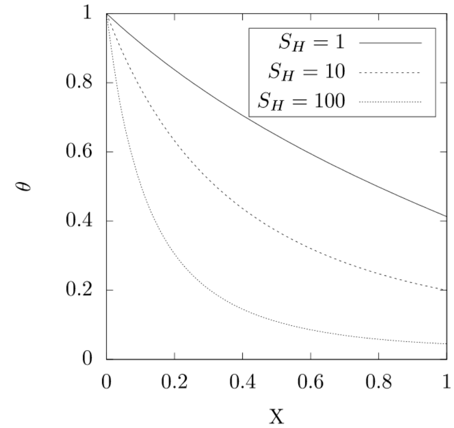
In Fig. 2, when an insulated tip boundary condition (Eq. (8)) is considered, the influence of varying S_H, C_T and G, keeping constants the two remaining parameters on the dimensionless temperature along the porous fin is clearly more pronounced than in the cases considered in Fig. 1. In Fig. 2(a), S_H, a porous parameter accounting for permeability and buoyancy effects varies from 1 to 100, while $G = 0.1$ and $C_T = 0.01$ are kept constant. Given an axial location the increase of S_H provokes a sharp decrease on θ. As in the previous case, there is good agreement with this work and the literature (Gorla and Bakier, 2011). The same occurs in Fig. 1(b), where the temperature ratio C_T, varies while $S_H = 1$ and $G = 0.1$. Unlike Fig. 1(b), the temperature ratio influences the dimensionless temperature θ along the porous fin, when it increases from $C_T = 0.01$ to $C_T = 0.5$. The radiation parameter G varied in Fig. 1(c) ($S_H = 1$ and $C_T = 0.01$) causing a sensible variation in the dimensionless temperature distribution along the axial porous fin, mainly when it increases from $G = 1$ to $G = 10$. Once again, good agreement with the literature (Gorla and Bakier, 2011) is noted.

$$\theta(0) = 1 \qquad \left.\frac{d\theta}{dX}\right|_{X=1} + Bi\theta(1) = 0 \tag{9}$$

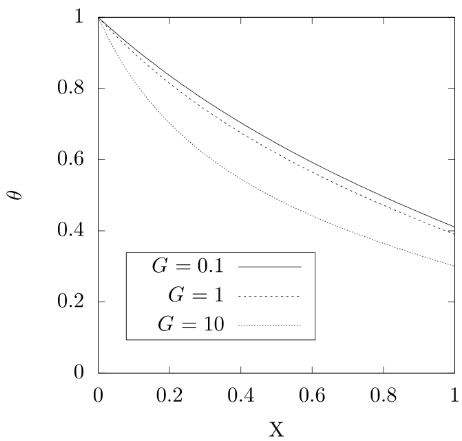
where Bi is the usual Biot number.



(a) Varying Bi



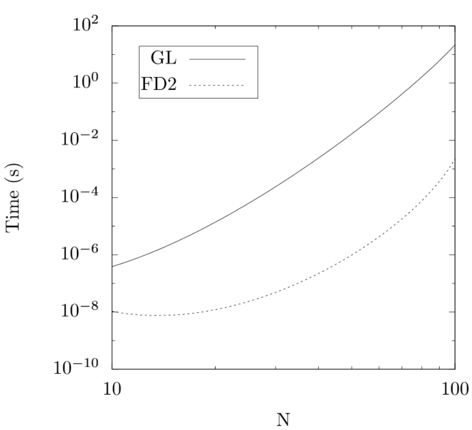
(b) Varying S_H



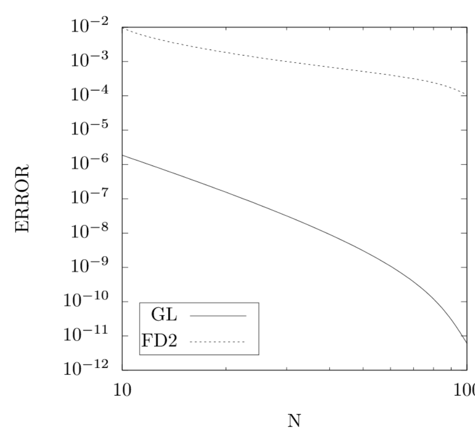
(c) Varying G

Figure 3. Dimensionless temperature distribution for the Convective Tip boundary condition

The results obtained closely resemble the results by Gorla and Bakier (2011) for low Biot numbers (Fig. 3 (a)). The variations of S_H and G are also depicted in Fig. 3 but is not compared due to the lack of similar results in the literature (Gorla and Bakier, 2011). The unvarying parameters in Fig. 3 have the same values as in Fig. 2, additionally for Fig. 3 (b) as well as Fig. 3 (c) the value for Biot's number is a constant $Bi = 1$.



(a) CPU Times versus Number of points



(b) Error versus Number of points

Figure 4. Computational data

Figure 4 (a) compares the CPU times for the second order finite difference (FD2) and the Gauss-Lobatto spectral (GL) methods, whereas Fig. 4 (b) compares the absolute error of the methods, which is defined as the difference between the current iteration and the previous one. It can be noted that although Gauss-Lobatto spectral methods requires more CPU time, it presents smaller error for the same number of points, when compared with the second order finite difference method.

4. CONCLUSIONS

The obtained results closely resemble those by Gorla and Bakier (2011), with the exception of the convective tip case. In this case the mechanical model could not be a good hypothesis for high Biot numbers. However, it is worth noting that the results using the Gauss-Lobatto-Chebyshev spectral method and second order finite difference method were both in good agreement for all considered Biot values as well as for all S_H and G for a convective tip. Also, it should be noted that a 4th order Runge-Kutta method although its simplicity is not more reliable that the employed methodologies.

5. REFERENCES

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