ENCIT 2018

17th Brazilian Congress of Thermal Sciences and Engineering
November 25th-28th, 2018, Águas de Lindóia, SP, Brazil

ENCIT-2018-XXXX
RADIATIVE TRANSFER IN TWO-DIMENSIONAL ANISOTROPIC MEDIA: AN ANALYTICAL-NODAL FORMULATION

Karine Rui
Programa de Pós-Graduação em Engenharia Mecânica, Universidade Federal do Rio Grande do Sul, Rua Sarmento Leite, 425, 90050-170 - Porto Alegre - RS, Brazil
karinerui@gmail.com

Liliane Basso Barichello
Instituto de Matemática e Estatística, Universidade Federal do Rio Grande do Sul, Av. Bento Gonçalves, 9500 - P.O. Box 15080, 91509-900 - Porto Alegre - RS, Brazil
lbaric@mat.ufrgs.br

Abstract. *In this work, the discrete ordinates approximation of the two-dimensional radiative transfer equation with anisotropic scattering is solved using nodal techniques along with the Analytical Discrete Ordinates Method (ADO Method). The main goal is the treatment of radiation problems in anisotropic media. The complexity of the model is reduced with the application of the nodal technique (integration process) and the one-dimensional transverse integrated equations are then solved by the ADO method, such that the resulting solution is explicit in terms of the spatial variables. The average density radiation is obtained for the case of second order expansion of the scattering law. Numerical results along of the centerline of the domain are presented and compared with results available in the literature with good agreement, in general achieving analogous accuracy based on the division of the domain into smaller number of nodes.*

Keywords: *Analytical Discrete Ordinates (ADO) method, anisotropic scattering, 2-D radiation*

1. INTRODUCTION

Radiative heat transfer in absorbing, emitting and scattering media plays an important role in many industrial and engineering applications (Howell *et al.*, 2016). Because of that, intense research is developed on the solution of the multidimensional radiative transfer equation, with special attention to effects of anisotropy that are relevant as natural part of the phenomena. In the literature there are many methods for solving the radiative transfer equation. Among them, the discrete ordinates method (S_N), proposed by Wick (1943) and Chandrasekhar (1950), is one of the classical methods. It has been applied to deal with different radiative transport problems, such as in radiation transfer in furnaces (Fiveland, 1984, 1988; Kim and Huh, 1999), conduction-radiation coupled problems (Zabihi *et al.*, 2017), radiation transfer in cylindrical high pressure discharges (Bouaoun *et al.*, 2005), in complex 2-D geometries (Aghanajafi and Abjadpour, 2016). This method is based on the discretization of the angular variable, such that the integral term of the equation is approximated by a numerical quadrature and the equation is transformed into a set of differential equations that can be solved with the use of numerical or analytical approaches. More details on the applications of the method in radiative transfer can be found in (Liu *et al.*, 2002; Godoy and DesJardin, 2010; Hunter and Guo, 2011; Efremenko *et al.*, 2013; Coelho, 2014; Ganapol, 2015).

In the present study, we analyze two-dimensional radiative heat transfer in a rectangular medium with special focus in anisotropic scattering, using the Analytical Discrete Ordinates (ADO) method (Barichello and Siewert, 1999) in conjunction with nodal schemes, for the solution of the radiative transfer equation. The ADO solves the system of unidimensional integrated equations derived from discrete ordinates equations. This method has been widely and successfully used to solve two-dimensional particle transport problems restrict to nuclear applications (Barichello *et al.*, 2011; Prolo Filho and Barichello, 2013, 2014; Picoloto *et al.*, 2015; Barichello *et al.*, 2017; Picoloto *et al.*, 2017). From the application of this method in coarser meshes, results of same order accuracy were obtained when compared with results provided by other schemes using more refined meshes (Barichello *et al.*, 2017). In this context, until the present date, solutions for the two-dimensional problems were developed only for the case of linear anisotropic scattering (Picoloto *et al.*, 2017) more usual to neutron applications. Our aim is to extend the ADO formulation to deal with higher order of anisotropy. However, since the method has analytical characteristic, this change requires the development of different eigenvalue problems. In this work, we obtain the average density radiation for different orders of the quadrature and degree of anisotropy $L = 2$ and we compare the results obtained by this methodology with results published in the literature by Kim and Lee (1988).

