

PERFORMANCE SIMULATION OF VERTICAL AXIS WIND TURBINE

André Jaques Ramos, jaquesramos92@hotmail.com¹

Carlos Eduardo de Souza, carlos.souza@ufsm.br²

Claiton Moro Franchi, claiton.franchi@gmail.com³

Frederico Menine Schaf, frederico.schaf@ufsm.br³

Humberto Pinheiro, humberto.ctlab.ufsm.br@gmail.com³

¹Electrical Engineering Graduate Program, Federal University of Santa Maria, Santa Maria-RS, Brazil

²Mechanical Engineering Department, Federal University of Santa Maria, Santa Maria-RS, Brazil

³Electrical Engineering Department, Federal University of Santa Maria, Santa Maria-RS, Brazil

Abstract: This paper presents a short review about the methods applied to simulate vertical axis wind turbine performance. Models based on blade element model like Single Streamtube Model, Multiple Streamtube Model and Double Multiple Streamtube Model are discussed. Equations using Double Multiple Streamtube Model are described to estimate the efficiency and power curve of 1.5 kW straight bladed vertical axis wind turbine. The obtained results will be compared with manufacturer data.

Keywords: Aerodynamic performance, Double multiple stream tube model, Power coefficient, Vertical axis wind turbines.

1. INTRODUCTION

Nowadays there are many ways to generate energy reducing negative impact on environment. Wind energy extraction is one of them and exist two turbine types to achieve this: horizontal (HAWT) and vertical axis wind turbine (VAWT). The first one is widely used because its power coefficient is higher compared with vertical axis wind turbine. However, horizontal axis wind type experience relatively higher load at the hub. Its blades suffer less with cyclic aerodynamic loading, but tower shadow or yaw error may cause dynamic stall effects to become significantly, even in moderate wind (Ashwill *et al.*, 2012). HAWT also needs a yaw mechanism to align the rotor with the wind. On the other hand, VAWT is independent from wind direction, and well behave in turbulence conditions and the generator could be located on the ground. Also, this model can be stall regulated, so no yaw mechanism is demand. Other advantage is the fact of the blades are cheaper to manufacture, but they should be long enough to required multiple joints on each blade. The main disadvantage is that, typically, is not self starting at low winds. Therefore, to start up, will be necessary install a pitch control mechanism, use its generator as motor or design different blades.

There are different methods that allow the estimation of the aerodynamic performance and loads that act on vertical axis wind turbines (Islam *et al.*, 2008; Jin *et al.*, 2015). The simplest model is the single streamtube, developed by Templin (1974) based on the actuator disk theory. Later, the multiple streamtube model was employed by Wilson *et al.* (1976) and also Strickland (1976). The double multiple streamtube model was proposed by Paraschivoiu (1981), combining the characteristics of those two previous models.

This paper will present a review of those methods and also apply double multiple stream tube model (DMST) to predict the power curve and power coefficient of a 1.5 kW straight bladed vertical axis wind turbine. A comparison from DMST and manufacturer data is presented.

2. Actuator disk theory

Initially, it is necessary to establish the actuator disk concept, that is important for wind turbines modeling in general, and establishes the control volume and its boundary conditions for the analysis. This theory is based on the follow assumptions (Hansen, 2015):

- the fluid is considered homogeneous, incompressible and at steady state condition;
- the pressure variation or the thrust by area unit is constant throughout the disk;
- the wind velocity through the disk is assumed constant;
- the disk has infinite blades.

With the above assumptions in mind, and based on Fig. 1, the model can be developed. Initially, it is considered that axial wind velocities before and after the disk are the same:

$$v_{up} = v_{dw} = v_{rotor}. \quad (1)$$

With the fluid in steady state, variation of mass the crosses the disk can be defined as

$$\frac{dm}{dt} = \rho A_{rotor} v_{rotor}, \quad (2)$$

where ρ , A_{rotor} and v_{rotor} are the air density, rotor area and rotor velocity.

