

Stochastic friction model with hysteresis: application to the dynamics of a drill-string

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Abstract: A probabilistic model is proposed to a hysteretic friction type model that might be used to model a bit-rock interaction. The application of interest is the nonlinear torsional dynamics of a drill-string. A nonstationary torque on bit, which is a function of the bit-speed, is constructed from a multiplicative stationary process.

Keywords: Stochastic friction model, hysteresis, nonlinear dynamics, drill-string

INTRODUCTION

This work proposes a stochastic model for a friction type model with hysteresis. The application of interest is the nonlinear bit-rock interaction of a drill string under torsional vibrations. The proposed model takes into account the fluctuations of the stick-slip oscillations and the hysteretic effect provided by the nonlinear bit-rock interaction observed during the drilling process. These fluctuations are modelled by introducing a stochastic process associated with the variations of the bit-rock interaction during the drilling. The parameters of the stochastic model are calibrated with field data.

DETERMINISTIC MODEL

The deterministic model: a constant angular speed Ω is imposed at the top and a reaction torque acts at the bottom due to the bit-rock interaction. In this paper, the torsional dynamics of the drill string is analysed by solving the torsional bar equation:

$$\rho I_p \frac{\partial^2 \theta(x,t)}{\partial t^2} - G I_p \frac{\partial^2 \theta(x,t)}{\partial x^2} = \mathcal{T}(x,t), \quad (1)$$

For a more complete model, with axial, lateral and torsional vibrations, check (Ritto et al 2009), for instance. After discretization by means of the finite element method, and model order reduction, using the normal models of the structure, we get:

$$[\tilde{M}]\ddot{\mathbf{q}}(t) + [\tilde{D}]\dot{\mathbf{q}}(t) + [\tilde{K}]\mathbf{q}(t) = \tilde{\mathbf{T}}(\dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t)), \quad (2)$$

Bit rock interaction model: The deterministic hysteretic bit-rock interaction model considered in the present paper (Real et al 2018) reads

$$T_{\text{bit}}(\dot{\theta}_{\text{bit}}, \ddot{\theta}_{\text{bit}}) = b_0 \left(\tanh(b_1 \dot{\theta}_{\text{bit}}) + \frac{b_2 \dot{\theta}_{\text{bit}}^{b_4}}{1 + b_3 \dot{\theta}_{\text{bit}}^{b_5}} (1 + H(\dot{\theta}_{\text{bit}} \ddot{\theta}_{\text{bit}})) \right), \quad (3)$$

for $\dot{\theta}_{\text{bit}} > 0$, in which $b_0, b_1, b_2, b_3, b_4, b_5$ are positive parameters to be fitted such that $0 < b_4 < b_5$. The bit-rock interaction model Eq. (3) [?] contains a hysteretic function H , which is a function of the bit angular acceleration (to separate forward and backward phases) and which is defined by

$$H(\dot{\theta}_{\text{bit}}, \ddot{\theta}_{\text{bit}}) = \beta_1 \tanh(\beta_2 \ddot{\theta}_{\text{bit}}), \quad (4)$$

where β_1 and β_2 are two positive parameters. Note that the hysteretic cycle is limited ($1 \pm \beta_1$), and if H equals to zero there is no hysteresis.

STOCHASTIC MDOEL

Proposed stochastic bit-rock interaction model: the present paper proposes to take into account the stochastic fluctuations of the bit-rock interaction model by including a multiplicative stochastic process to the deterministic model:

$$T_{\text{bit}}(\dot{\theta}_{\text{bit}}(t), \ddot{\theta}_{\text{bit}}(t)) = \underline{T}_{\text{bit}}(\dot{\theta}_{\text{bit}}(t), \ddot{\theta}_{\text{bit}}(t))(1 + \eta(t)), \quad (5)$$

where $\eta(t)$ is a centred stochastic process $\eta(t)$ which can be rewritten as

$$\eta(t) = \frac{T_{\text{bit}}(\dot{\theta}_{\text{bit}}(t), \ddot{\theta}_{\text{bit}}(t))}{\underline{T}_{\text{bit}}(\dot{\theta}_{\text{bit}}(t), \ddot{\theta}_{\text{bit}}(t))} - 1. \quad (6)$$

NUMERICAL RESULTS

Table 1 contains the parameters of the drill string used for the simulation.

