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STATISTICAL ANALYSIS OF THE ACCURACY OF EMPIRICAL LOSS OF LOAD EQUATION

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Abstract. *In repression pipes, it is utmost importance to measure load losses, since these phenomena have a direct influence on the hydraulic system sizing, mainly in the distribution lines. Diverse researchers and technicians of the hydraulic area use diverse empirical equations proposed in related literatures, with the intention of quantifying these losses. However, to obtain a more accurate parameter estimate, suitable equipment is necessary, such as differential manometers. With this scenario in mind, four empirical equations were analyzed, which quantify and evaluate the load loss indexes. They are: Hazen-Williams's equation, Flamant, Fair-Whiplle-Hsião and Darcy-Weisbach. The results obtained were compared with the load loss values measured by a Bourdon type differential manometer. The tests were carried out by varying a flow, where for each condition analyzed, to obtain the data, which allow to compare and evaluate the results obtained in the respective equations in this work. From the obtained results, it is possible to infer that the empirical equations indicate higher load losses than those observed in differential manometers. Darcy-Weisbach's equation was more accurate, with an accuracy above 20.00%, whereas the methodology presented by Fair-Whiplle-Hsião diverged around 45.00% of the loss indicated in the manometers.*

Keywords: *Load Loss, Empiric Equations, Precision.*

1. INTRODUCTION

Hydraulics is an area of knowledge of extreme relevance since antiquity. According to Netto (1973), there were irrigation canals in ancient Mesopotamia built on a plain lying between the Tigris and Euphrates rivers. Nevertheless, the theoretical and practical knowledge developed over the years about hydraulics is essential for the academic training of students in various engineering areas, as well as being responsible for improvements in the industry and machinery in general.

It should also be mentioned that hydraulics has always been a fertile field for mathematical investigations and analyzes and has given rise to theoretical studies that frequently depart from experimental results. According to Netto (1998), several expressions thus deduced had to be corrected by practical coefficients, which contributed to the hydraulics being called the "science of practical coefficients".

Due to the development of this science, and with the aid of new tools and technologies, the difference between values found empirically and real converges practically to zero. Thus, according to Lencastre 1972, the precision of the results depends on the correct choice (among a set of numerous formulas) of the model that best represents the reality of the system in question.

However, it is necessary that the parameters involved in hydraulic systems be perfectly understood, in order to guarantee and meet the specific needs of the system and its applications in a more appropriate way, avoiding or reducing the undesired effects of phenomena such as loss of charge. This knowledge brings greater reliability in the productive environment to which the hydraulic system is acting, increasing productivity and reducing costs.

2. LITERATURE REVIEW

2.1 Load Loss

Netto et al. (1973) points out that this is the energy lost per unit weight in the section of the pipeline under analysis. The load losses in a flow are due, among other factors, to the viscosity of the liquid, which opposes the movement of the particles, and this resistance must be overcome at the expense of the mechanical energy of the liquid.

2.2 Empirical Equations for Calculation of Load Loss

In designing a pipe, the main question is to determine the amount of energy needed to "push" the desired amount of liquid between one point and another of that pipe. Great efforts have been made to accurately measure load losses to make hydraulics safer (NETTO et al., 1998).

2.2.1. Darcy-Weisbach's Universal Equation (1845)

According to Netto et al., (1998), Darcy was the first researcher to consider the nature and state of the tubes. He obtained a formula whose usefulness and applications have been recognized for about 150 years. Follow Eq. (1).

$$h_f = f\frac{V^2}{2g}\frac{L}{D} \quad (1)$$

At where:

h_f = loss of load (m.c.a);
V = mean flow velocity of the fluid (m / s);
D = internal pipe diameter (mm);
L = pipe length (m);
g = acceleration of gravity (m / s-2);
f = coefficient of friction;

The factor is obtained through theoretical-experimental formulas in a function of the Reynolds number and the relative roughness of the studied pipeline.

