COMPARISON OF DIFFERENT EXPERIMENTAL DESIGNS, SAMPLE SIZES AND TECHNIQUES APPLIED TO METAMODELING THE INDUCED DRAG COEFFICIENT OF A BOX-WING AIRCRAFT

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Abstract: *This paper compares different metamodeling techniques applied to the design process of a box-wing aircraft, and also different space-filling designs and sample sizes. The metamodels were created in order to establish a relationship between different input parameters - such as lift coefficient, aspect ratio, taper ratio and the separation gap between the upper and lower wings - and the resulting induced drag coefficient. For this comparative work, an existing aerodynamic database of a box-wing aircraft, obtained with a low-order aerodynamic code (vortex lattice method) was used, allowing rapid evaluations. The design points were created with space-filling designs such as Sobol, Latin Hypercube and some variations, and the different metamodels were compared using the mean absolute error as metric. By comparing the techniques with CCD, which is a classic method for experiments, it was possible to examine the difference between a "modern" proposal for computational experiments and a classic formulation for experimental techniques. The main objective of this work is to initiate an analysis that correlates experimental designs with metamodeling accuracy, and it is expected that the obtained results will help to build a guide for future works in the development of box-wing aircraft, providing a benchmark for computational experimental techniques in this particular field.*

Keywords: *aerodynamics, metamodeling, box-wing, experimental design, computational experiments.*

1. INTRODUCTION

The increasing utilization of computational simulations in both the industrial and academic fields allowed engineering projects to reach greater levels of performance and reliability due to improved analysis and optimization. However, despite these improvements, there are still problems in conducting simulations within MDO (multidisciplinary design optimization) loops. Issues such as computational cost may render optimization processes unfeasible. MDO frameworks that consume more than one day per round are not uncommon according to (Santner *et al.*, 2010; Lyu *et al.*, 2014).

One possible way to solve such problems is to use metamodeling techniques (Corman & German, 2010; Forrester & Keane, 2008; Simpson *et al.*, 2001). The purpose of a metamodel is to obtain a simplified mathematical function that relates input to output data, avoiding the use of higher order computational simulations in the initial design phases. Its utilization is becoming common within multidisciplinary optimization loops and can significantly reduce the execution time.

The metamodeling process can be summarized in the flowchart of Fig. 2. An initial sample of design points – vectors containing different values of input data – is generated with a space-filling design. This part of the process is known as Design of Experiments (DOE). The design points are used in computational simulations based on true models, and the results are then processed with metamodeling techniques, such as kriging or evolutionary design. The resulting mathematical relations are the metamodels.

Although the values obtained from computational simulations are considered deterministic, the design of experiments does not employ classical techniques, like blocking and replication. Instead, different methods created for this purpose are used, such as techniques based on space filling designs, according to Santner *et al.* (2010).

Despite the large number of studies being conducted on metamodeling and metamodel based optimization, there is still no consensus about the adequate size of the initial sample in order to generate a properly-fitting metamodel. However, it is known that the number of samples required to construct good metamodels is exponentially dependent on the number of input variables and on the employed technique, but according to Corman and German (2010) when this number becomes very large, non-trivial responses are obtained.

The objective of this work is to study several DOE techniques as well as the number of samples required and to compare them in order to obtain the best relation of computational cost (number of points used to construct the surrogate) and accuracy of metamodels relating input parameters to the induced drag coefficient (C_{Di}). The experimental designs assessed in this work are widely used in the context of multidisciplinary design optimization, not only for obtaining metamodels, but also in other situations, for example the generation of an initial population for genetic algorithms.

In addition to basic space-filling techniques such as the Latin hypercube, Sobol and Random, the CCD method was also applied. By comparing the techniques with CCD, which is a classic method for experiments, it was possible to examine the difference between a "modern" proposal for computational experiments and a classic formulation for experimental techniques.

The different sample sizes analyzed follow the relations proposed by several references, according to Tab. 1, in which NI represents the number of input parameters.

Table 1. Analyzed sample sizes.

