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THEORETICAL ANALYSIS OF THE TEMPERATURE DYNAMICS FOR DIFFERENT CONCENTRATIONS OF THE WATER-ALCOHOL MIXTURE INSIDE A CYLINDRICAL TUBE

Pedro Granville Gonçalves
Dhiego Luiz de Andrade Veloso
Universidade Federal da Paraíba, Cidade Universitária - João Pessoa - PB - Brasil - CEP: 58051-900
pedrogranville@gmail.com
dhiego_vel@hotmail.com

Fábio Araújo de Lima
Universidade Federal da Paraíba, Cidade Universitária - João Pessoa - PB - Brasil - CEP: 58051-900
carloscabralsantos@yahoo.com.br

Carlos Antônio Cabral dos Santos
Universidade Federal da Paraíba, Cidade Universitária - João Pessoa - PB - Brasil - CEP: 58051-900
carloscabralsantos@yahoo.com.br

Francisco Antônio Belo
Universidade Federal da Paraíba, Cidade Universitária - João Pessoa - PB - Brasil - CEP: 58051-900
franciscoantoniobelo@gmail.com.br

Abstract. *This paper investigates heat transfer by conduction inside a circular tube filled with a fluid composed of different fractions of the water-ethyl alcohol mixture. The energy equation, which governs the problem, was solved by the generalized integral transform (GITT) technique, which presents itself as a well-established method for solving problems of heat and mass diffusion. With the aid of the engineering equation solver (EES) software, we obtained the thermodynamic properties of the studied fluid through pre-defined mathematical functions. The results presented in the form of graphs and tables allow us to analyze the influence of different concentrations of the studied fluid on the development of the temperature field.*

Keywords: *Mixture water-alcohol, temperature field, GITT.*

1. INTRODUCTION

Mixtures containing water and an amphiphilic and / or hydrophobic solute have been extensively studied in recent years. A brief bibliographic review can be found in (AKPA *et al*., 2012; HILSER, 2011; BALL, 2008 and PALO *et al*., 2007). The applications of this study play an important role in many biological, chemical and engineering applications, examples include protein folding, membrane self-assembly, electron transfer reactions, heterogeneous catalysis and fuel cell technology (LI *et al*. 2014).

The thermophysical properties of solutions containing water-ethyl alcohol have great influence as a function of their temperature. Temperature is one of the most measured quantities during industrial processes. Temperature monitoring and control is applied in manufacturing processes such as in the medical, chemical and food industries. Because it is a parameter capable of guaranteeing the quality of a product, temperature is a physical indicator of extreme relevance. From the use of sensors and actuators it is possible to control the amount of heat of a body or substance in a given space or region in order to maintain the temperature at a desired value (BELO, 2013 and 2014; NASCIMENTO, 2004).

In the present work, we approach the GITT solution of a heat - radial and transient diffusion formulation - describing the behavior of temperature dynamics inside a cylindrical tube filled with volumetric fractions of 10, 20, 30, 40, 50 and 60 percent of ethyl alcohol mixed with water. The theoretical values of thermal properties, adopted as parameters for computational simulation, such as: thermal conductivity, specific mass, specific heat and thermal diffusivity, were extracted from the engineering equation solver (EES) software, since the alcohol content also exerts a great interference on the thermophysical properties of alcohol-water mixtures (PEREZ, 2013; WAKISAKA, 2011).

2. PHYSICAL SYSTEM AND MATHEMATICAL MODELING

2.1 Model assumptions

In order to simplify the hydrodynamic model, the following considerations were adopted:

- Heat conduction equation in cylindrical coordinates;
- System is insulated, analysis neglects losses effects to environment;
- Initially in thermal equilibrium;
- Temperature gradients in longitudinal and angular directions weren´t took into account;
- Constant thermal properties;
- Cylinder’s length is considered much longer than its radio.

