X CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA 2018
20 A 24 DE MAIO DE 2018 SALVADOR – BA – BRASIL

COMPARISON BETWEEN SINGLE AND DOUBLE INTEGRAL TRANSFORMATION SOLUTIONS OF HEAT CONDUCTION IN SOLID-STATE ELECTRONICS

Lívia M. Corrêa, livcorrea@yahoo.com.br[1]
Daniel J. N. M. Chalhub, daniel.chalhub@uerj.br[1]
[1]Group of Environmental Studies in Reservoirs – GESAR, Departament of Mechanical Engineering, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, Brazil

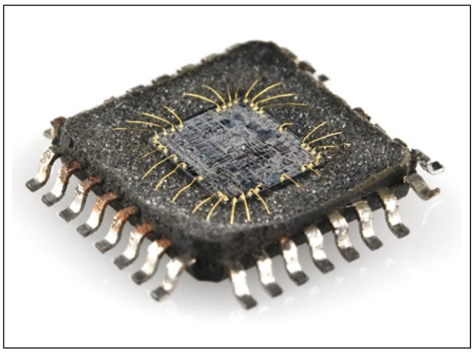
Abstract: *The design of modern electronic devices has been dealing with challenges on thermal control. In this work, it is proposed two different ways of predicting the temperature field in Solid State Electronics using integral transforms, with several heat generation on the domain of the chip and the external convection. First, it is proposed the single transformation of Classical Integral Transform Technique (CITT) and, later, the double transformation over the CITT. It will be that a single transformation is a more efficient methodology when compared to the double transformation of CITT.*
Keywords: *Thermal Control, Solid State Electronics, Multiple Heat Generation, Classical Integral Transform Technique*

1. Introduction

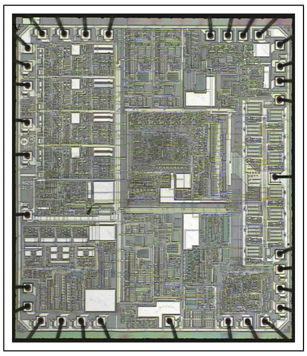
Solid-State Electronics (SSE) or devices are circuits built entirely from solid materials and in which the electrons or other charge carriers, are confined entirely within the solid material. They were responsible for the electronical revolution, marked by the Nobel prize in 1956 for the transistor invection, Chelikowsky and Franciosi (1991). The decrease of the size of integrated circuits and the need for a better and efficient power dissipation has motived several studies about the temperature control. Thermal control of electronic components the objective of maintain the component temperature equal to or below the manufacturer's maximum specified service temperature, typically between 85 and 100^{o}C, (Peterson and Ortega, 1994). The knowledge of those values and their ideal operational ranges would ensure an effective performance and long service life.

Figure 1: **Illustration of a SSE after plastic encapsulament remove**

(a) Microchip view



(b) Microchip on a microscope

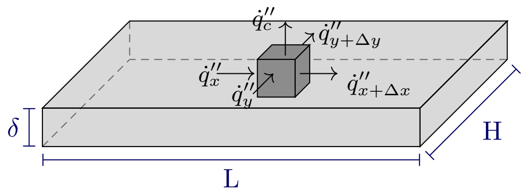


Previously, it was shown the solution of Solid State Electronics with one heat generation on its domain and solved by Classical Integral Transform Technique, (Corrêa and Chalhub, 2017). The integral transform technique provides a systematic, efficient, and straightforward approach for the solution of both homogeneous and nonhomogeneous, steady-state, and time-dependent boundary value problems of heat conduction. It is presented by Cotta (1993), Hahn and Özisik (2012) and the technique is applied by several authors. Sphaier and Cotta (2000) applied the Integral Transform Technique on the solution of a multidimensional partial differential models within irregularly shaped domains. da Silva *et al.* (2017) presented a formulation for the solution of the two-dimensional steady state heat conduction with heat generation, CITT was used to solve the problem in a semianalytical manner and a second order central finite difference method (FDM) was also implemented, the convergence analysis showed that CITT has a greater performance having no difficulties to obtain accurate results with very few terms in the solution summation while FDM struggled specially for the positions near the center and for high concentration of heat generation in the center of the plate. Chalhub *et al.* (2014) proposed a solution for the Poisson equation arising from the incompressible Navier-Stokes equations by two methodologies using the Classic Integral Transform Technique (CITT): a single transformation and a double transformation. Although both techniques presented a very similar convergence behavior and results, the double transformation has poorer performance in comparison with the single transformation scheme. Even though the performance of the double transformations was not so good, this formulation has a simpler analytical solution, which might be more interesting in some cases such as the problem that was presented. Dantas (1996) applied Integral Transforms for solving a heat transfer problem also in microchips with plastic encapsulament and different layouts of thermal conductivity. For this problem, the thermal conductivity is considered constant and is under natural convection effects. Braga Junior and Sphaier (2014) has applied the General Integral Transform Technique (GITT) for the solution of heat transfer in Functionally Graded Materials (FGM), with different thermal conductivity variation in the material.