2. FORMULATION OF THE PROBLEM

In this study, we consider the problem of solving the discrete ordinates approximation of the two-dimensional radiative transfer equation, in an emitting and anisotropic scattering medium, on a rectangular enclosure $x \in [0, a]$ and $y \in [0, b]$ shown in Fig. 1. We follow Chandrasekhar (1950) and Cacuci (2010) to express the phase function by a finite Legendre polynomial expansion in terms of the cosine of the scattering angle, here assuming the particular case $L = 2$ as degree of anisotropy. We write the equation for the radiation intensity $I(x, y, \mathbf{\Omega}_i)$ as (Modest, 2003; Howell *et al.*, 2016)

$$\mu_i \frac{\partial}{\partial x} I(x,y,\mathbf{\Omega}_i) + \eta_i \frac{\partial}{\partial y} I(x,y,\mathbf{\Omega}_i) + \beta I(x,y,\mathbf{\Omega}_i) = \frac{1}{4\pi}\Bigg\{ \sigma_{s0} \sum_{n=1}^{M} w_n I(x,y,\mathbf{\Omega}_n) + 3\sigma_{s1}\left[\mu_i \sum_{n=1}^{M} \mu_n w_n I(x,y,\mathbf{\Omega}_n) + \eta_i \sum_{n=1}^{M} \eta_n w_n I(x,y,\mathbf{\Omega}_n)\right] + 5\sigma_{s2}\left[\frac{1}{2}(3\xi_i^2 - 1)\sum_{n=1}^{M} \frac{1}{2}(3\xi_n^2 - 1) w_n I(x,y,\mathbf{\Omega}_n) + 3\mu_i\eta_i \sum_{n=1}^{M} \mu_n \eta_n w_n I(x,y,\mathbf{\Omega}_n) + \frac{3}{4}(\mu_i^2 - \eta_i^2)\sum_{n=1}^{M}(\mu_n^2 - \eta_n^2) w_n I(x,y,\mathbf{\Omega}_n)\right]\Bigg\}, \quad (1)$$

for $i = 1, \ldots, M$, in which M is the number of discrete directions defined in accordance with a numerical quadrature scheme; w_n are the weights (normalized to 4π) associated to the angular directions $\mathbf{\Omega} = (\mathbf{\Omega}_x, \mathbf{\Omega}_y, \mathbf{\Omega}_z)$, such that $\mathbf{\Omega}_x = \mu = (1 - \xi^2)^{1/2}\cos(\varphi)$; $\mathbf{\Omega}_y = \eta = (1 - \xi^2)^{1/2}\sin(\varphi)$ and $\mathbf{\Omega}_z = \xi = \cos(\theta)$, where θ is polar angle measured from the z-axis and φ azimuthal angle measured from the x-axis; $\beta = (\kappa + \sigma_s)$ is the extinction coefficient, κ and σ_s are the absorption and scattering coefficients of the medium, respectively. Here $\sigma_{sl} = \sigma_s C_l/(2l + 1)$, where C_l are the expansion coefficients for the phase functions listed in Kim and Lee (1988) and $l = 0, \ldots, 2$.