2.2 Matematical modeling

Energy equation:

$$\rho c_p \frac{\partial T(r,t)}{\partial t} = k \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T(r,t)}{\partial r}\right) \quad (1)$$

Boundary conditions:

$$r = 0 \;\rightarrow\; \frac{\partial T(r,t)}{\partial r} = 0 \quad (2)$$

$$r = r_0 \;\rightarrow T(r,t) = T_\infty \quad (3)$$

Initial condition:

$$t = 0 \;\rightarrow\; T(r,0) = T_0 \quad (4)$$

2.2.1 Dimensionless form

The dimensionless groups adopted for the model are:

$$R = \frac{r}{r_0} \;; \qquad \tau = \frac{\alpha t}{r_0^2} \;; \qquad \theta(R,\tau) = \frac{T(r,t)-T_\infty}{T_0-T_\infty} \quad (5\text{-}7)$$

Energy equation dimensionless:

$$\frac{\partial\theta(R,\tau)}{\partial\tau} = \frac{1}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,\tau)}{\partial R}\right] \quad (8)$$

Boundary conditions dimensionless:

$$R = 0 \;\rightarrow\; \frac{\partial\theta(R,\tau)}{\partial R} = 0 \quad (9)$$

$$R = 1 \;\rightarrow \theta(R,\tau) = 0 \quad (10)$$

Initial condition dimensionless:

$$\tau = 0 \;\rightarrow \theta(R,\tau) = 1 \quad (11)$$

2.2.2 Auxiliay eigenvalue problem in the radial direction

The auxiliary problem for the temperature field can be described by a system of second order differential equations, representing a classic Sturm-Liouville problem. The auxiliary problem chosen for the determination of the temperature field is written as follows:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{R d\psi_i R}{dR}\right) + \mu_i^2\psi_i(R) = 0 \quad (12)$$

$$R = 0 \rightarrow \frac{d\psi_i(R)}{d(R)} = 0 \quad (13)$$

$$R = 1 \rightarrow \psi_i(R) = 0 \quad (14)$$

The signal count method developed by (MIKHAILOV and VULCHANOV, 1983) was used for determine the eigenvalues (μ_i), the eigenfunctions, $\psi_i(R)$, and the norms ($N_i = \int_0^1 R\psi_i^2(R)dR$).

2.2.3 Integral transformation of the temperature field

GITT is based on the fact that a function can be written as an expansion in terms of eigenfunctions (COTTA, 1993). Thus, the transformed-inverse pair is given by:

$$\bar{\theta}_i(\tau) = \frac{1}{N_i^{1/2}} \int_0^1 R\psi_i(R)\theta(R,\tau)dR \quad (15)$$

$$\theta(R,\tau) = \sum_{i=1}^{\infty} \frac{\psi_i(R)\bar{\theta}_i(\tau)}{N_i^{1/2}} \quad (16)$$

After analytical treatment of Eq. (12) by integral operators, using the definition of the integral transform given by Eq. (17), as well as using the auxiliary problem defined by Eq. (14), (15) and (16), we can transform problem in the following ODE system:

$$\frac{d\bar{\theta}_i(\tau)}{d\tau} + \mu_i^2 \bar{\theta}_i(\tau) = 0 \quad (17)$$

This ODE has classic solution given by:

$$\bar{\theta}_i(\tau) = \bar{\theta}_i(0) e^{-\mu_i^2 \tau} \quad (18)$$

Such that:

$$\bar{\theta}_i(0) = \int_0^1 \frac{R\psi_i(R)}{N_i^{1/2}} dR = \bar{f}_i \quad (19)$$

Using Eq. (18) and (16), it is verified that the temperature field for the studied region takes the form:

$$\theta(R,\tau) = \sum_{i=1}^{\infty} \frac{\psi_i(R)\bar{f}_i e^{-\mu_i^2 \tau}}{N_i^{1/2}} \quad (20)$$

3. RESULTS

The mathematical model was implemented using computer code written in FORTRAN programming language to obtain the results. For the analysis of the convergence of results, 200 eigenvalues and 200 corresponding eigenfunctions were used. For the analysis performed in the present work we consider a tube with an internal radius of $4x10^{-3}$ m and that the water-alcohol mixtures have an initial temperature of 313.15 K and are cooled to 293.15 K.

The table below contains the values of the thermophysical properties of the water-ethanol blend provided by the software (EES). In the temperature range evaluated in the present work, from 293.15 K to 313.15 K, a small variation in the values of the parameters was noticed, so that we consider them constant from an average value.

Table 1. Thermophysical properties values for different fractions of the water-alcohol mixture obtained with the software (EES).