This work proposes a comparison between CITT single and double transformations in solid state electronics. It is considered an SSE and heat convection on the top of surface of the substrate. On the bottom of the SSE, the surface is in contact with the board, hence the boundary condition considered is insulation. Since the thickness of the device is small compared to other dimensions, a partial lumping approach in z-direction is performed and the final mathematical formulation is two dimensional. The heat generation is composed by several generation in piecewise constant function.

2. Problem Formulation

Figure 2: **Heat conduction in the SSE**



The mathematical formulation of the heat conduction in a electronic microchip is given by the energy equation in steady-state after applying a partial lumping approach in z-direction. For this work a few assumptions are made: the heat conductivity is constant inside the microchip. The formulation is showed bellow with its respective boundary conditions.

$$k\left(\frac{\partial^2 T(x,y)}{\partial x^2}+\frac{\partial^2 T(x,y)}{\partial y^2}\right)+\dot{g}'''(x,y)=\frac{h(T(x,y)-T_f)}{\delta} \quad \text{for} \quad 0\le x\le L \quad \text{and} \quad 0\le y\le H \tag{1a}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=0; \quad \left.\frac{\partial T}{\partial y}\right|_{y=0}=0; \tag{1b}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=L}=0; \quad \left.\frac{\partial T}{\partial y}\right|_{y=H}=0; \tag{1c}$$

where T is the temperature, k is the thermal conductivity, $\dot{g}'''$ is the heat generation, T_f is the environment air temperature, h is the convection heat transfer coefficient and L, H and δ are the dimensions of the chip in x, y and z directions respectively.

The nondimensionalization of the problem leads to the following mathematical formulation:

$$\frac{\partial^2\Theta}{\partial\xi^2}+\frac{\beta^2\partial^2\Theta}{\partial\eta^2}-\mathrm{Bi}_L\Theta\gamma=-G(\xi,\eta) \quad \text{for} \quad 0\le\xi\le 1 \quad \text{and} \quad 0\le\eta\le 1 \tag{2a}$$

$$\left.\frac{\partial\Theta}{\partial\xi}\right|_{\xi=0}=0; \qquad \left.\frac{\partial\Theta}{\partial\eta}\right|_{\eta=0}=0; \tag{2b}$$

$$\left.\frac{\partial\Theta}{\partial\xi}\right|_{\xi=1}=0; \qquad \left.\frac{\partial\Theta}{\partial\eta}\right|_{\eta=1}=0; \tag{2c}$$

The non-dimensional groups are defined as:

$$\xi=\frac{x}{L}; \qquad \eta=\frac{y}{H}; \tag{3a}$$

$$\beta=\frac{L}{H}; \qquad \gamma=\frac{L}{\delta}; \tag{3b}$$

$$\Theta=\frac{T-T_f}{T_0-T_f}; \qquad \mathrm{Bi}_L=\frac{hL}{k}; \qquad G(\xi,\eta)=\frac{\dot{g}'''L^2}{k\Delta T}; \tag{3c}$$

where β and γ are aspect ratios, Bi_L is the Biot number, Θ is the dimensionless temperature, ξ and η are the dimensionless versions of x and y; and G is composed by several generation in piecewise constant function over the domain of the chip, as can be seen in figure 3.