2.2.2. Flamant's Equation (1892)

According to Baptista (1953), Eq. (3) was originally tested for smooth wall tubes. However, these fit well to plastic tubes of small diameters.

$$h_f = 4bL.\sqrt[4]{\frac{V^7}{D^5}} \quad (2)$$

At where:

h_f = loss of load (m.c.a);
V = mean flow velocity of the fluid (m / s);
D= internal pipe diameter (mm);
L = pipe length (m);
b = correction coefficient;

2.2.3. Hazen-Willians's Equation (1902)

According to Netto et al. (1973), Eq. (2) considers previously obtained experimental data several researchers, as well as values of observations of the authors. This is recommended for pipes with a diameter greater than Ø 16 mm.

$$h_f = \frac{10{,}643 * Q^{1{,}85} * L}{C^{1{,}85} * D^{4{,}87}} \tag{3}$$

At where:

h_f = loss of load (m.c.a);
Q = Volumetric flow rate (m³ / s);
D = internal pipe diameter (mm);
L = pipe length (m);
C = coefficient depending on the nature and conditions of the materials;

2.2.4. Fair-Whipple-Hsião's Equation (1930)

According to Baptista (1953), Eq. (4) is indicated for the calculation of networks using light tubes and / or diameters smaller than 100.00 mm. They are also employed when the properties of the conduit material are not known.

$$h_f = 0{,}000859 * \frac{Q^{1{,}75} * L}{D^{4{,}75}} \tag{4}$$

At where:

h_f = loss of load (m.c.a);
Q = Volumetric flow rate (m³ / s);
D = internal pipe diameter (mm);
L = pipe length (m);

3. MATERIALS AND METHODS

The present work is divided in three stages: 1 - assembly of a hydraulic circuit; 2 - repeated testing to obtain an appreciable amount of data for further analysis; 3 - evaluation of the precision of the empirical equations in relation to the actual loss of the system.

3.1 Tests Workbench

The hydraulic system was designed in order to obtain an expressive loss of load in relation to the maximum hydraulic load offered by the pump used. As the test fluid, water was used at room temperature.

3.1.1. Base Structure and Measuring Instruments

The transport of fluids causes an intense vibration in the pipes, characteristic that can interfere in the precision of the variables measured in the present work. In order to minimize this effect, a SAE 1020 steel base was designed to install the hydraulic circuit. To validate the study, it is necessary to employ precision instruments. The following components were used:

- Flow sensor: a flow sensor of the volumetric type (turbine) model YF-5201, shown in Fig. 1a, was used to collect the flow data. Through a control algorithm, it was put into operation, and the values obtained were displayed on an LCD monitor, model Keypad Shield / Shield (Fig. 1b).

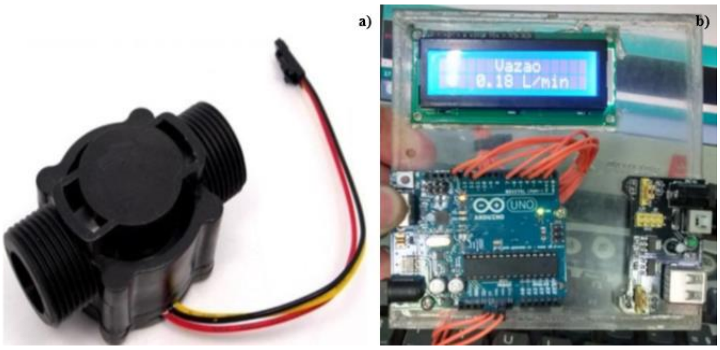

Figure 1. a) YF-5201 sensor; b) system flow visualization panel.

- Potentiometer: A potentiometer model 3590-S was used. The values of increments in its gradation were imputed through the computational interface generated by means of an Arduino micro controller.
- Hydraulic Circuit (Layout): was constructed from: 10 m of polyvinyl chloride (PVC) pipe, with a constant diameter of Ø 20 mm; 24 units of PVC curves, also with Ø 20 mm; 2 units of Ø 20 mm globe type valves; 2 units of analog gauges. The circuit is shown in Fig. 2.