Alcohol concentration [%]	k[W/m.K]	ρ[kg/m^3]	Cp[10^3 J/kg.K]	α[m^2/s]
10	0,5377	979,3	4,287	1,28x10^{-7}
20	0,4731	965,2	4,323	1,13x10^{-7}
30	0,4155	949,2	4,233	1,03x10^{-7}

Alcohol concentration [%]	k[W/m.K]	ρ[kg/m^3]	Cp[10^3 J/kg.K]	α[m^2/s]
40	0,3643	929,7	4,064	9,64x10^{-8}
50	0,3189	907,6	3,871	9,08x10^{-8}
60	0,2788	884,8	3,606	8,74x10^{-8}

The graphs depicted in Fig. 1-6 analyze the development of the thermal field for the different concentrations of the water-ethyl alcohol mixture. We can see that as we increase the alcohol concentration, it takes more time for the mixture to reach a temperature of 293.15 K. This fact can be justified by the decrease in thermal diffusivity values as the alcohol concentration increases, causing the low diffusivity to delay heat transfer.

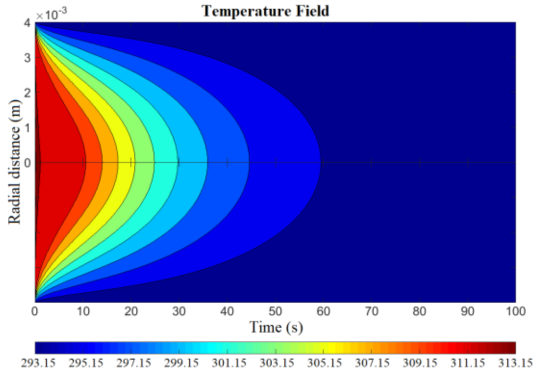

Figure 1. Evolution of the temperature field for the concentration of 10% alcohol.

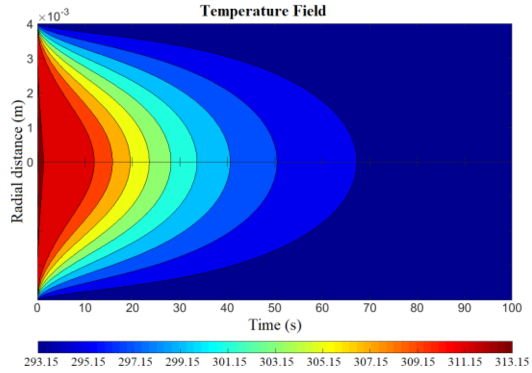

Figure 2. Evolution of the temperature field for the concentration of 20% alcohol.

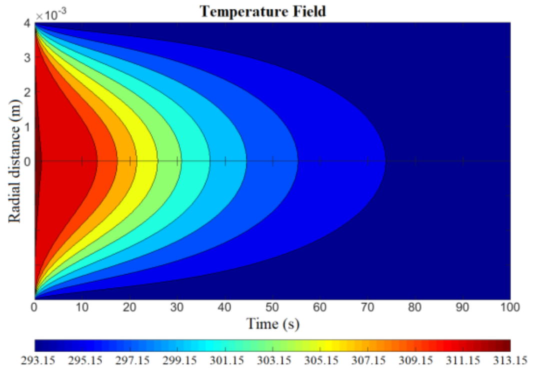

Figure 3. Evolution of the temperature field for the concentration of 30% alcohol.

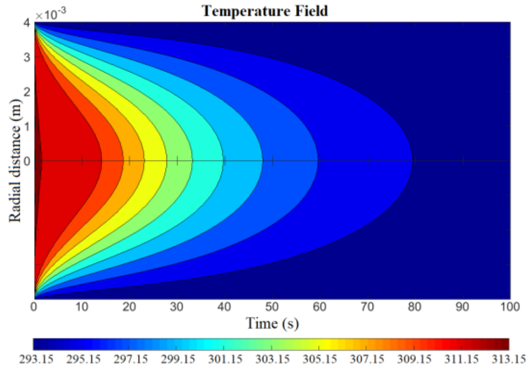

Figure 4. Evolution of the temperature field for the concentration of 40% alcohol.