2.1 Solution by Single Transformation of Classical Integral Transform Technique

The Superposition Method is introduced by Hahn and Özisik (2012) for solving non-homogen problems in linear problems. Because this particular problem is linear, the equation (2a) is rewritten and applied to different heat generation of the problem.

$$\frac{\partial^2\Theta_1}{\partial\xi^2} + \beta^2\frac{\partial^2\Theta_1}{\partial\eta^2} - \mathrm{Bi}\gamma\Theta_1 = -G_1 \tag{4a}$$
$$\frac{\partial^2\Theta_2}{\partial\xi^2} + \beta^2\frac{\partial^2\Theta_2}{\partial\eta^2} - \mathrm{Bi}\gamma\Theta_2 = -G_2 \tag{4b}$$
$$\frac{\partial^2\Theta_3}{\partial\xi^2} + \beta^2\frac{\partial^2\Theta_3}{\partial\eta^2} - \mathrm{Bi}\gamma\Theta_3 = -G_3 \tag{4c}$$
$$\frac{\partial^2\Theta_4}{\partial\xi^2} + \beta^2\frac{\partial^2\Theta_4}{\partial\eta^2} - \mathrm{Bi}\gamma\Theta_4 = -G_4 \tag{4d}$$
$$\frac{\partial^2\Theta_5}{\partial\xi^2} + \beta^2\frac{\partial^2\Theta_5}{\partial\eta^2} - \mathrm{Bi}\gamma\Theta_5 = -G_5 \tag{4e}$$

After summing the terms these equations:

$$\frac{\partial^2(\Theta_1+\Theta_2+\Theta_3+\Theta_4+\Theta_5)}{\partial\xi^2} + \beta^2\frac{\partial^2(\Theta_1+\Theta_2+\Theta_3+\Theta_4+\Theta_5)}{\partial\eta^2} - \mathrm{Bi}\gamma(\Theta_1+\Theta_2+\Theta_3+\Theta_4+\Theta_5) = -(G_1+G_2+G_3+G_4+G_5) \tag{5}$$

Θ and G may be written as:

$$\Theta = \Theta_1+\Theta_2+\Theta_3+\Theta_4+\Theta_5 \qquad \text{and} \qquad G = G_1+G_2+G_3+G_4+G_5 \tag{6}$$

Substituting these terms, the equation (2a) is found, proving that Superposition Method is applicable in this problem.

In order to solve the proposed problem, the Classical Integral Transform Technique (CITT) is applied. This is an analytical technique that uses expansions of the sought solution in terms of an infinite orthogonal basis of eigenfunctions, keeping the solution process always within a continuous domain. In order to establish the transformation pair, the temperature field is written as function of an orthogonal eigenfunctions obtained from the following auxiliary eigenvalue problem known as the Helmholtz classic problem in cartesian coordinates, where $\Psi(\eta)$ are the eigenfunctions and λ_n are the eigenvalues. For this particular problem, the case where $\lambda = 0$ also exists.

$$\Psi_n''(\eta) + \lambda_n^2\Psi_n(\eta) = 0 \tag{7a}$$
$$\Psi_n'(0) = 0; \quad \Psi_n'(1) = 0. \tag{7b}$$

where the solution of the eigenvalue problem is given by, for $\lambda = 0$:

$$\Psi_0(\eta) = 1; \qquad \lambda_0 = 0; \tag{8}$$

and for $\lambda > 0$:

$$\Psi_n(\eta) = \cos(\lambda_n\eta); \qquad \lambda_n = n\pi, \quad \text{for} \quad n = 1, 2, 3, \ldots \tag{9}$$

To apply the CITT, the transformation pair is defined.

$$\text{Transformation} \quad \Rightarrow \quad \bar{\Theta}_n(\xi) = \int_0^1 \Theta\Psi_n(\eta)\mathrm{d}\eta \tag{10}$$

$$\text{Inversion} \quad \Rightarrow \quad \Theta = \sum_{n=0}^{\infty}\frac{\bar{\Theta}_n(\xi)\Psi_n(\eta)}{N_n} \tag{11}$$

where $\bar{\Theta}_n$ is the transformed version of Θ and the norm N_n is defined by:

$$N_n = \int_0^1 \Psi_n^2\mathrm{d}\eta \tag{12}$$

To obtain the transformed equation, both sides of the heat conduction equation (2a) are multiplied by the eigenfunction Ψ_n and integrated in the domain. Two transformed equations are obtained: one for $\lambda > 0$ and one for $\lambda = 0$:

For $\lambda > 0$:

$$\bar{\Theta}_n'' - (\beta^2\lambda_n^2 + \mathrm{Bi}\gamma)\bar{\Theta}_n = -\bar{G}_n(\xi) \tag{13a}$$
$$\bar{\Theta}_n'(0) = 0; \quad \bar{\Theta}_n'(1) = 0; \tag{13b}$$

where $\bar{G}_n$ is given by:

$$\bar{G}_n(\xi) = \int_0^1 G\Psi_n(\eta)\mathrm{d}\eta = G\int_0^1 \Psi_n(\eta)\mathrm{d}\eta \tag{14}$$

The transformed equation admits an analytical solution for $\lambda > 0$.

For $\lambda = 0$:

$$\bar{\Theta}_0'' - (\mathrm{Bi}\gamma)\bar{\Theta}_0 = -\bar{G}_0(\xi) \tag{15a}$$
$$\bar{\Theta}_0'(0) = 0; \quad \bar{\Theta}_0'(1) = 0; \tag{15b}$$

where $\bar{G}_0$ is given by:

$$\bar{G}_0(\xi) = \int_0^1 G\Psi_0(\eta)\mathrm{d}\eta = G\int_0^1 \mathrm{d}\eta \tag{16}$$

The transformed equation for admits the following analytical solution $\lambda = 0$.

Finally, in order to obtain the final temperature field, the inversion formula (11) must be utilized and the summation must be truncated to a finite value ($n_{\max}$).

2.2 Solution by Double Transformation of Classical Integral Transform Technique

The Classical Integral Transform Technique may be a single transformation or double transformation. The difference between them is that double transformation presents a transformation for each direction.

$$\text{Inversion} \quad \Rightarrow \quad \Theta = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{\bar{\bar{\Theta}}_{nm}(\xi)(\eta)\Psi_n(\eta)\Xi_m(\xi)}{N_{y_n}N_{xm}} \tag{17}$$

$$\text{Transformation} \quad \Rightarrow \quad \bar{\bar{\Theta}}_{nm}(\xi)(\eta) = \int_0^1\int_0^1 \Theta\Psi_n(\eta)\Xi_m(\xi)\mathrm{d}\eta\mathrm{d}\xi \tag{18}$$

where $\Psi_n(\eta)$ and $\Xi_m(\xi)$ are the functions to be solved and are also eigenfunctions. $\bar{\bar{\Theta}}_{nm}$ is the transformed version of Θ. N_{y_n} and N_{xm} are the norms and are defined in (19).

$$N_{xm} = \int_0^1 \Xi_m^2\mathrm{d}\xi; \quad N_{y_n} = \int_0^1 \Psi_n^2\mathrm{d}\eta \tag{19}$$

The greatest advantage of this approach is that it requires a lot less analytical effort and the final solution is simpler and more compact. The double transformation presents, however, the disadvantage of having a double summation, having a poorer convergence performance. The consequence is that the solution requires more terms for its convergence.

The eigenfunctions $\Psi(\eta)$ and $\Xi(\xi)$ are solved. λ_n are the eigenvalues of $\Psi(\eta)$ and μ_m are the eigenvalues of $\Xi(\xi)$.

$$\Psi_n''(\eta) + \lambda_n^2\Psi_n(\eta) = 0 \tag{20a}$$
$$\Psi_n'(0) = 0; \quad \Psi_n'(1) = 0. \tag{20b}$$

$$\Xi_m''(\xi) + \mu_m^2\Xi_m(\xi) = 0 \tag{21a}$$
$$\Xi_m'(0) = 0; \quad \Xi_m'(1) = 0. \tag{21b}$$

For $\lambda = 0$:

$$\Psi_0(\eta) = 1; \qquad \lambda_0 = 0; \tag{22}$$

For $\lambda > 0$:

$$\Psi_n(\eta) = \cos(\lambda_n\eta); \qquad \lambda_n = n\pi, \quad \text{para} \quad n = 1, 2, 3, \ldots \tag{23}$$