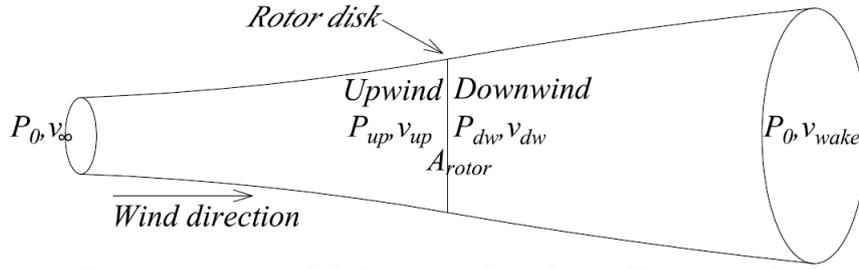


Figure 1: Actuator disk diagram, adapted from Hansen (2015).

The force that acts on disk due to variation of pressure, also known by wind force applied on disk or thrust, is given by:

$$F_T = -\frac{dm}{dt} (v_\infty - v_{wake}), \quad (3)$$

where F_T , v_∞ and v_{wake} are the thrust force, velocity at freestream and velocity at wake region.

Assuming there is not work being realized, it is possible to use the Bernoulli's equation in both sides of the disk as presented below. For the left side:

$$P_0 + \frac{1}{2}\rho v_\infty^2 = P_{up} + \frac{1}{2}\rho v_{rotor}^2, \quad (4)$$

with P_0 , P_{up} being the pressure at freestream and pressure at upstream, and

$$P_{dw} + \frac{1}{2}\rho v_{rotor}^2 = P_0 + \frac{1}{2}\rho v_{wake}^2, \quad (5)$$

with P_{dw} being pressure at downstream.

Another way to calculate thrust force is by summing the forces on each side of disk, such as:

$$F_T = A_{rotor} \Delta P_{rotor} \quad (6)$$

$$\Delta P_{rotor} = P_{up} - P_{dw}, \quad (7)$$

From here, isolating P_{up} and P_{dw} in (4) and (5); and replace them in (7), then

$$F_T = \frac{1}{2}\rho A_{rotor} (v_\infty^2 - v_{wake}^2). \quad (8)$$

Replacing equation (2) in (3) and comparing with equation (8):

$$\frac{1}{2}(v_\infty + v_{wake}) = v_{rotor}. \quad (9)$$

So that, it is possible to define an induction fact a , that describes the relation between freestream and upstream velocities as

$$a = \frac{v_\infty - v_{rotor}}{v_\infty} = 1 - \frac{v_{rotor}}{v_\infty}, \quad (10)$$

what takes to

$$v_{rotor} = v_\infty (1 - a), \quad (11)$$

In the same way to right side

$$v_{wake} = v_\infty (1 - 2a). \quad (12)$$

The power extracted from the wind by wind turbine is defined by:

$$P_{wt} = F_T v_{rotor} = \frac{1}{2}\rho A_{rotor} v_{rotor} (v_\infty^2 - v_{wake}^2). \quad (13)$$

Combining (11) and (12) in (13), the following definition is obtained

$$P_{wt} = \frac{1}{2}\rho A_{rotor} (v_\infty^2 - (v_\infty (1 - 2a))^2) v_\infty (1 - a) \quad (14)$$

$$= \frac{1}{2}\rho A_{rotor} v_\infty^3 4a(1 - a)^2. \quad (15)$$

The term $\frac{1}{2}\rho A_{rotor}v_{\infty}^3$ is known as the total power available in the wind P_{∞} . So, rewriting (15),

$$\frac{P_{wt}}{P_{\infty}} = 4a(1-a)^2 = C_P, \tag{16}$$

where it is obtained the efficiency of wind turbine or the power coefficient C_P . Its limit is calculated deriving (16) in relation a , resulting in a $C_P = 0.5926$ to $a = \frac{1}{3}$. It means, the maximum C_P to any type of wind turbine is 59.26 %.

In the same way, applying this sequence to thrust force, its coefficient, C_T is defined as

$$C_T = \frac{F_T}{F_{T\infty}} = \frac{2\rho A_{rotor}v_{\infty}^2 a(1-a)}{\frac{1}{2}\rho A_{rotor}v_{\infty}^2} = 4a(1-a), \tag{17}$$

with its maximum equals to $C_T = 1$ to $a = \frac{1}{2}$, however when $a = \frac{1}{3}$, $C_T = 0.889$.