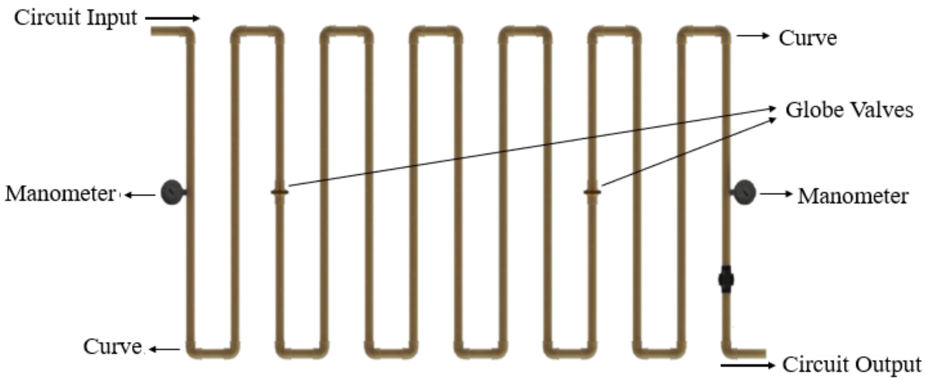

Figure 2. Hydraulic circuit layout.

3.1.1. Workbench Mounting

The workbench was assembled as shown in Fig. 3. A centrifugal pump model QB 60 was used as the source of power.

Figure 3. Mounted workbench layout.

3.2 Tests

The test is based on the variation of the flow through the potentiometer and the observation of the influence that the action prints on the values observed in the differential manometers installed in the hydraulic circuit. It is important to note that the size of the sample space of the measurements was calculated using Minitab software, which indicated a total of 17 replicates. To optimize the adhesion of the obtained data, the tests were replicated 20 times for each flow value. Fig. 4 shows a simplified flowchart of the script followed for each test performed on the circuit.

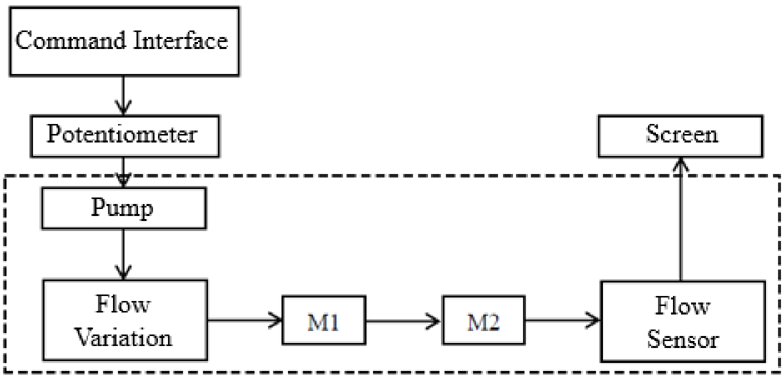

Figure 4. Test script flowchart.

4. RESULTS AND DISCUSSIONS

All the iterative calculations were solved by the software C ++, while the others through the software MS - Excel.

4.1 Flow Rate Indicated by Potentiometer

By means of increments in the initial flow (Q_0), it was possible to obtain different load losses using the same hydraulic circuit. The control of this flow was carried out by the potentiometer. It is important to emphasize that the motorcycle pump was only able to recharge water after reaching a flow of 22 l/min, and the maximum flow rate was 32 l / min, which is equivalent to about 92% of the maximum flow indicated in the manufacturer's manual. Values obtained for different valve opening percentages are shown in Table 1.

Table 1. Variation of flow through the potentiometer.

Opening (%)	**Flow rate (l/min)**
80	24,00
86	26,00
91	28,00
95	30,00
100	32,00

After the measurement of the flow values, the measurements measured by differential manometers M_1 e M_2. Table 2 shows the mean values of the pressure variation observed in the experiments.

Table 2. Variation of pressure in differential pressure gauges.