For $\mu = 0$:

$$\Xi_0(\xi) = 1; \qquad \mu_0 = 0; \tag{24}$$

For $\mu > 0$:

$$\Xi_m(\xi) = \cos(\mu_m \xi); \qquad \mu_m = m\pi, \quad \text{para} \quad m = 1, 2, 3, \ldots \tag{25}$$

The equation (2a) is rewritten, multiplied by Ψ_n and by Ξ_m and integrated in the domain of ξ and η.

$$\int_0^1 \int_0^1 \frac{\partial^2 \Theta}{\partial \xi^2} \Xi_m \Psi_n \mathrm{d}\xi \mathrm{d}\eta + \beta^2 \int_0^1 \int_0^1 \frac{\partial^2 \Theta}{\partial \eta^2} \Xi_m \Psi_n \mathrm{d}\xi \mathrm{d}\eta - \mathrm{Bi}\gamma \int_0^1 \int_0^1 \Theta \Xi_m \Psi_n \mathrm{d}\xi \mathrm{d}\eta = - \int_0^1 \int_0^1 G \Xi_m \Psi_n \mathrm{d}\xi \mathrm{d}\eta \tag{26}$$

Solving the equation, it is obtained the values of $\bar{\bar{\Theta}}_{nm}$:

$$\bar{\bar{\Theta}}_{nm} = \frac{\bar{\bar{G}}_{nm}}{(\mu_m^2 + \beta^2 \lambda_n^2 + \mathrm{Bi}\gamma)} \quad \text{para} \quad n \neq 0 \quad \text{e} \quad m \neq 0 \tag{27a}$$

$$\bar{\bar{\Theta}}_{0m} = \frac{\bar{\bar{G}}_{0m}}{(\mu_m^2 + \mathrm{Bi}\gamma)} \quad \text{para} \quad n = 0 \quad \text{e} \quad m \neq 0 \tag{27b}$$

$$\bar{\bar{\Theta}}_{n0} = \frac{\bar{\bar{G}}_{n0}}{(\beta^2 \lambda_n^2 + \mathrm{Bi}\gamma)} \quad \text{para} \quad n \neq 0 \quad \text{e} \quad m = 0 \tag{27c}$$

$$\bar{\bar{\Theta}}_{00} = \frac{\bar{\bar{G}}_{00}}{(\mathrm{Bi}\gamma)} \quad \text{para} \quad n = 0 \quad \text{e} \quad m = 0 \tag{27d}$$

This algebraic system of equations is more compact than the analytical solution of single transformation of CITT.

To obtain the final temperature, a inversion formula (17) must be applied. The double summation is reduced to one as can be seen in the equation (28) by the procedure called Reordenament, where the higher values of $\bar{\bar{\Theta}}$ by combinations of n and m are selected for the convergence of the solution.

$$\Theta = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\bar{\bar{\Theta}}_{nm}(\xi)(\eta)\Psi_n(\eta)\Xi_m(\xi)}{N_{y_n} N_{xm}} \quad \rightarrow \quad \Theta = \sum_{q=0}^{\infty} \frac{\bar{\bar{\Theta}}_q(\xi)(\eta)\Psi_n(\eta)\Xi_m(\xi)}{N_{y_n} N_{xm}} \tag{28}$$

where q is the decreasing position of $\bar{\bar{\Theta}}$ after the reordering scheme. The summation must be truncated for a finite value ($n_{\max}$).

The transformed heat generations are defined as:

$$\bar{\bar{G}}_{nm} = \sum_{k=1}^{k_{max}} G_k \int \int_{\Omega_k} \Xi_m \Psi_n \mathrm{d}\xi \mathrm{d}\eta \quad \text{para} \quad n \neq 0 \quad \text{e} \quad m \neq 0 \tag{29a}$$

$$\bar{\bar{G}}_{0m} = \sum_{k=1}^{k_{max}} G_k \int \int_{\Omega k} \Xi_m \Psi_0 \mathrm{d}\xi \mathrm{d}\eta \quad \text{para} \quad n = 0 \quad \text{e} \quad m \neq 0 \tag{29b}$$

$$\bar{\bar{G}}_{n0} = \sum_{k=1}^{k_{max}} G_k \int \int_{\Omega_k} \Xi_0 \Psi_n \mathrm{d}\xi \mathrm{d}\eta \quad \text{para} \quad n \neq 0 \quad \text{e} \quad m = 0 \tag{29c}$$

$$\bar{\bar{G}}_{00} = \sum_{k=1}^{k_{max}} G_k \int \int_{\Omega_k} \Xi_0 \Psi_0 \mathrm{d}\xi \mathrm{d}\eta \quad \text{para} \quad n = 0 \quad \text{e} \quad m = 0 \tag{29d}$$

where G_K represents each heat generation, which are constant values. The domain Ω_k is defined by the ocupated area of each component responsible for heat generation on the microchip. k_{max} is the total of generations in the microchip.

