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SENSITIVITY OF THE SIZE AND POSITION OF PIEZOCERAMICS IN THE STRUCTURAL VIBRATIONS PASSIVE CONTROL

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Abstract: *Piezoelectric materials have been extensively studied in recent years for the development of electromechanical harvesting devices. Therefore, this work presents a numerical modelling technique, known as the Finite Element Method, for the analysis and optimization of the best positioning and size of piezoelectric ceramics (PZT5H), which are acting on the control of structural vibrations of a crimped-free beam. These ceramics are connected to a resonant shunt circuit to act in the passive control of the structure, in order to dissipate the generated electric energy coming from the structural vibrations of the beam, where the ceramics are coupled. With the analysis of the frequency responses of the generated systems, for each size variation of the piezoelectric ceramics, it is possible to determine the most efficient size of the ceramics in questions of vibration suppression efficiency. Thus, it was possible to optimize the inductance and resistance values of the electrical circuit associated to the ceramic pad, through a genetic algorithm, to maximize the passive control effect of the system. As a parameter of comparison, a normalization of the output data using the area of the piezoelectric ceramics was done, which demonstrates that the cost function of the optimization does not increase linearly. Therefore, the optimal size and positioning values were found.*
Palavras-chave: *Vibration control; piezoelectric materials; optimization; energy harvesting.*

1. INTRODUCTION

The vibrations to which the structures are subjected are often an undesirable factor, as they can cause several negative effects in structures, such as noise and a decrease in the mechanical resistance due to fatigue efforts. However, studies made (Trindade e Maio, 2006; Trindade, Benjeddou and Ohayon, 2001), show that it is possible to obtain a satisfactory control of these vibrations through systems that use piezoelectric materials, with passive, active or active-passive techniques, in order to attenuate the vibration peaks and avoid their negative effects. The active techniques are characterized by the use of actuators to carry out the control. On the other hand, passive techniques utilize only sensors that absorb part of the deformation energy, which is later dissipated by some dissipation mechanism.

According to Clark et al. (1998), piezoelectricity is the property that certain materials have: they deform mechanically in the presence of an electric field, and also in the opposite case, they produce an electric charges when mechanically deformed. Therefore, it is possible to use these types of materials in the absorption of mechanical energy associated with vibrations, since this energy will be converted into electrical energy, which can be stored or dissipated through some types of circuits.

Among the types of circuits that are connected to the piezoelectric patches, in order to dissipate the energy, the resonant type, formed by an inductor and a resistor, stand out for having the capacity to act in several frequency bands. This fact makes possible for this type of circuit, to damp one or more vibration modes by simply tuning on the wanted frequency. The performance of piezoelectric patches for these types of applications are very much dependent on the adequate tuning between resonant, circuit and operation frequencies and on the effective electromechanical coupling between patches and host structure.(Santos and Trindade, 2012; Godoy and Trindade, 2012).

Hence, the objective of the work is to present an analysis of the Euler-Bernoulli beam model as a function of size and positioning in the system to get the best control efficiency of vibrations and performance piezoelectric sensors and actuators with application to passive damping and energy harvesting.

2. EQUATIONS OF THE SYSTEM

For the study of the structure was adopted the classic beam model (piezoceramic-host).(Santos 2008). In this case was considered only the presence of flexion, disregarding the shear, thus Bernoulli Euler theory can be developed the equation of the movement for the structure. For the analysis of the output mechanical the resistance and inductance were optimization through on genetic algorithm in function of the tension applied. Were is applied only in first mode of the vibration.

2.1 Equation of movement

With the applied theory of the Bernoulli can be developed the equation of movement for the structure, this form the structure-patches-circuits coupled equations of motion can be written as

$$\begin{bmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{M}_q \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{D}}_p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{C}_q \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{D}}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_m & -\bar{\mathbf{K}}_{me} \\ -\bar{\mathbf{K}}_{me}^t & \bar{\mathbf{K}}_e \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{D}_p \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{F}_q \end{Bmatrix}, \tag{1}$$

where $\mathbf{M}_q$ is the inertial vector due to the presence of resistance and inductance, $\mathbf{u}$ and $\mathbf{D}_p$ are the vectors global mechanical displacement and electric displacement dofs. $\mathbf{M}$, $\mathbf{K}_m$, $\bar{\mathbf{K}}_{me}$, $\bar{\mathbf{K}}$ are the mass and mechanical, piezoelectric and dielectric stiffness matrices and $\mathbf{F}$ is the mechanical excitation force vector. $\mathbf{C}_q$ and $\mathbf{F}_q$ are the matrix of the damping and the vector of force dues the presence of resistance and inductance, but how in this work is studded only the output mechanical the value of the vector of electric voltage is equal zero.