Flow rate (l/min)	$\Delta \boldsymbol{P}$ ($\boldsymbol{Kgf/cm^2}$)	$\Delta \boldsymbol{P}$ (k$\boldsymbol{Pa}$)
24	0,30	30,30
26	0,36	34,77
28	0,41	40,32
30	0,48	47,14
32	0,51	50,33

It is important to emphasize that the similarity of the dispersion values found between the flows allowed to simplify the analysis to a flow rate equivalent to 24 l / min.

The frequency histogram obtained shows a high probability that the values are within the reliability index stipulated in 95%. Another factor that increases the credibility of the data obtained in the normality test is the critical value of p = 0.63. According to Schneider (2000), Delme (2003), Berger (2010) among several other authors, for a value of p> 0.5, the Gaussian (normal) curve can be considered as having an expressive representation of the distribution of the data evaluated. For the data dispersion analysis, the low values of kurtosis, variance and asymmetry show that the measurements have low amplitude. Table 3 presents these values.

Table 3. Descriptive data of loss of load (m.c.a).

Analyze	**Flow rate (l/min)**				
Parameters	**24**	**26**	**28**	**30**	**32**
Average	3,10	3,55	4,15	4,80	5,16
Medium	3,07	3,50	4,20	4,85	5,10
Mode	3,00	3,40	4,20	4,90	5,00
Standard deviation	0,17	0,19	0,21	0,22	0,23
Sample variance	0,03	0,04	0,02	0,05	0,03
Curtose	0,04	-0,20	-1,50	-1,28	-1,66
Asymmetry	-0,24	1,20	-0,65	-0,09	0,42
Interval	0,70	0,50	0,60	0,60	0,40
Minimum	2,70	3,40	3,80	4,50	5,00
Maximum	3,40	3,90	4,40	5,10	5,40
Average	3,10	3,55	4,15	4,80	5,16
Medium	3,07	3,50	4,20	4,85	5,10

Table 3 indicates a good accuracy of the data, highlighting the low standard deviation between them. This error has been minimized by the use of electronic sensors, which reduce systematic and / or accidental errors. According to Schneider (2000), the control of experimental processes through sensors can reduce the error rate by up to 30%.

It is necessary to make a statistical treatment that allows the adoption of a reference range where the data have good reliability. The results obey a normal (Gaussian) statistical distribution. We adopted a normal curve 4σ, whose limits are set to ∓2 standard deviations of the mean, that is, we took as reference range the interval [-2σ; + 2σ]. Therefore, it is possible to affirm, with 95% reliability, that the actual value of loss of load is contained within this range, as shown in Table 4.

Table 4. Loss of load for the different values of evaluated flow.

Flow rate (l / min)	**Loss (m.c.a)**	**±**
24,00	3,10	0,34
26,00	3,50	0,40
28,00	4,20	0,26
30,00	4,80	0,44
32,00	5,20	0,34

4.2 Load Loss

The total pipe length was calculated using the virtual length method, which replaces the singularities of the circuit with equivalent straight lengths. In order to estimate the load loss values, the empirical equations presented in the literature review are used. As the study was conducted exclusively on PVC pipes, tabulated data are considered to be constants of the material used in the calculations. The flow data of Table 1 in m³ / s is used for calculations.

4.2.1. Real Load Loss (Differential Pressure Manometers)

It is necessary to make some considerations based on the layout of the test workbench:

- The transport of the fluid takes place in a horizontal section, the flow and the section area are constant. Thus, we have Eq. (5).

$$h_{f(1-2)} = \frac{P_1 - P_2}{\rho g} \qquad (5)$$

- The specific mass is adopted as $\rho = 10^3\, kg.m^{-3}$ and the gravity $g = 9{,}81\ m.s^{-2}$.

Substituting the pressure variation values of Table 2 in Eq. (5), we find the loss of charge, which is represented graphically in Fig. 5.