Sample Size	Reference
(4/3)*NI	Manache & Melching (2007)
2*NI	Chang *et al.* (1993)
10*NI	(Chapman *et al.*, 1994; Loeppky *et al.*, 2009)

Although the sample sizes analyzed in the references refer to the Latin hypercube design, these values were also employed in this work to evaluate the other selected methodologies. Several metamodel classes were trained – Kriging, anisotropic Kriging, DACE-Kriging, radial basis functions, neural networks and evolutionary design (all of them implemented in Esteco's Mode Frontier®) – and evaluated according to the mean absolute error criterion and the physical coherence when compared to the results exposed in Ribeiro (2017).

2. EXPERIMENTAL ANALYSIS PROCEDURE

The experiment involves the aerodynamic analysis of a box-wing aircraft (Fig. 1) with a low order Vortex Lattice method proposed by Drela and Youngren (2017). The chosen configuration, as well as the proposed aerodynamic analysis was based on the work of Ribeiro (2017), in which the feasibility of such aircraft is analyzed. Further details on the aerodynamic analysis and database creation, which are not the main objective of this work, can be found in this reference.

Figure 1. Example of a box-wing aircraft – Ribeiro (2017).

The use of such a lower order method for the aerodynamic analysis allows a larger number of simulations to be conducted and greatly simplifies the whole process. The automation of the experiment generation process and the aerodynamic analysis of a given configuration were conducted in Esteco's ModeFrontier. Figure 2 shows an overview of the workflow used for the study.

As shown in Fig. 2, the software automatically generates the design table, which are in turn analyzed in the aerodynamic code. The results are used to train the metamodels, which are then compared according to the MAE metric. It becomes possible to relate space-filling designs, number of samples and quality of the metamodels.

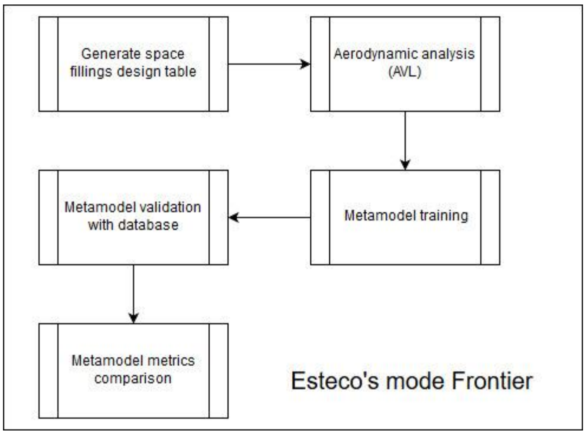

Figure 2. Workflow overview.

The process of analysis was structured according to the design. For each design a different run was planned, in which all metamodeling techniques were applied, each resulting in a different metamodel. Also, different sample sizes were tested in different runs. Table 2 shows the experimental planning, with the selected designs and sample sizes.

Table 2. Experimental planning, with the selected design and sample size for each run.

Run	Design	Sample size
1	ULH	(4/3)*NI
2	SOBOL	(4/3)*NI
3	ULH	2*NI
4	SOBOL + Random	2* (4/3)*NI
5	ULH	10*NI
6	CCD	25
7	Random	10*NI
8	SOBOL	(4/3)*NI
9	SOBOL	10*NI
10	Full Factorial	81 (3 levels per variable)

The selected input parameters for the models were: wing taper ratio (tip chord divided by root chord), design lift coefficient (C_L), wing aspect ratio (wing span squared and divided by wing planform area), and the ratio between wing vertical spacing and wing span (H/B).

3. RESULTS AND DISCUSSION

In each run, the best metamodel was selected according to the MAE criterion, that is, within each run, the best metamodel is the one with the lowest MAE. The results of the planned runs are shown in Tab. 3.

Table 3. Best metamodels for each run, with respective MAE.

Run	Best-fit metamodeling technique	MAE
1	Evolutionary design	7,26e-4
2	DACE Kriging	1,02e-3
3	DACE Kriging	5,57e-4
4	DACE Kriging	4,80e-4
5	DACE Kriging	3,45e-4
6	Evolutionary design	6,34e-4
7	Evolutionary design	9,23e-3
8	Evolutionary design	3,47e-3
9	DACE Kriging	8,43e-4
10	Kriging	3,37e-4

Figure 3 shows results obtained in run 1 for two different metamodeling techniques: DACE Kriging and evolutionary design, which presented better results. Both charts show an induced drag coefficient versus induced drag coefficient relationship. The blue points represent the baseline data, from Ribeiro (2017), related to itself. The red points relate the baseline data (x axis) to data obtained from the metamodel.