Those results said that if an ideal rotor was built, so that, velocity of wind at rotor region would be $\frac{2}{3}$ of free stream velocity. Considering the wind turbine operating at its maximum power power point tracking, its C_P is known by Betz's Limit. From the presented theory three ways of predicting the performance were developed: single streamtube model (SSTM), multiple streamtube model (MSTM) and double multiple streamtube model (DMSTM), as presented in Fig. 2 and Fig. 3 .

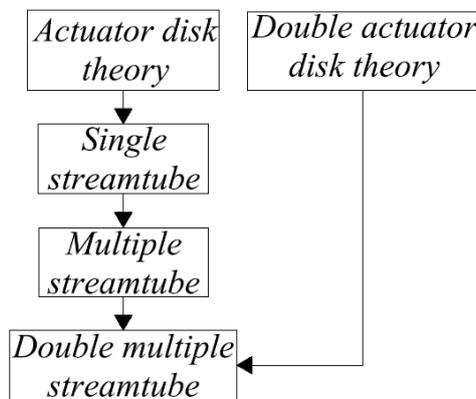


Figure 2: Actuator disk development, adapted from Biadgo *et al.* (2013).

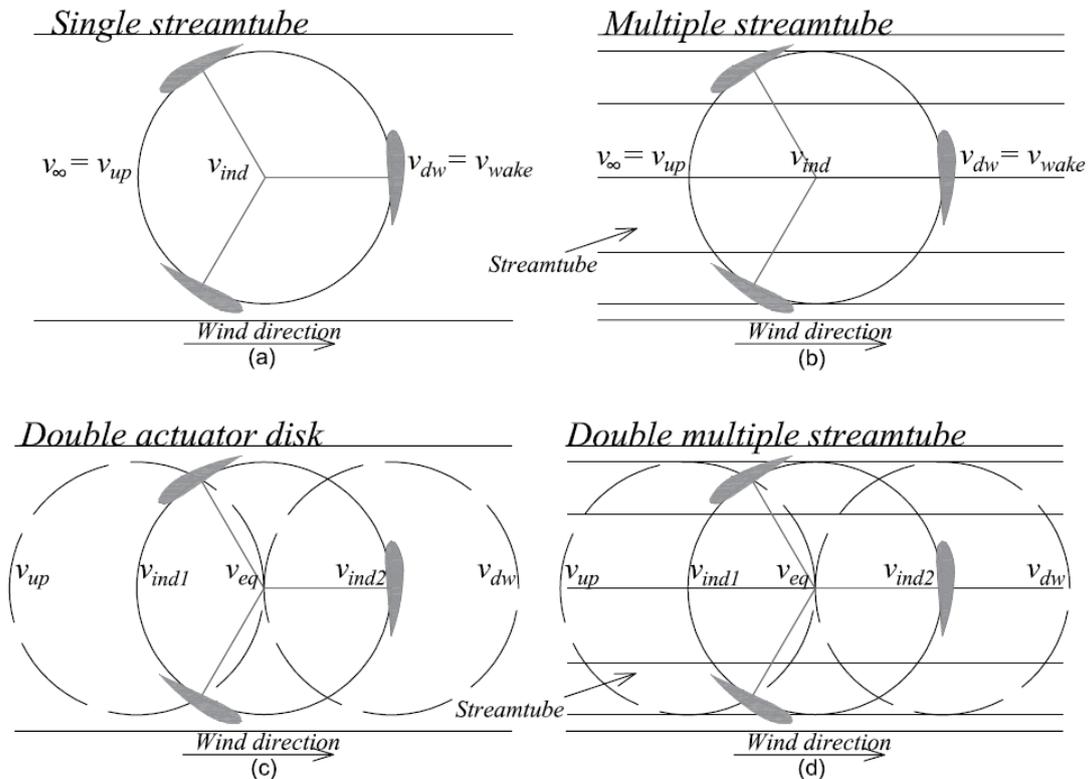


Figure 3: Disk velocities diagram for each model.