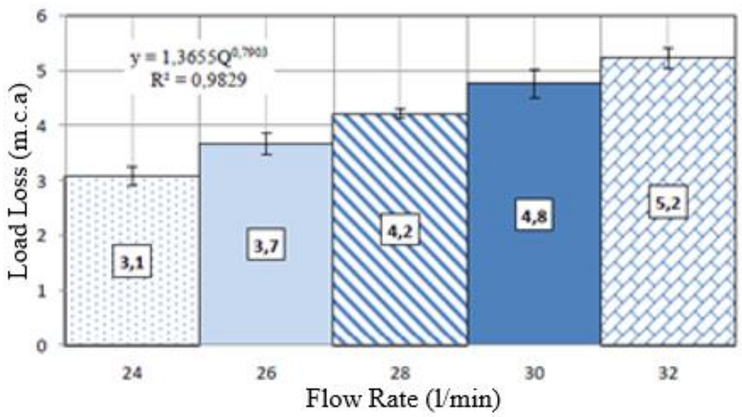

Figure 5. Actual load loss graph (differential manometers).

Referring to Fig. 5, it is noted that the circuit has a low head loss in relation to the overall length. An increase of 25% in flow compared to Q_0 resulted in an increase in load loss of 41%. The trend line shows an exponential equation with high correlation.

4.2.2. Estimation of Load Loss by the Darcy-Weisbach Formula (Universal Method)

The equation depends on characteristics of the hydraulic circuit and properties of the transported fluid. In order to perform the calculations, the internal diameter of pipes Ø 17 mm, roughness of 0.0015 mm and kinematic viscosity of 9.0E-6 (LENCASTRE, 1972) were used. The Reynolds number was obtained by replacing these values in the specific equation. The friction factor was solved by the Newton-Raphson numerical method. Table 5 shows the values obtained for each flow value.

Table 5. Friction Factor (Colebrook) and Reynolds.

Flow rate (l/min)	Reynolds (Re)	f
24	2814,55	0,021234
26	3096,00	0,021136
28	3339,93	0,021055
30	3565,10	0,021015
32	3752,73	0,021000

As observed by Assy (1977), the increase in the number of Reynolds gives a decrease in friction factor f. In addition, it also implies a more turbulent regime where this random flow of the fluid decreases the influence that the imperfections of the inner surface of the pipes impress in their flow, causing an even greater reduction of the friction factor.

After obtaining the terms listed in Table 5, the friction values in Eq. (1) are substituted, determining the load loss values represented in the graph of Fig. 6.

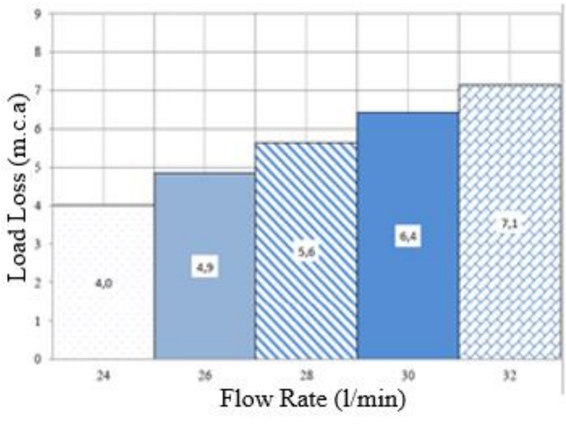

Figure 6. Loss of charge by the Universal Formula (Darcy-Weisbach).

4.2.3. Calculation by Empirical Equations

The calculation of the loss of load through the empirical method is based on replacing the values of the flows and specific constants of the pipes in the corresponding equations. The calculations were performed in MS - Excel software. Table 6 lists the constants and their equations. Table 7 presents the results.

Table 6. Calculation of implicit equations.

Method	Constant		Equations
Hazen - Williams	C	140	Equation 17
Flamant	b	0,000135	Equation 18
Fair - Whipple - Hsião	-	-	Equation 19

Table 7. Loss of load in m.c.a obtained by empirical methods.

Flow rate (l/min)	Hazen Williams	Flamant	Fair - Whiplle - Hsião
24	5,54	5,48	5,78
26	6,42	6,30	6,65
28	7,37	7,18	7,57
30	8,37	8,10	8,54
32	9,43	9,06	9,56

For better visualization, the values of Table 7, obtained by the empirical equations, differential manometers and the Darcy-Weisbach method, are plotted in the graph of Fig.