	DP	BHA
Elastic Modulus [GPa]	220	220
Poisson's coefficient	0.29	0.29
Volumetric mass density [kg/m ³]	7,800	7,800
Length [m]	4,733.60	466.45
Inner radius [m]	0.0595	0.0363
Outer radius [m]	0.070	0.0803

Table 1 – Drill string characteristics.

The non-linear equation (2) is solved using a modified Euler scheme with a time step 0.512 ms. For one realization of the stochastic bit-rock interaction model, Figure 1 shows the stochastic response of the drill string in the stationary regime.

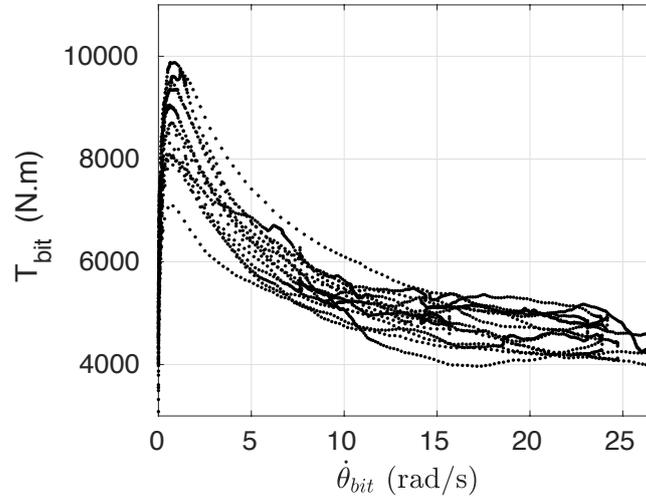


Figure 1 – One realization of the simulated bit-rock interaction.

The torsional stability of a drill string can be quantified through the stick-slip severity factor, defined by

$$\gamma_{\text{SS}}(\Omega) = \frac{\dot{\theta}_{\text{bit}}^{\text{max}}(\Omega) - \dot{\theta}_{\text{bit}}^{\text{min}}(\Omega)}{2\Omega}, \quad (7)$$

The statistical envelope shown in Fig. 2, due to the stochastic bit-rock interaction, yields large fluctuation in the random stick-slip severity factor. This means that bit-rock interaction variability has a direct impact on the drill string stability and should therefore be taken into account for a robust analysis of the drill string dynamics. The results show that when the top speed is about 20.5 rad/s the system has 5% probability of having the value of γ_{SS} greater than 0.5. The probability of instability increases as the speed decreases. This result brings much more information comparing with the deterministic result, where, for the same threshold of $\gamma_{\text{SS}} < 0.5$, the limit speed of 16 rad/s was obtained.

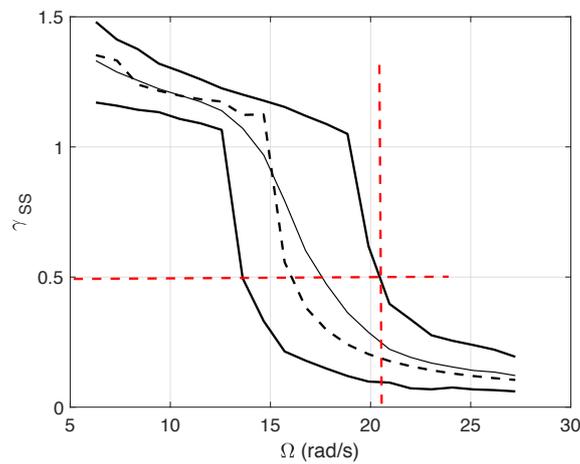


Figure 2 – Random stick-slip severity factor for a stochastic bit-rock interaction. Solid lines: mean response and 90% confidence region. Dashed line: deterministic case (black), stability threshold (red).

CONCLUSIONS

A probabilistic model is proposed for friction type mode with hysteresis. A nonstationary process is constructed using an additive stationary process. The random vibration of a torsional drill string is analyzed and a stochastic analysis is performed.

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