2.1 Single streamtube model

Developed by Templin (1974), this model is the simplest form to estimate vertical axis wind turbine performance. It is considered that the rotor is inside a unique streamtube and applying the actuator disk principle and blade element for estimate the overall wind turbine performance, as shown in Fig. 3(a). This model always predicts higher power than the experimental results. Meanwhile, it cannot be used to predict the wind velocity distribution across the rotor (Islam *et al.*, 2008). The SSTM considers that there is not loss of velocity from freestream to rotor, as well as at wake region. So, $a = 1$ for any position of blades during them rotation. In order to calculated the output power of wind turbine using SSTM take a look in Fig. 4.

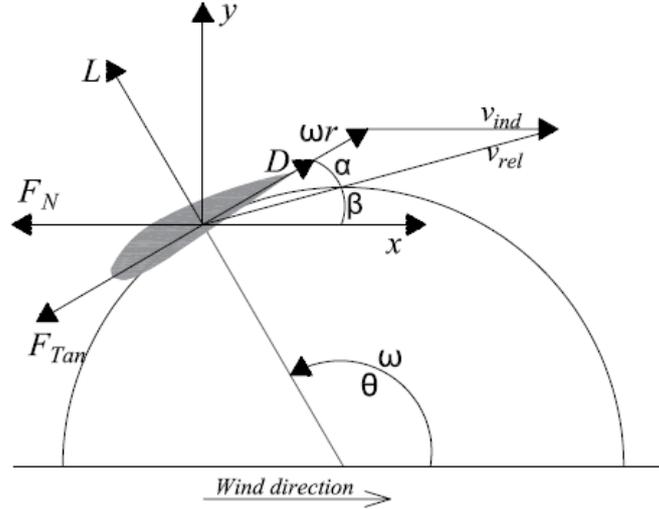


Figure 4: Velocities and forces in a streamtube.

It is possible to defined the velocity in x-axis and y-axis as:

$$v_x = (\omega r) \sin(\theta), \quad (18)$$

$$v_y = (\omega r) \cos(\theta) + v_{ind}, \quad (19)$$

where r , ωr , θ and v_{ind} are the radius of wind turbine rotor, blade rotational velocity, azimuthal angle and induced velocity on the blade, respectively.

The velocity of wind saw from blade point of view is called relative velocity and can be given by

$$v_{rel} = \sqrt{(v_x^2 + v_y^2)}. \quad (20)$$

The flight path angle is the angle between x-axis and relative velocity and is defined as:

$$\beta = \arctan\left(\frac{v_x}{v_y}\right). \quad (21)$$

Computing the flight path and azimuthal angle, is possible to calculated the angle of attack as being angle between relative velocity and blade rotational velocity like:

$$\alpha = \left| \frac{\beta - \theta + \pi}{2\pi} \right| - \pi + \Omega, \quad (22)$$

with Ω as pitch angle.

After to define velocities, it is possible to calculate the loads that act on the blades. There are two main forces: the first one is the normal or radial force, also new as thrust and the tangential force and they are defined as

$$F_N = D \cos(\beta) - L \sin(\beta), \quad (23)$$

$$F_{Tan} = (D \cos(\beta) - L \sin(\beta)) + (D \sin(\beta) - L \cos(\beta)), \quad (24)$$

where L and D are lift and drag forces described by:

$$L = \frac{1}{2} \rho c h v_{rel}^2 c_l, \quad (25)$$

$$D = \frac{1}{2} \rho c h v_{rel}^2 c_d, \quad (26)$$

with c_l and c_d are the lift and drag coefficients. These coefficients depends on what airfoil is utilized. c is the chord length and h is the blade length.

Between normal and tangential forces, the only one that contributes to generate torque on blades putting them in movement is the tangential force. The produced torque is obtain by:

$$F_Q = F_{Tan}r, \quad (27)$$

Thus, the power generated by all blades is:

$$P_{wt} = \left(\sum_{j=1}^{N_{blades}} F_Q \right) \omega, \quad (28)$$

where N_{blades} is number of blade at wind turbine rotor.

The power coefficient can be calculated by:

$$C_P = \frac{P_{wt}}{\frac{1}{2}\rho A_{rotor}v_{up}^2}, \quad (29)$$

where A_{rotor} for a straight bladed VAWT is $2rh$.