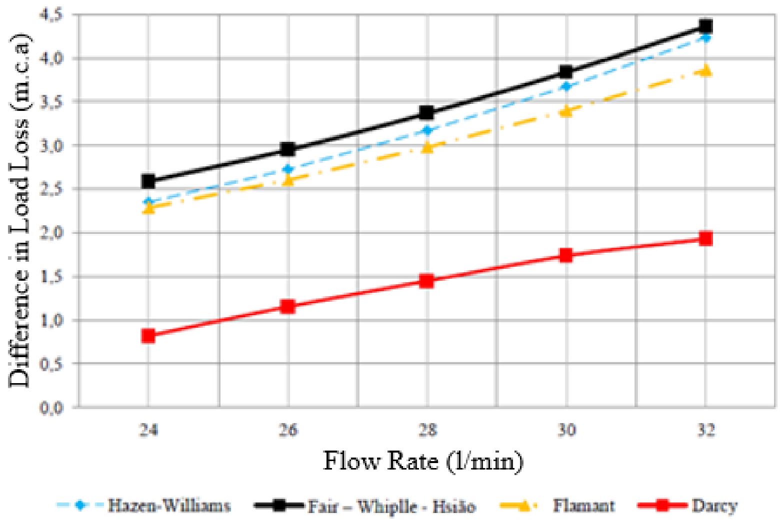

Figure 7. Comparison between the methods of calculating the load loss evaluated.

The graph of Fig. 7 shows the divergence between the actual values and those obtained by the equations, using the loss measured in the manometers as reference. It is verified that the equations are oversized, and that increasing the flow increases the divergence in relation to the comparison equation (differential manometers). This disparity can be explained by the limitation of the recommended use of each equation. The incorporation of these formulas is all the greater the broader the intended field of application.

The graph of Fig. 8 also highlights a certain similarity between the values obtained by the Flamant and Hazen Williams equations, which are commonly recommended by the Brazilian Association of Technical Standards (ABNT) for light tubes, and diameters smaller than 50.00 mm. Assy (1977) reports that the empirical equation of Hazen Williams converges to the real value in his experiment for Reynolds numbers greater than 5x105.

However, Darcy-Weisbach's formula proved to be the most accurate, obtaining in a given flow a 20.50% error. On the other hand, the Fair-Whipple-Hsião equation presented unsatisfactory results for use in the analyzed model. Although this is used for tubes with small diameters, the absence of a correction factor (constant in relation to the material) corroborates to the divergence of the values, causing an error of 45.60%, being twice that obtained through the method of Darcy -Weisbach.

The fact that the equations are simplified models of the phenomenon entails a succession of cumulative errors, such as: error in the constants relative to the material, in the precision of the flow meters, pressure and others. It is important to point out that the analysis was done in a system with low flow velocity, not allowing the development of the flow, which would generate less errors of simplification in relation to the roughness of the material. Netto et al. (1973), comments that the errors of the empirical equations are 60.00% embedded in the characteristics of the material, that is, in the adjustment factor of the tabulated constants.

Another important point to be made is that the pressure loss propagates along the pipe. Since the method of analysis of the calculation was simplistic, converting the whole pipe into virtual lengths, there is a propagation of errors related to each simplification of the function of loss of load.

4.3. Relative Error Analysis of Differential Manometers

The analysis of the disparity was made through the relative error in relation to differential manometers. Table 8 summarizes the errors of the equations evaluated.

Table 8. Relative error of the equations.

Flow rate (l/min)	Hazen-Williams	Flamant	Fair-Whipple-Hsião	Darcy-Weisbach
24,00	42,30%	41,30%	44,30%	20,50%
26,00	42,40%	41,50%	44,50%	23,70%
28,00	43,00%	41,80%	44,60%	25,60%
30,00	43,90%	41,90%	44,90%	26,90%
32,00	44,90%	42,60%	45,60%	27,10%

Table 8 shows that, for low flow rates, the Hazen-Williams and Flamant equations show similarity between the values obtained. However, for high flows the values of these equations tend to diverge, presenting a variation of 2.30% relative error. The Darcy-Weisbach equation presented homogeneity of results, with errors much smaller than the other equations. This fact is explained because the calculation of the friction factor is quantified and corrected for each model, through the Colebrook-White equation, corroborating the values verified in the hydraulic circuit. The other equations, in turn, present a constant value for the correction factor.