2.2 Analysis of the equations for harmonic vibrations

For analysis of harmonic vibration, the proposed model (Santos 2008) is used to evaluate the mobility (velocity/force) frequency response function of the base structure. The resistive (R) or resonant (RL) shunt circuit affects both the passive control performance.In this way, it became necessary to use the circuit that will dissipate the energy or to storage for later use.

How this work analyze a purely mechanical excitation, such as $\mathbf{F}_q = 0$ and F = $bfe^{j\omega t}$, the amplitude of a displacement output y = c_pu can be written as $y = G\,(\omega)\,f$, where the FRF $G(\omega)$ is

$$G(\omega) = c_p\{(-\omega^2 M + K_m - \bar{K}_{me}(\omega^2 M_q + i\omega C_q + \bar{K}_e)^{-1}\bar{K}_{me}^t)\}^{-1} b \tag{2}$$

Analyzing the equation 2 it can be noted that the resistance and the inductance have the capacity to change the rigidity properties of the piezoelectric material, in this way it will be applied to the case types i) open-circuit when R_c tending to infinity and ii) short-circuit when $Lc = Rc = 0$. For the open circuit it has

$$G^{oc}(\omega) = c_p\{-\omega^2 M + K_m\}^{-1} b \tag{3}$$

To the closed circuit

$$G^{sc}(\omega) = c_p\{-\omega^2 M + [K_m - \bar{K}_{me}\bar{K}_e^{-1}\bar{K}_{me}^t]\}^{-1} b \tag{4}$$

You may note that no structural modification is observed in the open circuit box, whereas in the case of a short circuit, the rigidity of the piezoelectric patches is reduced.

2.3 Vibration Control using piezoelectric actuators and state feedback

This way is necessary to rewrite the motion equations in the form of state space, containing the displacements and modal velocities of the piezoelectric patches and their derivatives of time.

$$\dot{\mathbf{z}} = \hat{\mathbf{A}}\mathbf{z} + \hat{\mathbf{B}}\mathbf{V}_c + \hat{\mathbf{B}}_f\mathbf{f}\,,\ \mathbf{y} = \hat{\mathbf{C}}_y\mathbf{z}, \tag{5}$$

where

$$\mathbf{z} = \begin{bmatrix} \alpha \\ \mathbf{q}_p \\ \dot{\alpha} \\ \dot{\mathbf{q}}_p \end{bmatrix},\ \hat{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\boldsymbol{\Omega}^2 & \mathbf{K}_p & -\boldsymbol{\Lambda} & \mathbf{0} \\ \mathbf{L}_c^{-1}\mathbf{K}_p^{\mathrm{t}} & -\boldsymbol{\Omega}_e^2 & \mathbf{0} & -\boldsymbol{\Lambda}_e \end{bmatrix},\quad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{L}_c^{-1} \end{bmatrix},\ \hat{\mathbf{B}}_f = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{b}_\phi \\ \mathbf{0} \end{bmatrix},\ \hat{\mathbf{C}}_y = \begin{bmatrix} \mathbf{c}_\phi & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{6}$$