Each heat generation, called G_K, has its own domain in the chip. For this work, the heat generations are indicated by the lighter color in figure 3 and the intensities are defined in table 1. k_{max} in this case is 5.

3. Results and Discussion

After describing the problem formulation and solution methodology, in this section the results are shown. There were selected 1, 0.1 and 0.01 as values to $\mathrm{Bi}\gamma$ and 1 and 0.8 as values to β in different points of the microchip. Each combination of $\mathrm{Bi}\gamma$ and β admited a different solution for $\bar{\Theta}_0$ and $\bar{\Theta}_n$.

Figure 3: **Layout of the heat generation in the SSE**

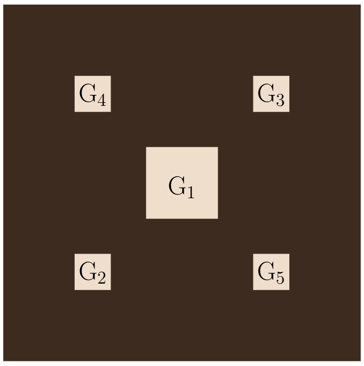


Table 1: **Heat Generation's Intensity**

G_1	1.5
G_2	1.2
G_3	1
G_4	0.6
G_5	2

The figure 4 presents the thermal analysis of the SSE. The figure 4a shows the different heat generation in each component of the SSE and its distribution on the chip. The dark part of the chip indicates that there isn't any heat generation in the region. The solution of the problem is indicated in figure 4b for $\mathrm{Bi}\gamma = 0.01$ and $\beta = 1$.

The figure 4b exposes the effect of a symmetrical layout but different intensity of its components generates an assymetrical thermal profile. Even though G_1 have a lower intensity than G_5, its bigger area causes a higher heating where this component G_1 is located. The reason for it is that heat generations depends on its volume, and in this case because of partial lumping on z-direction, depends on the components's area. G_5 presents the highest heat generation intensity in this SSE, which can be noticed when comparing its thermal effect with the other same sized components, such as G_2, G_3 and G_4.

The results of convergence of CITT by double transformation are presented on table 2.

Analyzing table 2 of CITT's double transformation, it can be noticed that the lower $\mathrm{Bi}\gamma$ favored a faster convergence of the solution, as can be seen in for $\mathrm{Bi}\gamma = 0.01$ where all the selected points have convergenced within 300 terms. In other cases, such as $\mathrm{Bi}\gamma = 1$ and β=1 required 3000 terms for its convergence on position (0.9,0.9). Although the solution using the double transformation of CITT is simpler than single transformation, the solution required many terms for its convergence.