2.2 Multiple streamtube model

This method was presented by Wilson *et al.* (1976), Strickland (1976). The rotor swept volume was divided into a series of adjacent and aerodynamically independent parallel streamtubes, Jin *et al.* (2015), represented by straight lines in Fig. 3(b).

To evaluate turbine efficiency blade element theory is applied in each streamtube. The algorithm is fast, it more precisely predicts measured C_P and distributed effects such as blade forces and rotor wake velocity distributions, Strickland (1976), however estimated power is lower than experimental results and also it is not able to differentiate upwind part from downwind part of the rotor as well as single streamtube. The development of equations for this model is based in Fig. 5.

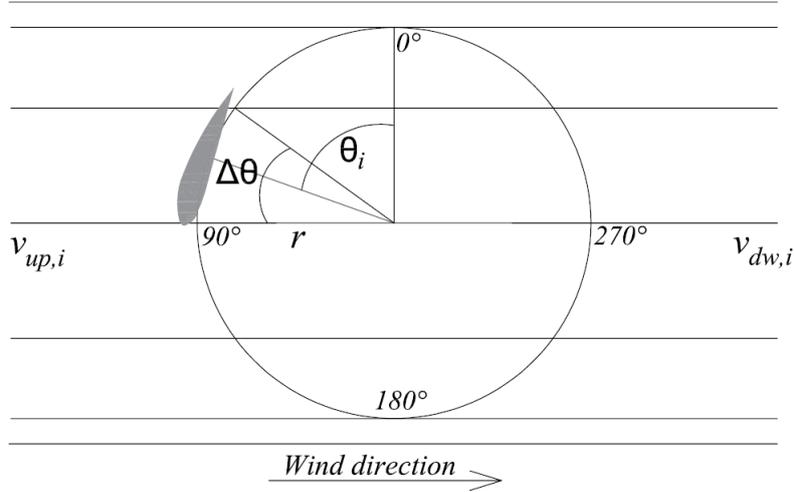


Figure 5: Rotor divided into several minor streamtubes.

Now, the total span of tube is separate in i small streamtubes and then the equations presented earlier are applied to each streamtube. When calculus are done, it is obtain the average thrust, torque and power of wind turbine.

After defined the number of streamtubes N_{st} , is calculated $\Delta\theta$ like:

$$\Delta\theta = \frac{\pi}{N_{st}}. \quad (30)$$

The number of θ_i 's is given by $N_{\theta_i} = 2N_{st}$. Other modification is in relation to area used in equations, because now will be utilized each streamtube area instead rotor area, and it can be obtained with $A_{st,i} = rh\Delta\theta\sin(\theta_i)$.

The instantaneous normal force on a single blade at one azimuthal location is given by (23). the average thrust considering N_{blades} and noting that each blade spend $\frac{\Delta\theta}{\pi}$ percent of their time in each streamtube, thus:

$$F_{N,avg} = 2 \left(N_{blades} \frac{\Delta\theta}{\pi} F_N \right). \quad (31)$$

The thrust coefficient can be calculated as presented below:

$$C_T = \frac{F_{N,avg}}{\frac{1}{2}\rho A_{st} v_{up}^2}. \quad (32)$$

The average torque can be obtained following equation:

$$F_{Q,avg} = N_{blades} \sum_{i=1}^{N_{\theta_i}} \frac{F_{Q_i}}{N_{\theta_i}}, \quad (33)$$

and the torque coefficient and power coefficient are defined by:

$$C_Q = \frac{F_{Q,avg}}{\frac{1}{2}\rho v_{up}^2 (2rh) r}, \quad (34)$$

$$C_P = \lambda C_Q, \quad (35)$$

where λ is the tip speed ratio (TSR) given by

$$\lambda = \frac{\omega r}{v_{\infty}}. \quad (36)$$

2.3 Double multiple streamtube model

This method was proposed by Paraschivoiu (1981) combines multiple streamtube and double actuator disk theory (Laplin, 1975), Fig. 3(c). As presented in Fig. 3(d), it is possible to observe multiple streamtubes as presented by MSTM, but in addition the possibility to identify upwind and downwind part of the rotor. This model allows for variations of the aerodynamic loads and the torque with the blade position for each part of the rotor (Paraschivoiu, 1981).