The modal displacements are such that $\mathbf{u} = \boldsymbol{\phi}\boldsymbol{\alpha}$ and, for mass normalized vibration modes, $\boldsymbol{\Omega}^2 = \boldsymbol{\phi}^{\mathrm{t}}\mathbf{K}_m\boldsymbol{\phi}$ and $\boldsymbol{\Lambda} = \boldsymbol{\phi}^{\mathrm{t}}\mathbf{C}\boldsymbol{\phi}$. $\boldsymbol{\Omega}$ is a diagonal matrix which elements are the undamped natural frequencies of the structure with piezoelectric patches in open-circuit. $\boldsymbol{\Omega}_e^2 = \mathbf{L}_c^{-1}\bar{\mathbf{K}}_e$ and $\boldsymbol{\Lambda}_e = \mathbf{L}_c^{-1}\mathbf{R}_c$ are both diagonal matrices which elements stand, respectively, for the squared natural frequencies of the electric circuits and the ratio between the resistances and inductances. The electromechanical coupling stiffness matrix projected in the undamped modal basis is defined as $\mathbf{K}_p = \boldsymbol{\phi}^{\mathrm{t}}\bar{\mathbf{K}}_{me}$. Input $\mathbf{b}$ and output $\mathbf{c}_y$ distribution vectors are also defined, with modal projections $\mathbf{b}_\phi = \boldsymbol{\phi}^{\mathrm{t}}\mathbf{b}$ and $\mathbf{c}_\phi = \mathbf{c}_y\boldsymbol{\phi}$, and $\mathbf{f}$ is a vector of the amplitudes of each mechanical force applied to the structure (Santos and Trindade 2016).

A linear state feedback for the applied voltages $\mathbf{V}_c$ is assumed such that $\mathbf{V}_c = -\mathbf{g}\mathbf{z} = -\mathbf{g}_{dm}\boldsymbol{\alpha} - \mathbf{g}_{de}\mathbf{q}_p - \mathbf{g}_{vm}\dot{\boldsymbol{\alpha}} - \mathbf{g}_{ve}\dot{\mathbf{q}}_p$, where $\mathbf{g}$ is a matrix of control gains for each state variable. Therefore, the state space equation (5) becomes

$$\dot{\mathbf{z}} = (\hat{\mathbf{A}} - \hat{\mathbf{B}}\mathbf{g})\mathbf{z} + \hat{\mathbf{B}}_f\mathbf{f}\ ,\ \mathbf{y} = \hat{\mathbf{C}}_y\mathbf{z}. \tag{7}$$

For a single-input mechanical excitation f, the closed-loop or controlled amplitude of a single displacement output y can be written such that $\tilde{y} = G_h(\omega)\tilde{f}$, where the FRF $G_h(\omega)$ is

$$G_h(\omega) = \hat{\mathbf{C}}_y(\mathrm{j}\omega\mathbf{I} - \hat{\mathbf{A}} + \hat{\mathbf{B}}\mathbf{g})^{-1}\hat{\mathbf{B}}_f, \tag{8}$$

which can also be derived from the second order equations of motion projected into the undamped modal basis leading to

$$G_h(\omega) = \mathbf{c}_\phi\{-\omega^2\mathbf{I} + \mathrm{j}\omega(\boldsymbol{\Lambda} + \mathbf{K}_p\mathbf{D}_{cc}^{-1}\mathbf{g}_{vm}) + [\boldsymbol{\Omega}^2 + \mathbf{K}_p\mathbf{D}_{cc}^{-1}(\mathbf{g}_{dm} - \mathbf{K}_p^{\mathrm{t}})]\}^{-1}\mathbf{b}_\phi, \tag{9}$$

where the closed-loop dynamic stiffness of the electric circuit $\mathbf{D}_{cc}$ is

$$\mathbf{D}_{cc} = -\omega^2\mathbf{L}_c + \mathrm{j}\omega(\mathbf{R}_c + \mathbf{g}_{ve}) + (\bar{\mathbf{K}}_e + \mathbf{g}_{de}). \tag{10}$$

In this work, the control gain $\mathbf{g}$ is calculated using the standard optimal LQR control theory applied to a single-input/single-output case, that is with only one active-passive patch-circuit pair for the control to minimize the vibration amplitude at one specific location of the structure, such that the following objective function is minimized

$$J = \frac{1}{2}\int_0^\infty \left(\dot{y}^2 + rV_c^2\right)\,\mathrm{d}t, \tag{11}$$

where $\dot{y}$ is the velocity at one location of interest and V_c is the control voltage applied to the active-passive shunt circuit in all cases following an iterative routine proposed in (Trindade, Benjeddou and Ohayon, 1999).