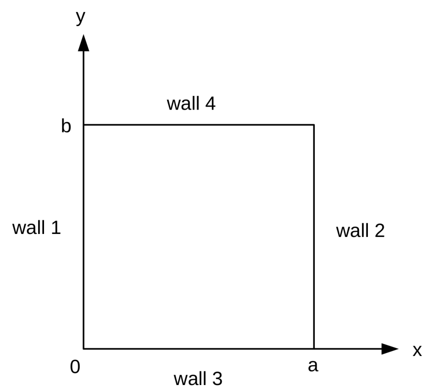

Figure 1. Two-dimensional geometry

We consider the domain subdivided in $r = 1, \ldots, R$ rectangular regions defined by $x \in [a^r_{h-1}, a^r_h]$ and $y \in [b^r_{k-1}, b^r_k]$ with $0 \leq a^r_{h-1} < a^r_h \leq a$ and $0 \leq b^r_{k-1} < b^r_k \leq b$, where $h = 1, \ldots, H$ and $k = 1, \ldots, K$ indicate a number of subdivisions considered in the axes x and y, respectively (Barichello *et al.*, 2017). We follow Barichello *et al.* (2011) to solve Eq. (1) using nodal methods, in which we consider that the directions $\mathbf{\Omega}_i$, for indices $i = 1, \ldots, M/2$, have coordinates $\eta_i > 0$; and for $i = M/2 + 1, \ldots, M$ have coordinates $\eta_i < 0$. Thus, to obtain the one-dimensional transverse integrated equations in y direction, we integrate Eq. (1) for all $x \in [a^r_{h-1}, a^r_h]$

$$\eta_i \frac{\mathrm{d}}{\mathrm{d}y} I_{xr}(y, \mathbf{\Omega}_i) + \frac{\mu_i}{\alpha_h^r}\left[I_r(a_h^r, y, \mathbf{\Omega}_i) - I_r(a_{h-1}^r, y, \mathbf{\Omega}_i)\right] + \beta_r I_{xr}(y, \mathbf{\Omega}_i) =$$
$$\frac{1}{4\pi}\Bigg\{\sigma_{s0}\sum_{n=1}^{M/2} w_n[I_{xr}(y,\mathbf{\Omega}_n) + I_{xr}(y,\mathbf{\Omega}_{n+M/2})] + 3\sigma_{s1}\Bigg[\mu_i \sum_{n=1}^{M/2}\mu_n w_n[I_{xr}(y,\mathbf{\Omega}_n) + I_{xr}(y,\mathbf{\Omega}_{n+M/2})] +$$
$$\eta_i\sum_{n=1}^{M/2}\eta_n w_n[I_{xr}(y,\mathbf{\Omega}_n) - I_{xr}(y,\mathbf{\Omega}_{n+M/2})]\Bigg] + 5\sigma_{s2}\Bigg[$$
$$\frac{1}{2}(3\xi_i^2-1)\sum_{n=1}^{M/2}\frac{1}{2}(3\xi_n^2-1)w_n[I_{xr}(y,\mathbf{\Omega}_n)+I_{xr}(y,\mathbf{\Omega}_{n+M/2})]+3\mu_i\eta_i\sum_{n=1}^{M/2}\mu_n\eta_n w_n[I_{xr}(y,\mathbf{\Omega}_n)-I_{xr}(y,\mathbf{\Omega}_{n+M/2})]+$$
$$\frac{3}{4}(\mu_i^2-\eta_i^2)\sum_{n=1}^{M/2}(\mu_n^2-\eta_n^2)w_n[I_{xr}(y,\mathbf{\Omega}_n) + I_{xr}(y,\mathbf{\Omega}_{n+M/2})]\Bigg]\Bigg\}, \quad (2)$$