Compared with the previous two models, double multiple streamtube model gives better correlation between the calculated and experiment results, however under high solidity and high tip speed ratios, the model shall have its precision further improved, (Vallverdú, 2014; Jin *et al.*, 2015). In the following sections this model will be discussed in detail.

From double actuator disk theory it is possible to notice that upstream wind speed is not equal to wind velocity on blade and also differ from velocity at central part of rotor as well as downstream. In order to obtain these velocities Fig. 3(c) is used. As result:

$$v_{ind1} = a_1 v_{up}, \quad (37)$$

$$v_e = 2a_1 - 1, \quad (38)$$

$$v_{ind2} = a_2 v_e, \quad (39)$$

where a_1 , a_2 and v_e are upstream induction factor, downstream induction factor and equilibrium velocity, respectively.

In this model the upstream are defined by $(0, \pi)$ and downstream $(\pi, 2\pi)$. So, each streamtube swept area is different from one side to another and they are defined by:

$$A_{1,st} = rh\Delta\theta \sin(\theta_i), \quad (40)$$

$$A_{2,st} = rh\Delta\theta \sin(-\theta_i). \quad (41)$$

Using the equations presented in section 2., in each side of rotor, thrust force coefficients are defined as:

$$C_{T,1,i} = \frac{\sigma}{\pi \sin(\theta)} \frac{v_{ind1}^2}{v_{up}^2} (c_d \cos(\beta) - c_l \sin(\beta)), \quad (42)$$

$$C_{T,2,i} = \frac{-\sigma}{\pi \sin(\theta)} \frac{v_{ind2}^2}{v_{up}^2 (2a_1 - 1)} (c_d \cos(\beta) - c_l \sin(\beta)), \quad (43)$$

where σ is known as solidity ($\sigma = \frac{N_{blades} c}{2r}$).

Finally the total power coefficient can be given by sum of power coefficient from both sides of rotor as showed below:

$$C_{P1} = \frac{\sigma \lambda}{2\pi} \sum_0^{\pi} C_{Q,1}, \quad (44)$$

$$C_{P2} = \frac{\sigma \lambda}{2\pi} \sum_{\pi}^{2\pi} C_{Q,2}, \quad (45)$$

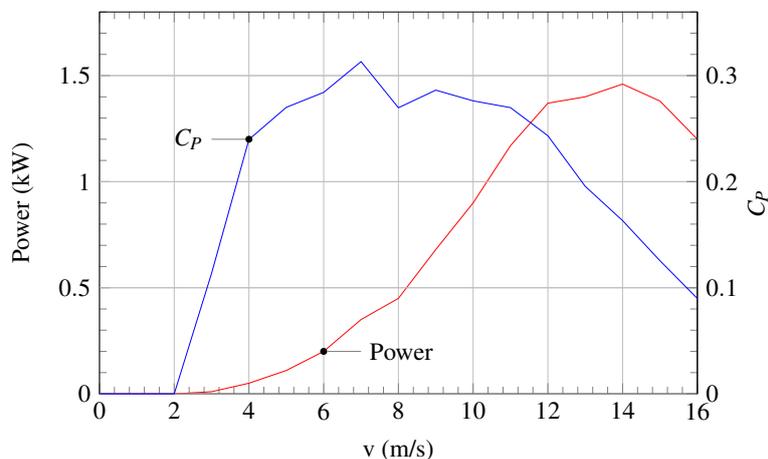
$$C_P = C_{P1} + C_{P2}. \quad (46)$$

Table 1: Razec features.

	Description/value
Power	1.5 kW (at 12 m/s)
Rotational velocity	60 to 180 RPM
Cut-in	2.2 m/s
Start generate at	4.0 m/s
Diameter	2.0 m
Blade length	2.66 m
Chord length	0.32595 m

3. NUMERICAL STUDIES

The methodology is applied to a 1.5 kW H-Darrieus type wind turbine using a PMSG. It was designed to deal with turbulent winds from any direction and low noise production, what allows it to be installed on top of buildings, for urban energy production. Figure 6 presents a photo of the turbine installed on top of a building at the UFSCM Campus in Santa Maria, Brazil. Table 1 presents its main characteristics. The manufacturer supplies the power and efficiency curves, seen in Fig. 7. The blade profile is approximated by NACA 4418. Its lift and drag coefficient curves is present below, as obtained using the XFOIL webpage.