3. RESULTS

The structure is a fixed beam of the aluminum of dimension 220 mm in length, width of 25mm and thickness of 3 mm, the piezoelectric has a variable length, width of 25mm and thickness of 0.5 mm, as we can see in the figure 1. The extension piezoceramics are made of PZT-5H material whose properties are: $\bar{C}_{11}^D = 97.767$ GPa, $\rho = 7500Kg.m^{-3}$, piezoelectric coupling constants $\bar{h}_{31} = -1.3520x10^9$ N.C^{-1}, and dielectric constants $\bar{\beta}_{33}^{\epsilon} = 57.830x10^6 m.F^{-1}$. For the beam has: $\rho = 2700Kg.m^{-3}$ and $E = 70x10^9 MPa$. (Santos, 2008).

The resistance and inductance were tuned to the first resonance frequency, using an optimization algorithm, leading to Rc and Lc. For the general case, the inductance and resistance not only modify the dynamic stiffness of the structure, leading to damping and/or absorption but also affects the harvesting effect.(Santos and Trindade, 2016)

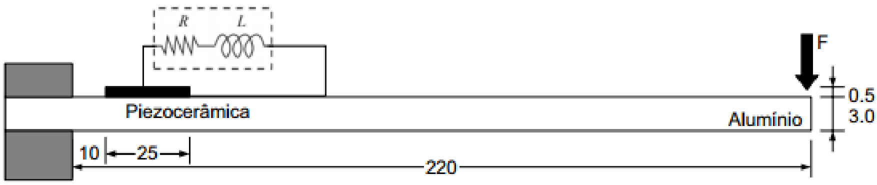

Figure 1: Representation of cantilever beam with bonded extension piezoceramic patch.

For the analysis of the output mechanical the resistance and inductance were optimization through on genetic algorithm in function of the a tension applied. The structure, figure 2, has a continuous system were is present several modes of vibrations, but in this work is applied only in first mode of the vibration size the focus of the work is optimize vibration methods in passive control.

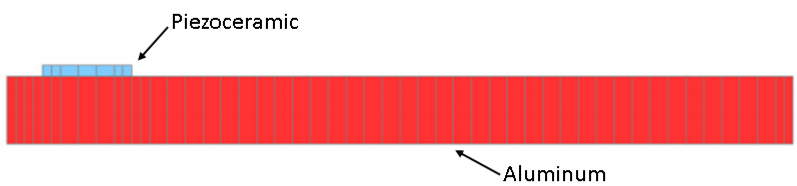

Figure 2: Mesh considered for the simulation – 50 elements.

Optimization of control parameters

The optimization had the focus in resistance and inductance values of the circuit, wherever the resistance (R) is responsible in damping by means of Joule effect and the inductance (L) is responsible to control resonant frequency of the structure, with can see in figure 3.

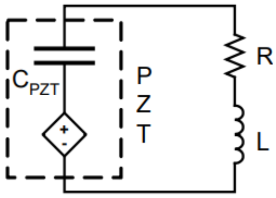

Figure 3: Configuration of the RLC circuit together with the piezoelectric.

For the optimization the proposed model (Santos 2008; Den Hartog 1972 and Viana 2005) were used, so was possible to set the optimized values for $\boldsymbol{R_{opt}}$ and $\boldsymbol{L_{opt}}$, wherever $\boldsymbol{R_{opt}} = 53517\Omega$ and $\boldsymbol{L_{opt}} = 171,85\text{H}$.

This way it was analyzed changes in dimensions and positions of the piezoceramic. For the length begin in 5mm with the addition of 5mm to the end of the beam, totaling 50mm, and was variated the position wherever in combination with the variation of the length it was possible to obtain values for $\boldsymbol{R_{opt}}$ and $\boldsymbol{L_{opt}}$, as can see in table 1.

The dissipation has greater efficiency when it acts where has greater deformation of the structure occurs, instead of the greater displacement (Costa, 2006; Hagood and von Flotow, 1991), therefore is no necessary to make verification of the length on all area of the beam.

Table 1: Optimized parameter values.

	Position (*mm*)								
	15			35			50		
Length (*mm*)	15	35	50	15	35	50	15	35	50
$R_{opt}(\Omega)$	40489	24009	17455	29588	17541	12835	24007	15719	11879
$L_{opt}(H)$	695	261	169	712	276	184	703	276	183

Frequency Response obtained for variation of the length of piezoceramic

Varying the length (since the width of the piezoceramic is constant) and based on the values $\boldsymbol{R_{opt}}$ and $\boldsymbol{L_{opt}}$ for the first mode were defined damping in first vibration mode. You can see in the figure 4.