and

$$-\eta_i \frac{\mathrm{d}}{\mathrm{d}y} I_{xr}(y, \mathbf{\Omega}_{i+M/2}) + \frac{\mu_i}{\alpha_h^r}\left[I_r(a_h^r, y, \mathbf{\Omega}_{i+M/2}) - I_r(a_{h-1}^r, y, \mathbf{\Omega}_{i+M/2})\right] + \beta_r I_{xr}(y, \mathbf{\Omega}_{i+M/2}) =$$
$$\frac{1}{4\pi}\Bigg\{\sigma_{s0}\sum_{n=1}^{M/2} w_n[I_{xr}(y,\mathbf{\Omega}_n) + I_{xr}(y,\mathbf{\Omega}_{n+M/2})] + 3\sigma_{s1}\Bigg[\mu_i \sum_{n=1}^{M/2}\mu_n w_n[I_{xr}(y,\mathbf{\Omega}_n) + I_{xr}(y,\mathbf{\Omega}_{n+M/2})] -$$
$$\eta_i\sum_{n=1}^{M/2}\eta_n w_n[I_{xr}(y,\mathbf{\Omega}_n) - I_{xr}(y,\mathbf{\Omega}_{n+M/2})]\Bigg] + 5\sigma_{s2}\Bigg[$$
$$\frac{1}{2}(3\xi_i^2-1)\sum_{n=1}^{M/2}\frac{1}{2}(3\xi_n^2-1)w_n[I_{xr}(y,\mathbf{\Omega}_n)+I_{xr}(y,\mathbf{\Omega}_{n+M/2})]-3\mu_i\eta_i\sum_{n=1}^{M/2}\mu_n\eta_n w_n[I_{xr}(y,\mathbf{\Omega}_n)-I_{xr}(y,\mathbf{\Omega}_{n+M/2})]+$$
$$\frac{3}{4}(\mu_i^2-\eta_i^2)\sum_{n=1}^{M/2}(\mu_n^2-\eta_n^2)w_n[I_{xr}(y,\mathbf{\Omega}_n) + I_{xr}(y,\mathbf{\Omega}_{n+M/2})]\Bigg]\Bigg\}, \quad (3)$$

for $i = 1, \ldots, M/2$ and $r = 1, \ldots, R$. The average intensity along the x direction in region r is defined by

$$I_{xr}(y, \mathbf{\Omega}_i) = \frac{1}{\alpha_h^r}\int_{a_{h-1}^r}^{a_h^r} I_r(x, y, \mathbf{\Omega}_i)\mathrm{d}x, \quad (4)$$

with $\alpha_h^r = a_h^r - a_{h-1}^r$ and $i = 1, \ldots, M$.

In a similar way, to obtain the one dimensional transverse integrated equations in x direction, we integrate Eq. (1) for all $y \in [b_{k-1}^r, b_k^r]$. Then, following Barichello *et al.* (2011), we associate indices $i = 1, \ldots, M/2$ to directions where $\mu_i > 0$ and indices $i = M/2+1, \ldots, M$ to directions with coordinates $\mu_i < 0$ and we obtain the average intensity along the y direction in region r: $I_{yr}(x, \mathbf{\Omega}_i)$.

From the integration process additional unknowns on contours of each region were introduced in the system, in the second term of the left hand side in Eqs. (2) and (3). From the boundary conditions of the problem these variables can be known for some (incoming) directions, whereas for other directions these terms have to be approximated. Thus, as usual in nodal schemes, auxiliary equations are required to approximate the unknown fluxes at the boundaries. In this work we express the unknown intensities in each region r by constant functions, although other approximations have been tested (Prolo Filho and Barichello, 2014).

2.1 A discrete-ordinates solution in a region r

From the application of nodal schemes in the two-dimensional radiation equation it is possible to obtain a system of ordinary differential equations, which we solve by the ADO method. For the y direction in region r we propose solutions of the homogeneous problem, that are constructed in terms of eigenvalues and eigenfunctions, as (Picoloto *et al.*, 2015)

$$I_{xr}^H(y, \mathbf{\Omega}_i) = \Phi_{xr}(\gamma_r, \mathbf{\Omega}_i)e^{-y/\gamma_r}, \quad (5)$$

for $i = 1, \ldots M$, with $r = 1, \ldots, R$, where γ_r is a separation constant in region r, associated with eigenfunction $\Phi_{xr}(\gamma_r, \mathbf{\Omega}_i)$. Substituting the Eq. (5) in the Eqs. (2) and (3) and extending previous work (Picoloto *et al.*, 2017), after some manipulations, we derive the eigenvalue problem

$$[\mathbf{A}_{xr}\mathbf{B}_{xr}]\,\mathbf{U}_{xr} = \lambda_{xr}\mathbf{U}_{xr}, \tag{6}$$

with

$$\lambda_{xr} = \frac{1}{\gamma_r^2}, \tag{7}$$

where $\mathbf{A}_{xr}$ and $\mathbf{B}_{xr}$ are $M/2 \times M/2$ matrices defined as