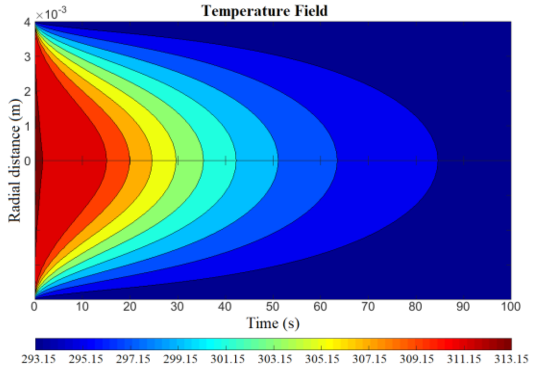

Figure 5. Evolution of the temperature field for the concentration of 50% alcohol.

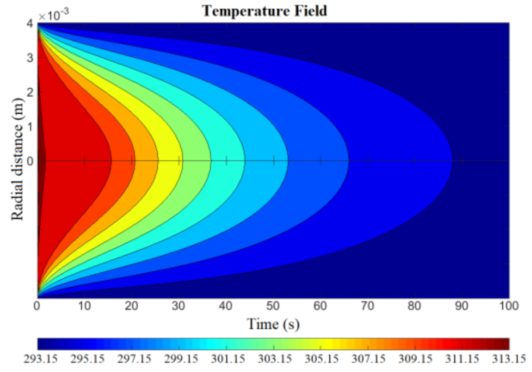

Figure 6. Evolution of the temperature field for the concentration of 60% alcohol.

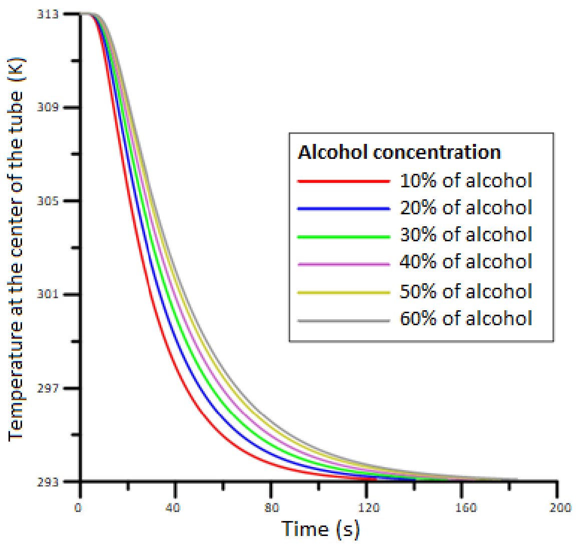

Figure 7. Temperature in the center of the tube as a function of time for different concentrations of alcohol.

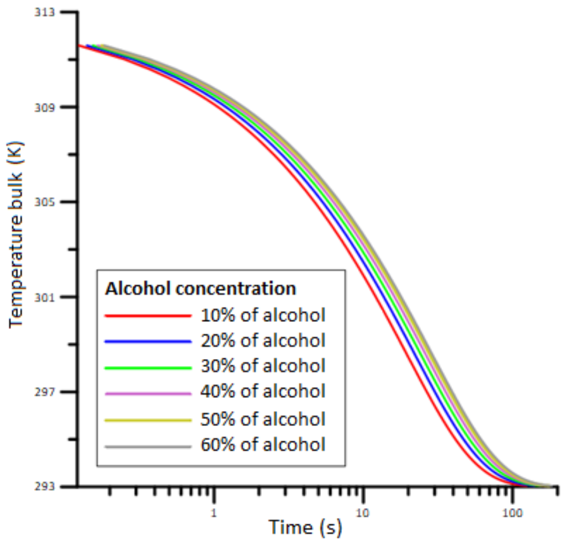

Figure 8. Bulk temperature versus time for different concentrations of alcohol.

In Fig. 7 and 8, we present the temperature bulk and temperature in the center of the tube graphs for the various fractions of water-ethyl alcohol. It can be seen that the temperature rapidly decreases with lower alcohol concentration.

4. CONCLUSIONS

In this work, it is concluded from the analysis of the results obtained that the application of GITT is effective in solving the proposed problem. In this way, the objectives were satisfactorily achieved, showing the influence of the alcohol on the temperature dynamics of the mixture. For each case analyzed it was possible to observe the time of thermal development, as well as the variations of the bulk temperatures and at the center of the tube. The results showed that it is possible to detect changes in the development of the thermal field when variations occur in the concentration values of the water-alcohol mixture. The data show the potentiality of the method used in the characterization of liquid mixtures.

5. ACKNOWLEDGEMENTS

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