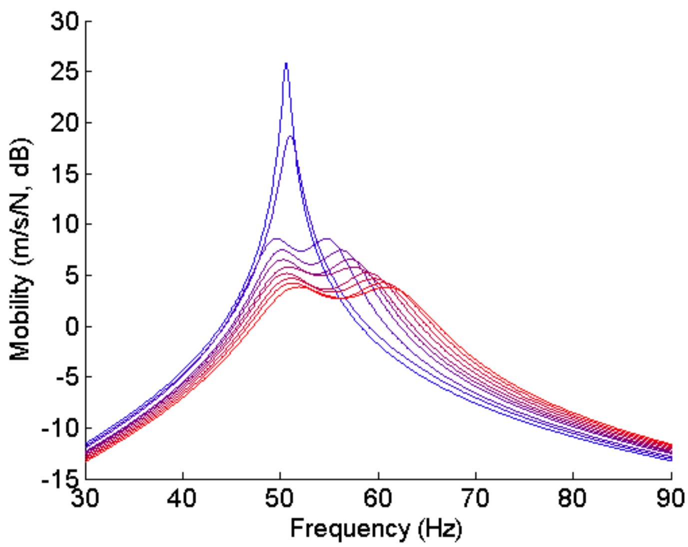

Figure 4: Damping for first vibration mode of the system.

As it can be observed in figure 4, it is possible to conclude that in the method of determination of $\boldsymbol{R_{opt}}$ and $\boldsymbol{L_{opt}}$ for the first mode there was a damping in the peak of vibration of the structure according to the increase of the size of the piezoceramic, of the blue with 5 mm for the red with 50 mm.

Considering the variation of the length of piezoceramic and the position of the patch, a analysis was done for a normalized response. In figure 5(a) can be observed the better ratio occurs for ceramics with reduced size, but when observed the figure 5(b) the better size for piezoceramic patch occurs in 10*mm* using a resistance and inductance values optimized.

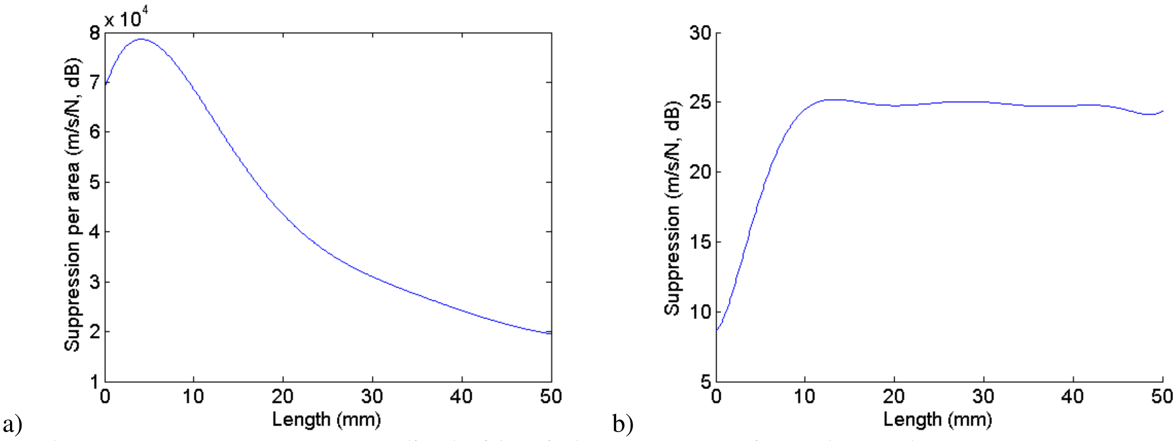

Figure 5: (a) Frequency Response normalized with ratio between attenuation and area. (b) Frequency Response with the real value of attenuation present for first mode..

4. CONCLUSION

An analysis of the size of the piezoceramic was performed. This analysis show the influence of the size and position on tuning of the circuit electric for passive control. A analysis considering a normalization of the attenuation in function of the area was presented, showing for the first mode of vibration, a better ratio for piezoceramic bounded in a defined position.Future works can be performed for new results using a genetic algorithm for a new configuration, considering a cost of piezoceramic and a attenuation in the structure.

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