$$\mathbf{A}_{xr}(i,j) = \begin{cases} \dfrac{2}{\eta_i}\dfrac{1}{4\pi}\Big[3\sigma_{s1}\eta_i\eta_j w_j + 5\sigma_{s2}3\mu_i\eta_i\mu_j\eta_j w_j\Big] - \dfrac{\beta_r}{\eta_i}, & \text{if} \quad i=j, \\ \dfrac{2}{\eta_i}\dfrac{1}{4\pi}\Big[3\sigma_{s1}\eta_i\eta_j w_j + 5\sigma_{s2}3\mu_i\eta_i\mu_j\eta_j w_j\Big], & \text{otherwise,} \end{cases} \tag{8}$$

and

$$\mathbf{B}_{xr}(i,j) = \begin{cases} \dfrac{2}{\eta_i}\dfrac{1}{4\pi}\Big[\sigma_{s0}w_j + 3\sigma_{s1}\mu_i\mu_j w_j + 5\sigma_{s2}\Big[(3\xi_i^2-1)(3\xi_j^2-1)\frac{w_j}{4} + \frac{3}{4}(\mu_i^2-\eta_i^2)(\mu_j^2-\eta_j^2)w_j\Big]\Big] - \dfrac{\beta_r}{\eta_i}, & \text{if} \quad i=j, \\ \dfrac{2}{\eta_i}\dfrac{1}{4\pi}\Big[\sigma_{s0}w_j + 3\sigma_{s1}\mu_i\mu_j w_j + 5\sigma_{s2}\Big[(3\xi_i^2-1)(3\xi_j^2-1)\frac{w_j}{4} + \frac{3}{4}(\mu_i^2-\eta_i^2)(\mu_j^2-\eta_j^2)w_j\Big]\Big], & \text{otherwise,} \end{cases} \tag{9}$$

for $i = 1, ..., M/2$, $j = 1, ..., M/2$ and $r = 1, ..., R$. Thus, we can be write the homogeneous solution in the y direction, in a region r, as

$$I_{xr}^H(y, \mathbf{\Omega}_i) = \sum_{j=1}^{M/2}\Big[A_{j,r}\Phi_{xr}(\gamma_{jr}, \mathbf{\Omega}_i)e^{-(y-b_{k-1}^r)/\gamma_{jr}} + A_{j+M/2,r}\Phi_{xr}(\gamma_{jr}, \mathbf{\Omega}_{i+M/2})e^{-(b_k^r-y)/\gamma_{jr}}\Big], \tag{10}$$

$$I_{xr}^H(y, \mathbf{\Omega}_{i+M/2}) = \sum_{j=1}^{M/2}\Big[A_{j,r}\Phi_{xr}(\gamma_{jr}, \mathbf{\Omega}_{i+M/2})e^{-(y-b_{k-1}^r)/\gamma_{jr}} + A_{j+M/2,r}\Phi_{xr}(\gamma_{jr}, \mathbf{\Omega}_i)e^{-(b_k^r-y)/\gamma_{jr}}\Big], \tag{11}$$

for $i = 1, ..., M/2$; $x \in [a_{h-1}^r, a_h^r]$ and $y \in [b_{k-1}^r, b_k^r]$. Where, $A_{j,r}$ and $A_{j+M/2,r}$ are the coefficients of the homogeneous solution in region r.

In order to obtain the homogeneous solution by the ADO method to the problem in x direction we proceed analogously. The steps to obtain the eigenvalue problem in x will not be repeated here, but it is of the same type of the problem obtained in y. The matrices $M/2 \times M/2$ are defined by the expressions given in Eqs. (8) and (9) exchanging η_i by μ_i and vice-versa. Then, in a similar way, we can be write the solution of the homogeneous problem in the x direction, as $I_{yr}^H(x, \mathbf{\Omega}_i)$.