Figure 4: **Analysis of the SSE**

(a) heat generation distribution

(b) CITT solution for $\mathrm{Bi}\gamma = 0.01$ and $\beta = 1$

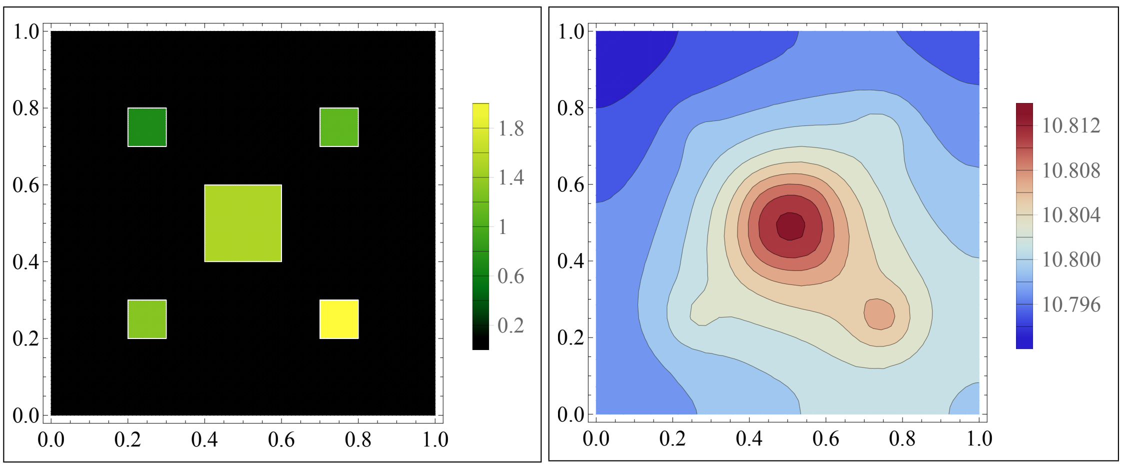


Table 2: Temperature $\Theta(\xi, \eta)$ Convergence solved CITT-double Transformation

$n_{\max}$	Θ(0.2,0.6)	Θ(0.5,0.5)	Θ(0.7,0.3)	Θ(0.9,0.9)	Θ(0.2,0.6)	Θ(0.5,0.5)	Θ(0.7,0.3)	Θ(0.9,0.9)
	Biγ =1 & β= 1				Bi γ =1 & β= 0.8			
300	0.106165	0.120257	0.113477	0.104136	0.106776	0.123308	0.114909	0.102245
500	0.106165	0.120255	0.113475	0.104139	0.106772	0.123304	0.114907	0.102249
1000	0.106165	0.120256	0.113474	0.104138	0.106773	0.123305	0.114908	0.102248
2000	0.106165	0.120257	0.113474	0.104139	0.106773	0.123306	0.114908	0.102248
3000	0.106165	0.120257	0.113474	0.104138	0.106773	0.123306	0.114908	0.102248
	Biγ =0.1 & β= 1				Biγ =0.1 & β= 0.8			
300	1.07802	1.09243	1.08572	1.07599	1.07862	1.09559	1.08734	1.07380
500	1.07802	1.09242	1.08572	1.07600	1.07862	1.09559	1.08733	1.07380
1000	1.07802	1.09243	1.08572	1.07599	1.07862	1.09559	1.08733	1.07380
2000	1.07802	1.09243	1.08572	1.07600	1.07862	1.09559	1.08734	1.07380
3000	1.07802	1.09243	1.08572	1.07600	1.07862	1.09559	1.08733	1.07380
	Biγ =0.01 & β=1				Biγ =0.01 & β= 0.8			
300	10.7980	10.8124	10.8058	10.7960	10.7986	10.8156	10.8074	10.7938
500	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938
1000	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938
2000	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938
3000	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938

The results of convergence of CITT by single transformation are presented on table 3.

Table 3: Temperature $\Theta(\xi, \eta)$ Convergence solved CITT-single Transformation

$n_{\max}$	Θ(0.2,0.6)	Θ(0.5,0.5)	Θ(0.7,0.3)	Θ(0.9,0.9)	Θ(0.2,0.6)	Θ(0.5,0.5)	Θ(0.7,0.3)	Θ(0.9,0.9)
	Biγ =1 & β= 1				Bi γ =1 & β= 0.8			
5	0.106241	0.119493	0.113402	0.104149	0.10689	0.122211	0.114841	0.102272
10	0.106118	0.120422	0.113522	0.104138	0.106701	0.123558	0.11498	0.102245
20	0.106167	0.120227	0.113465	0.104138	0.106776	0.123259	0.114894	0.102248
40	0.106165	0.120252	0.113473	0.104138	0.106773	0.123299	0.114906	0.102248
50	0.106165	0.120259	0.113475	0.104138	0.106772	0.123309	0.114909	0.102248
DT-CITT	0.106165	0.120257	0.113474	0.104138	0.106773	0.123306	0.114908	0.102248
	Biγ =0.1 & β= 1				Biγ =0.1 & β= 0.8			
5	1.07809	1.09166	1.08565	1.07601	1.07873	1.09449	1.08727	1.07383
10	1.07797	1.09259	1.08577	1.07599	1.07854	1.09584	1.08741	1.07380
20	1.07802	1.09240	1.08571	1.07600	1.07862	1.09554	1.08732	1.07380
40	1.07802	1.09242	1.08572	1.07600	1.07862	1.09558	1.08733	1.07380
50	1.07802	1.09243	1.08572	1.07600	1.07862	1.09559	1.08734	1.07380
DT-CITT	1.07802	1.09243	1.08572	1.07600	1.07862	1.09559	1.08733	1.07380
	Biγ =0.01 & β=1				Biγ =0.01 & β= 0.8			
5	10.7981	10.8117	10.8057	10.7960	10.7987	10.8145	10.8073	10.7938
10	10.7980	10.8126	10.8058	10.7960	10.7985	10.8159	10.8075	10.7937
20	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938
40	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938
50	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938
DT-CITT	10.7980	10.8124	10.8057	10.7960	10.7986	10.8156	10.8074	10.7938