The Fair-Whipple-Hsião equation presents a practically constant error, with a small variation in relation to the flow. However, this was the one that presented the greatest divergence in relation to the values measured in the hydraulic circuit, about 45.60%.

Through the analysis of the error, it is verified that the inadequate choice of the method can cause a disparity of approximately 24.00%, which emphasizes the need for a good delineation of the system to be dimensioned. This discrepancy shows an oversizing of the equations. This overestimation of the load loss occurs due to the increase in the diameter of the polyethylene pipes with the increase in operating pressure. Vilela et al. (2003), when evaluating tubes of this type, observed a significant influence of the operating pressure on the diameter of the tubes, and reported that these changes can cause variations greater than 40.00% in the determination of the pressure loss.

The graph of Fig. 8 shows the disparity of the equations of loss of charge of the equation that most converged (Darcy-Weisbach) and the one that most diverged (Fair-Whiplle-Hsião).

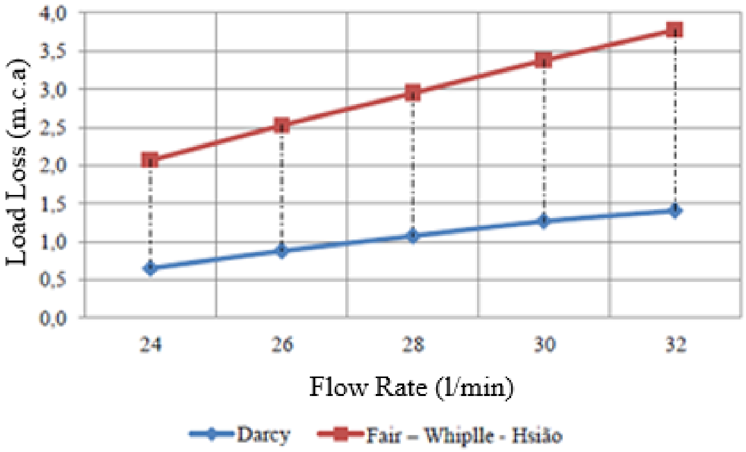

Figure 8. Load Loss (extremes).

It is observed that, although the empirical equations show a high disparity between the theoretical values and the real value, the percentage loss of the circuit does not increase significantly, adding a maximum loss of 2.40 mca in the hydraulic system, which represents about 0.24 bar.

5. CONCLUSIONS

This work allowed to measure and classify the empirical equations of loss of load analyzed in the study, through their respective levels of precision in relation to the actual load loss of the hydraulic circuit. The Darcy-Weisbach equation showed to be the most accurate, with percentage errors lower than 27.00%, followed by the Flamant equation with 42.70% and Hazen-Williams with 44.90%. Fair-Whiplle-Hsião emerged as the most divergent equation in relation to the analyzed circuit, with a relative error of 45.60%.

The study allowed to summarize the main equations used for load loss, listing their uses and limitations in relation to the analyzed system. The variety of existing models for the analysis of the phenomena, and their advancement over the years, became noticeable. The literature presents the Hazen-Williams, Flamant, Fair-Whiplle-Hsião and Darcy-Weisbach equations as the most accurate for PVC pipes.

The dimensioning of the sample space allowed the tests to be performed reliably, because few data could introduce errors that would hinder the analysis of the problem in question. The number of measurements performed was satisfactory, representing well the dispersion and fidelity of the data, reducing the standard deviation measured in the initial tests for the flow rate of 24 l / min.

The workbench allowed to carry out the tests as expected, presenting a reliability of 95%, thus generating a good repeatability of the results. The good accuracy of the obtained values allowed to minimize the effects of external interferences in the analyzes of the different evaluated equations.

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