The approximation on the boundary for the unknown variables, wich appear in the second term of the left hand side in Eqs. (2) and (3), generates a source term representing the nonhomogeneous part of the problem, so a particular solution is required. Following the derivations presented in the work by Prolo Filho and Barichello (2013, 2014) we define the particular solution for the one-dimensional transverse integrated equations, by making use of Green's functions. Once the particular solution of the problem is established, we determine the general solution as the sum of the homogeneous solution with the particular solution. To establish the general solution for y direction we need to determine the arbitrary coefficients in Eqs. (10) and (11) and in the particular solution. Similarly, the arbitrary coefficients of the solution of the homogeneous problem and the particular solution in x direction also need to be determined to estabillish the general solution for x direction. Then, we consider the integrated form of the boundary conditions making use of the auxiliary equations as well as continuity conditions on the interface to generate a linear system, and thus obtain the coefficients. In this step, the coupling between the one-dimensional problems integrated in the variables x and y occurs.

3. NUMERICAL RESULTS

In order to establish numerical comparisons with results provided in the literature by Kim and Lee (1988) we assume as boundary conditions in two-dimensional geometry, Fig. 1, that the wall designated as "wall 3" is kept hot and their constant emissive power is fixed at unity, $E_{b3} = 1.0$, while the other walls and the medium are kept cold and have prescibed emissive powers of zero, $E_{b1} = E_{b2} = E_{b4} = 0$. The enclosure contains pure scattering medium, $\beta = 1$, $\kappa = 0$ and the wall reflectivity is zero. We consider a square enclosure with dimension $a = b = 1.0\ cm$.

In this work, we are interested in obtaining the average density radiation in some regions r of the domain, that is given by

$$\phi_{xr}(y) = \frac{1}{4} \sum_{n=1}^{M/2} w_n [I_{xr}(y, \mathbf{\Omega}_n) + I_{xr}(y, \mathbf{\Omega}_{n+M/2})]. \tag{12}$$

The ordinates set and their respective angular weights are obtained from the Level Symmetric quadrature scheme (LQ_N) (Lewis and Miller, 1984). Using the LQ_N for different orders, average density radiation data for degree of anisotropy $L = 2$ are generated and are compared with the results available in the literature reported by Kim and Lee (1988). Here, the square enclosure is subdivided into 16 nodes (4×4). In the work of Kim and Lee (1988) the solution of the radiative transfer equation was obtained through the S-N discrete ordinates method along with finite volume method, the domain was subdivided into 26×26 control volumes and the average density radiation was calculated using the S-14 approximation, we note, however, that is not possible to guarantee if the quadrature scheme is exactly the same.

In Tab. 1 we present the estimated values for the average density radiation, $\phi_{xr}(y)$, along the centerline $(x = 0.5)$ for various orders of the LQ_N and anisotropic scattering with degree $L = 2$. We also list the results of the average density radiation along the centerline generated by Kim and Lee (1988) using S-14 approximation. The values calculated in this study are similar, in general we observe one to two digits of agreement, with the results obtained by Kim and Lee (1988). For being consistent with the range our solution is applicable we use $\sigma_s = 0.999$ instead of 1. In addition to the fact that different methodologies, analitical and numerical, were used to deal with the transverse integrated equations, the fact that the quadrature scheme is possibly not the same also may cause some minor variation in the results.

Table 1. Average density radiation $\phi_{xr}(y)$ along the centerline $(x = 0.5)$ for anisotropic scattering with degree $L = 2$, for different quadrature order.