A graphical analysis can evidence the disparity in the results shown in Tab. 3 if Fig. 3 is compared with Fig. 4, which shows the best-fitting metamodel for run 2, where a larger number of design points were employed, almost seven times larger than that of run 1.

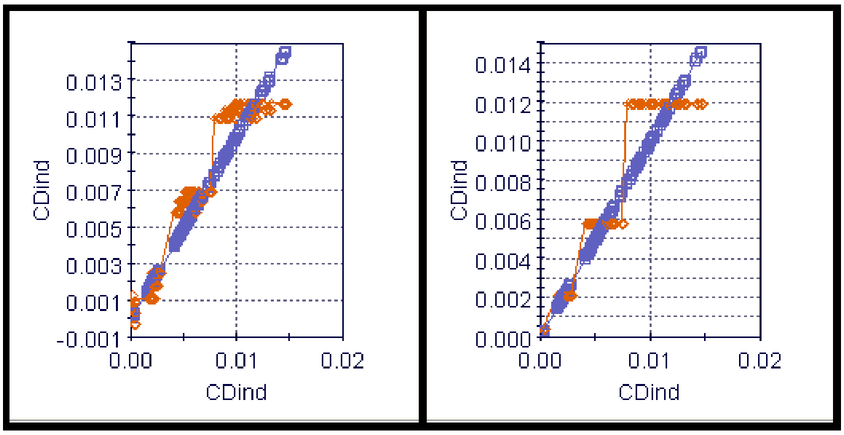

Figure 3. Results for DACE Kriging (left) and evolutionary design (right) in run 1.

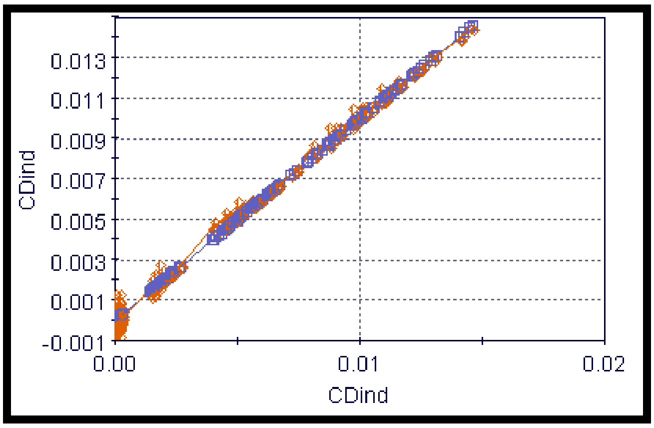

Figure 4. Results for the best fitting metamodel in run 2 (DACE Kriging).

In addition to the MAE evaluation, the residuals were analyzed to verify if they were normally and independent distributed. Figure 5 shows the residuals obtained from run 2, evidencing their distribution.

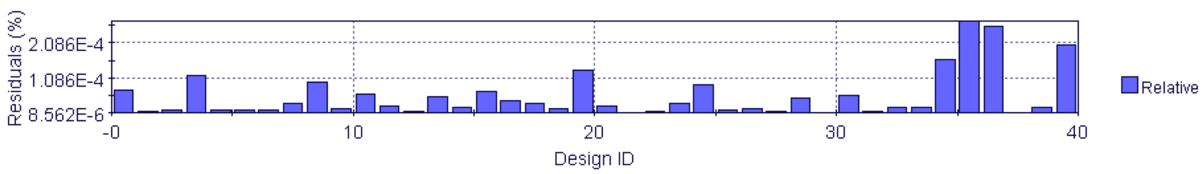

Figure 5. Relative residuals from run 2.

Although it was not a technique originally intended to computational experiments, the CCD presented good results when compared to the others, as can be seen in Fig. 6 and Fig. 7. This is likely due to the fact that the number of design points was significantly higher than those of the other runs.

The best fitting metamodel of all was obtained in run 10, with the Kriging technique. Figure 7 shows the resulting curves of the metamodel, relating the output (C_{Di}) to the selected inputs. The residuals of this technique in run 10 are shown in Fig. 8, where one can see their independence and normal distribution. It is worth noting that the curves are coherent with physical observations. For example, an increase in the aspect ratio reduces the influence of the wing wake and wingtip vortices along the wing itself, resulting in a lower induced drag coefficient, as can be seen in Fig. 7. It is also worth noting that when the separation gap becomes too low, the induced drag coefficient increases.