**Figure 6: Razec wind turbine.****Figure 7: Manufacturer power and power coefficient curves.**

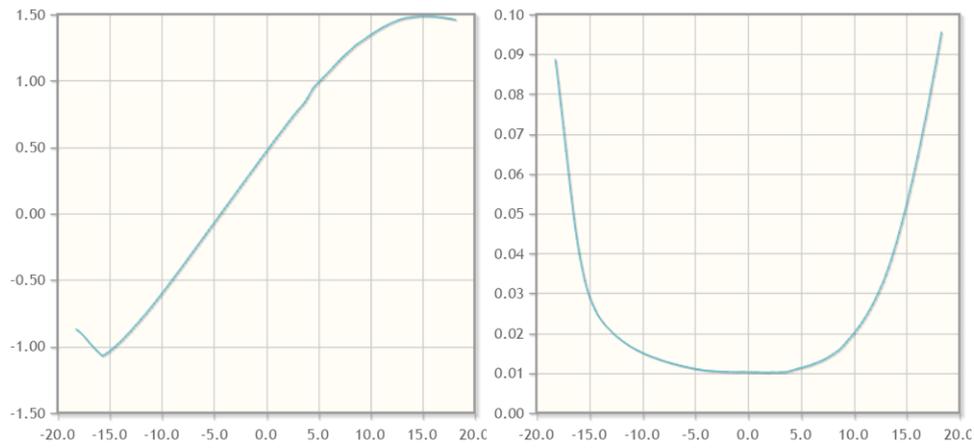


Figure 8: Lift and drag coefficient from NACA 4418.

Figure 9 presents the resulting angle of attack in relation to azimuthal angle for different tip speed ratios, as computed with the implemented methodology. It is possible to see that the angle of attack remains between $+15$ and -25 degrees for TSR equal to 2. This range indicates that the input data is not adequate to cover the required values of angle of attack. There is no information below and above the limits shown in Fig. 8. Moreover, the dynamic stall effects are not taken into consideration in the implemented methodology.

The power curve considering the tip speed ratio equal 3 varying the wind velocity from 1 m/s to 16 m/s is shown in Fig. 10, compared to the manufacturer curve. It is observed that the manufacturer shows a higher value for most part of the velocity range. Some points that influence the differences are: the stall characteristics is not included in simulation and convergence aspects in relation to induction factor.

In order to obtain the power coefficient in relation to tip speed ratio earlier presented equation are used. The number of streamtube was choice as 50, use bigger number of streamtubes does not produce significant changes. Figure 11 shows the power coefficient of wind turbine. It is possible to notice that the upstream part of the wind turbine C_{P1} contributes positively with C_P , but the downstream C_{P2} part contributes negatively. This happens because when blades are between 90° and 270° they accelerate, there is a positive torque contribution. However, between 270° and 90° the wind speed and torque influence are against blade direction, so the blade velocity falls down. Adding those contributions it is possible to obtain the overall efficiency of wind turbine. The maximum value of power coefficient is 0.3978 (it harnesses 39.78% of the available wind energy) for tip speed ratio equals to 1.8. Thus, when turbine is operating, it keeps the ratio between wind speed and rotor velocity near of 1.8 in order to extract maximum power of turbine is an objective.