y	N = 4	N = 8	N = 12	N = 16	Kim and Lee (1988)
0.00	0.663846	0.662990	0.662654	0.662523	0.660444
0.02	0.644982	0.639907	0.637725	0.636468	0.632666
0.06	0.608667	0.596589	0.591757	0.589076	0.585263
0.10	0.574186	0.556800	0.550432	0.547135	0.538400
0.14	0.541485	0.520264	0.513213	0.509850	0.496928
0.18	0.510516	0.486748	0.479678	0.476628	0.461070
0.22	0.481234	0.456063	0.449503	0.447041	0.428843
0.26	0.448199	0.424048	0.418817	0.417295	0.398681
0.30	0.401524	0.382833	0.379889	0.379467	0.369998
0.34	0.357932	0.344500	0.343409	0.343712	0.342773
0.38	0.317358	0.308977	0.309388	0.310148	0.317122
0.42	0.279745	0.276217	0.277856	0.278895	0.293117
0.46	0.245041	0.246189	0.248864	0.250089	0.270764
0.50	0.213203	0.218883	0.222486	0.223888	0.249999
0.54	0.199646	0.207507	0.210118	0.210900	0.230725
0.58	0.186364	0.195618	0.197095	0.197237	0.212815
0.62	0.173394	0.183417	0.183717	0.183256	0.196136
0.66	0.160776	0.171086	0.170247	0.169261	0.180560
0.70	0.148546	0.158786	0.156910	0.155509	0.165955
0.74	0.136738	0.146665	0.143911	0.142233	0.152204
0.78	0.125102	0.133598	0.131129	0.129825	0.139191
0.82	0.113519	0.120498	0.118683	0.117885	0.126801
0.86	0.102065	0.107774	0.106608	0.106250	0.114907
0.90	0.090765	0.095412	0.094864	0.094870	0.103343
0.94	0.079643	0.083400	0.083411	0.083703	0.091917
0.98	0.068723	0.071725	0.072217	0.072707	0.080447
1.00	0.063345	0.066012	0.066708	0.067261	0.074333

4. CONCLUDING REMARKS

In this work, the ADO-nodal method was used to solve the two-dimensional radiative transport equation with anisotropic scattering. Different orders of the quadrature LQ_N were implemented and the degree of anisotropy considered was $L = 2$. The main goal was to extend the ADO methodology to deal with orders of anisotropy higher than one. From the analysis performed we verified that the values obtained in this study with coarser mesh are similar to the results found in the literature using a more refined mesh. Also, with the use of the ADO method we have analytical solution in the spatial variables for the integrated (average) quantities.

The generalization of the present approach for any degree of anisotropic scattering is under derivation and we aim to report results on that in future works.

5. ACKNOWLEDGEMENTS

The authors would like to thank CAPES and CNPq of Brazil for partial financial support of this work.

6. REFERENCES

Aghanajafi, C. and Abjadpour, A., 2016. "Discrete ordinates method applied to radiative transfer equation in complex geometries meshed by structured and unstructured grids". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 38, No. 3, pp. 1007–1019.

Barichello, L.B., Cabrera, L.C. and Prolo Filho, J.F., 2011. "An analytical approach for a nodal scheme of two-dimensional neutron transport problems". *Annals of Nuclear Energy*, Vol. 38, pp. 1310–1317.

Barichello, L.B., Picoloto, C.B. and da Cunha, R.D., 2017. "The ADO-nodal method for solving two-dimensional discrete ordinates transport problems". *Annals of Nuclear Energy*, Vol. 108, pp. 376–385.

Barichello, L.B. and Siewert, C.E., 1999. "A discrete-ordinates solution for a non-grey model with complete frequency redistribution". *Journal of Quantitative Spectroscopy & Radiative Transfer*, Vol. 62, pp. 665–675.

Bouaoun, M., Elloumi, H., Charrada, K., Rhouma, M.B.E.H. and Stambouli, M., 2005. "Discrete ordinates method in the analysis of the radiative transfer in high intensity discharge lamps". *Journal of Physics D: Applied Physics*, Vol. 38, No. 22, pp. 4053–4065.

Cacuci, D.G., 2010. *Handbook of Nuclear Engineering*. Springer Science & Business Media, New York.

Chandrasekhar, S., 1950. *Radiative transfer*. Oxford University Press, London.

Coelho, P.J., 2014. "Advances in the discrete ordinates and finite volume methods for the solution of radiative heat transfer problems in participating media". *Journal of Quantitative Spectroscopy & Radiative transfer*, Vol. 145, pp. 121–146.