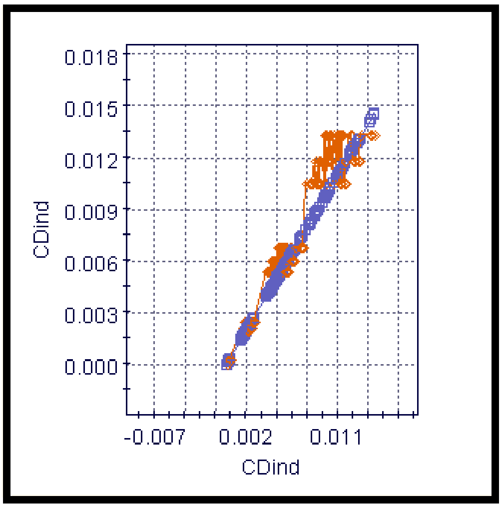

Figure 6. Results for the evolutionary design metamodel in run 6 (CCD Design).

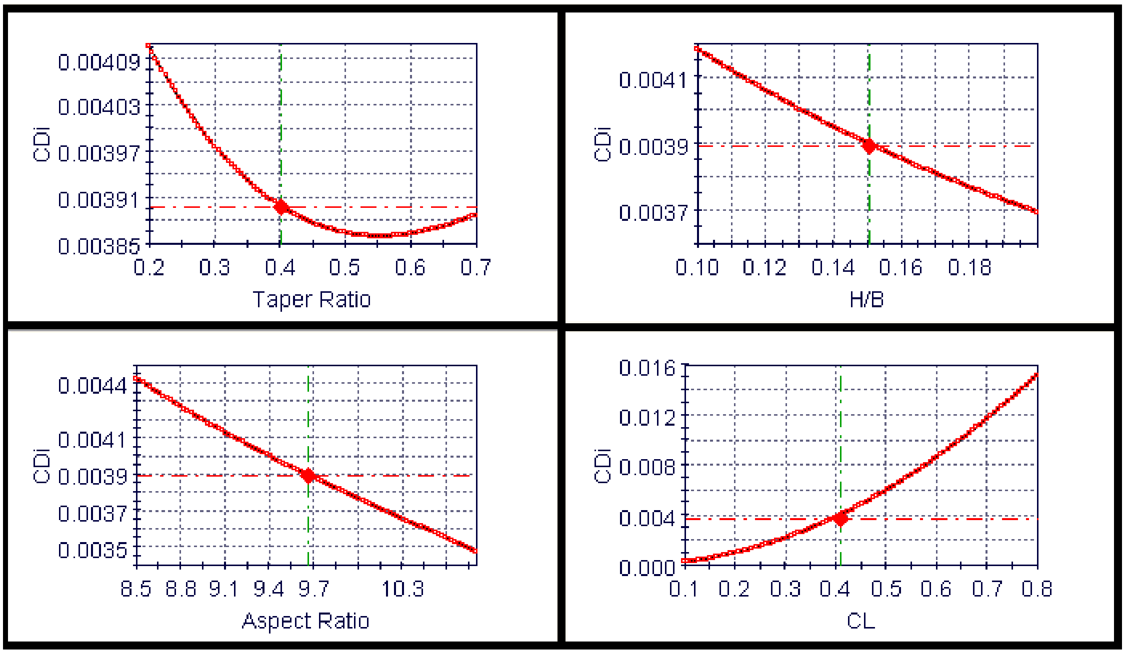

Figure 7. Results for the Kriging metamodel obtained in run 10: induced drag coefficient in function of different input parameters of the box-wing design.

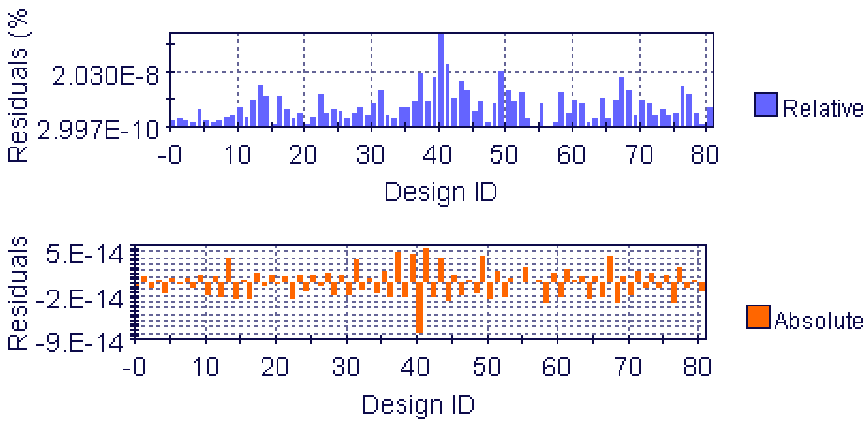

Figure 8. Relative residuals from the Kriging metamodel from run 10.