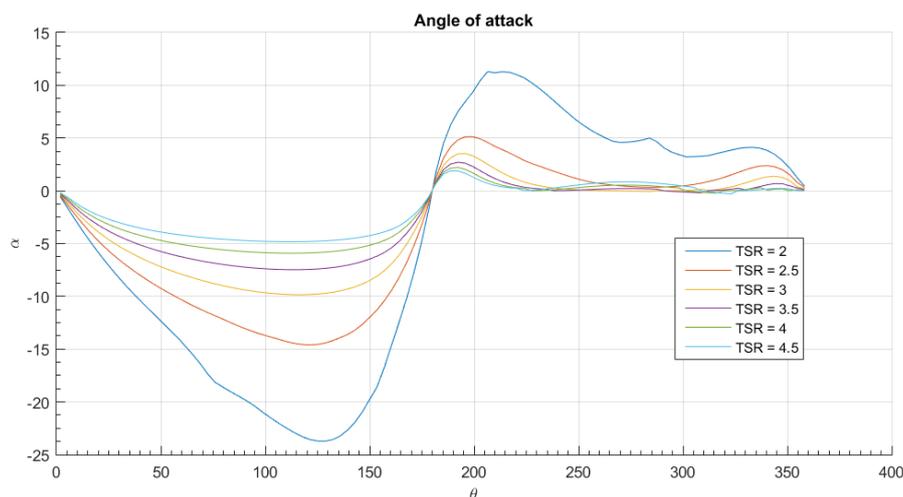


Figure 9: Angle of attack versus azimuthal angle for different TSR.

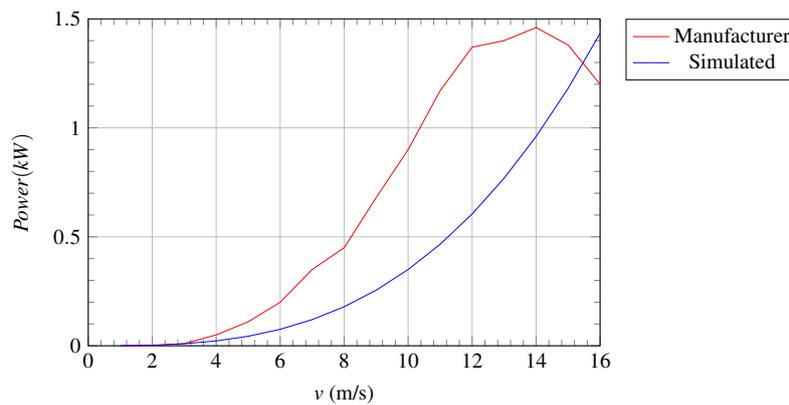


Figure 10: Simulated power curve compared to the provided by the manufacturer .

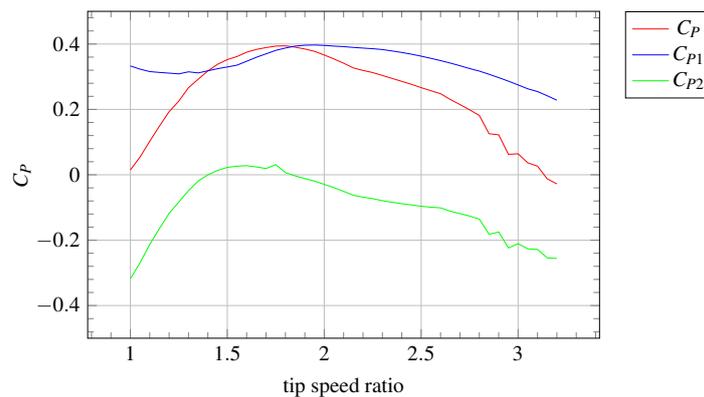


Figure 11: Power coefficient vs TSR.

4. CONCLUSION

This paper presented a review about the methods, based on actuator disk theory, applied to simulate vertical axis wind turbine characteristics. Also power and power coefficient curves for an 1.5 kW vertical axis wind turbine were presented to demonstrate double multiple streamtube model algorithm. From comparison of simulating and manufacturer curve, some points that can influence the differences are: the stall characteristics is not included in simulation and convergence aspects about induction factor. For most TSR conditions, the angle of attack is well behaved between -15 and $+15$ that is available data range. In relation to power coefficient, it can be noticed the difference of energy efficient between upstream and downstream rotor parts. In the front the turbine it is accelerated by the wind, but on the other side the torque is negative and thus there is loss of speed. The maximum power coefficient is 0.3978 at TSR equal to 1.8. So, during its operation to obtain the maximum power point tracking this turbine should rotate 1.8 times faster than wind speed. This turbine can be classify considering INMETRO N° 168/2015 normative as category *B*. The next steps of this work aims to obtain experimental values for the turbine performance, allowing improvement in the numerical model and future control techniques implementation.

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