The table 3 shows that the single transformation of CITT converges faster than the double transformation. DT-CITT indicates the converged value using double transformation of CITT. For Biγ=0.01 and β=1 the summation of 20 terms in single transformation gets the same value of double transformation, which required 300 terms for its convergence in position (0.5,0.5). For Biγ=0.1 and β=1 also in position (0.5,0.5) required 50 terms using single tranformation against 1000 from double.

Single transformation shows being more efficient to obtain the solution for heat conduction in Solid State Electronics.

4. Conclusion

This paper presented the comparison of the solution by Classical Integral Transform Technique of a solid-state device with heat generation and natural convection by single and double transformation. It was presented an SSE with a symmetrical layout of components but with different heat generations in each one of the components. It was shown the convergence for different values of Biγ and β in different points of the plate and single and double transformations were compared by its solution methodology and the convergence analysis.

Solving the problem by double transformation presents a more compact solution, in a algebraic system of equations, than by single transformation of CITT with a complex analytical solution. The convergence analysis, however, showed that CITT-single transformation has a greater performance then the double transformation for the case with multiple heat generation over the chip. Single transformation required very few terms and obtained a fully converged six-digits solution.

Finally, the integral transform technique has shown to be a good alternative method for this kind of problem and the single transformation of CITT is a better methodology of solution in this case than double transformation.

5. References

Braga Junior, N.R. and Sphaier, L.A., 2014. "Effective thermal conductivity of composite materials using the integral transform technique". In *15th Brazilian Congress of Thermal Sciences and Engineering - ENCIT*.

Chalhub, D.J.N.M., Sphaier, L.A. and de B Alves, L.S., 2014. "Semi-analytical method for the solution of the poisson equation derived from the navier-stokes using integral transform". In *ASME 2014 12th International Conference on Nanochannels, Microchannels and Minichannels collocated with the ASME 2014 4th Joint US-European Fluids Engineering Division Summer Meeting*.

Chelikowsky, J.R. and Franciosi, A., 1991. *Electronic Materials: A New Era in Materials Science*. Springer-Verlag.

Corrêa, L.M. and Chalhub, D.J.N.M., 2017. "Solution of the heat conduction in solid-state electronics by integral transforms". In *24th ABCM International Congress of Mechanical Engineering*.

Cotta, R., 1993. *Integral Transforms In Computacional Heat*. CRC Press.

da Silva, L.C., Chalhub, D.J.N.M., Calil, A.L.O. and de Moura, R.S., 2017. "Semi-analytical solution of the heat conduction in a plate with heat generation". *Engenharia Térmica (Thermal Engineering), Vol. 16, No. 1, pp. 58–65*.

Dantas, L.B., 1996. *Estudo da Transferência de Calor por Encapsulamento Plástico Usando a Técnica da Transformada Integral Generalizada*. Master's thesis, Universidade Federal do Rio de Janeiro.

Hahn, D.W. and Özisik, M.N., 2012. *Heat Conduction*. John Willy & Sons.

Peterson, G.P. and Ortega, A., 1994. "Thermal control of electronic equipment and devices". *Advances in Heat Transfer, vol 20, pg 181-314*.

Sphaier, L. and Cotta, R., 2000. "Integral transform analysis of multidimensional eigenvalue problems within irregular domains". *Numerical Heat Transfer Part B*.