Efremenko, D., Doicu, A., Loyola, D. and Trautmann, T., 2013. "Acceleration techniques for the discrete ordinate method". *Journal of Quantitative Spectroscopy & Radiative Transfer*, Vol. 114, pp. 73–81.

Fiveland, W.A., 1984. "Discrete-ordinates solutions of the radiative transport equation for rectangular enclosures". *Journal of heat transfer*, Vol. 106, No. 4, pp. 699–706.

Fiveland, W.A., 1988. "Three-dimensional radiative heat-transfer solutions by the discrete-ordinates method". *Journal of Thermophysics and Heat Transfer*, Vol. 2, No. 4, pp. 309–316.

Ganapol, B.D., 2015. "The response matrix discrete ordinates solution to the 1D radiative transfer equation". *Journal of Quantitative Spectroscopy & Radiative Transfer*, Vol. 154, pp. 72–90.

Godoy, W.F. and DesJardin, P.E., 2010. "On the use of flux limiters in the discrete ordinates method for 3D radiation calculations in absorbing and scattering media". *Journal of Computational Physics*, Vol. 229, No. 9, pp. 3189–3213.

Howell, J.R., Mengüç, M.P. and Siegel, R., 2016. *Thermal radiation heat transfer*. CRC Press, New York, 6th edition.

Hunter, B. and Guo, Z., 2011. "Comparison of the discrete-ordinates method and the finite-volume method for steady-state and ultrafast radiative transfer analysis in cylindrical coordinates". *Numerical Heat Transfer, Part B: Fundamentals*, Vol. 59, No. 5, pp. 339–359.

Kim, S.H. and Huh, K.Y., 1999. "Assessment of the finite-volume method and the discrete ordinate method for radiative heat transfer in a three-dimensional rectangular enclosure". *Numerical Heat Transfer: Part B: Fundamentals*, Vol. 35, No. 1, pp. 85–112.

Kim, T.K. and Lee, H., 1988. "Effect of anisotropic scattering on radiative heat transfer in two-dimensional rectangular enclosures". *International Journal of Heat and Mass Transfer*, Vol. 31, No. 8, pp. 1711–1721.

Lewis, E.E. and Miller, W.F., 1984. *Computational methods of neutron transport*. John Wiley & Sons, New York.

Liu, L.H., Ruan, L.M. and Tan, H.P., 2002. "On the discrete ordinates method for radiative heat transfer in anisotropically scattering media". *International Journal of Heat and Mass Transfer*, Vol. 45, No. 15, pp. 3259–3262.

Modest, M.F., 2003. *Radiative heat transfer*. Academic Press, 2nd edition.

Picoloto, C.B., da Cunha, R.D., Barros, R.C. and Barichello, L.B., 2017. "An analytical approach for solving a nodal formulation of two-dimensional fixed-source neutron transport problems with linearly anisotropic scattering". *Progress in Nuclear Energy*, Vol. 98, pp. 193–201.

Picoloto, C.B., Tres, A., da Cunha, R.D. and Barichello, L.B., 2015. "Closed-form solutions for nodal formulations of two dimensional transport problems in heterogeneous media". *Annals of Nuclear Energy*, Vol. 86, pp. 65–71.

Prolo Filho, J.F. and Barichello, L.B., 2013. "An analytical discrete ordinates solution for a nodal model of a two-dimensional neutron transport problem". In *Proceedings of the 23rd International Conference on Mathematics & Computational Methods Applied to Nuclear Science & Engineering*. Sun Valley, USA, pp. 2350–2360.

Prolo Filho, J.F. and Barichello, L.B., 2014. "General expressions for auxiliary equations of a nodal formulation for two-dimensional transport calculations". *Journal of Computational and Theoretical Transport*, Vol. 43, No. 1-7, pp. 352–373.

Wick, G.C., 1943. "über ebene Diffusionsprobleme". *Zeitschrift für Physik*, Vol. 121, No. 11-12, pp. 702–718.

Zabihi, M., Lari, K. and Amiri, H., 2017. "Coupled radiative-conductive heat transfer problems in complex geometries using embedded boundary method". *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 39, No. 7, pp. 2847–2864.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.