The correlation between input and output variables is shown in Fig. 9, with the Pearson correlation matrix. The strongest correlation is observed between the aspect ratio and the output (C_{Di}), in an inverse fashion, that is, an increase in aspect ratio leads to a decrease in the induced drag coefficient. The vertical separation ratio (H/B) comes in second, also in an inverse fashion, and the taper ratio is the least correlated value, yet still significant. These correlations are coherent with the results of the finite wing theory. It is also worth noting that the input variables are not correlated as can be seen by the null terms in the matrix. This is also coherent with the theory, since the input parameters do not interfere in each other, and can be independently chosen.

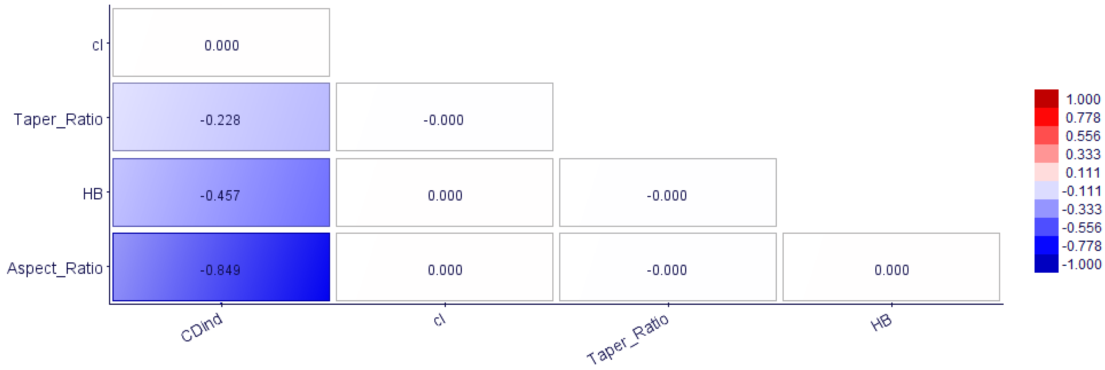

Figura 9. Pearson correlation matrix for the Kriging metamodel from run 10.

4. CONCLUSIONS

By observing the obtained data, one can notice that the values proposed by (Manache and Melching, 2007; Chang *et al.*, 1993) resulted in higher values of MAE, which is coherent with the fact that the resulting metamodels were based on less data points.

Traditional experimental design techniques, namely CCD and full factorial showed good results regarding the MAE. In the CCD case, this result can be attributed to the greater number of points employed, compared to the other techniques, as well as to the CCD method itself, which uses greater spacing between the points, leading to a behavior similar to the space filling technique. The full factorial design, which resulted in the metamodel with the best MAE, employed a large number of data points, twice as large as the design used in run 5, which resulted in the second best metamodel.

Therefore best computational performance however was obtained with the ULH design, employed according to (Chapman *et al.*, 1994; Loeppky *et al.* 2009), and the Kriging metamodeling technique, in run 5. The resulting metamodel shows the second lowest MAE, close to the one obtained with the full factorial design in run 10, but employed only half of the points.

The authors were not able to reach a defining conclusion regarding which sample size should be used in the metamodeling process. One can conclude that this sample size is very dependent on the nature of the modeled process, and its context within a given project. One must also take into account the available data and the easiness of working with a large sample size. In initial design phases, such as conceptual design, the required level of accuracy is lower, so it is possible to employ smaller samples for the sake of speed. However, in more advanced stages in the development of an airplane, like detailed design, the required accuracy is significantly higher, so the sample sizes must be also be increased.

5. REFERENCES

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6. AUTHORAL RESPONSIBILITY

The authors are solely responsible for the content of this work.