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PIEZOELECTRIC ENERGY HARVESTING BY USING A TWO-DEGREES-OF-FREEDOM PORTAL FRAME

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Abstract: *In this work, a piezoelectric energy harvesting application will be considered using a two-degrees-of-freedom portal frame. The piezoelectric material will be considered as a linear device, however, using an electric voltage mathematical model. The portal frame is a two-degrees-of-freedom structure considering a quadratic coupling in its coupling and a two-to-one internal resonance, which is the majority condition to present saturation phenomenon. Saturation phenomenon is when the vibration energy of a coordinate saturates and partially transfer from it to the other one, which allows the energy harvesting, in this case, due to the transfer from beam's vibration to the columns. The governing equations of motion of the system were obtained by applying the energy method of Lagrange. The results showed a considerably amount of power due the saturation phenomenon.*

Key-Words: *Piezoelectric energy harvesting, nonlinear dynamics, electric voltage model, portal frame, saturation phenomenon*

1. INTRODUCTION

Nowadays, the world demand for energy has increased intensively. Therefore, the search for new energy sources have been of great importance (Rocha, 2016 and Abdelkefi, 2012).

Due to the development of technology, it has become possible to explore new energy sources from many different ways, from higher to smaller ways, that is, high scale energy sources and low-power consumption devices. Such low-power devices have the advantage to be smaller, cheaper and of easy application.

Piezoelectric (PZT) materials have becoming one of the most used device to energy harvesting of low-power consumption from environmental vibration. Many studies involving energy harvesting using piezoelectric material has been developed, as for example, the works of Stephen (2006); Jalili (2009); Preumont (2006); Erturk (2009); Rocha (2016); Priya and Inman (2009) and Abdelkefi (2012), among others.

There are many kinds of structures that are in constant vibration motion due to many kinds of external excitation. Depending on such structure, it may possess some kind of phenomenon that can directly affect the structural integrity of an engineering structure. In special, portal frames of two-degrees-of-freedom has shown that, with a structural quadratic stiffness, when externally excited near the resonance possessing a 2:1 internal resonance between its coordinates, the system may present a well-known phenomenon called saturation. Such phenomenon has been study by many authors in nonlinear and non-ideal dynamics, chaos, control and energy harvesting, for example, in the works of Rocha (2016); Nayfeh (2000); Nayfeh and Mook (2008); Mook *et al.* (1985); Rocha *et al.* (2017a,b); Golnaraghi (1991); Oueini (1999); Oueini *et al.* (1997); Pai *et al.* (1998, 2000); Balthazar *et al.* (2003); Felix *et al.* (2005, 2014) and Tusset *et al*. (2015, 2017).

The structural integrity of a engineering structure may be maintained by suppressing the vibrations that may be of high amplitudes. The introduction of a PZT transducer on such vibrating structure is a good solution and way to suppress part of vibration energy and convert it into electric energy.

Therefore, this work presents a piezoelectric energy harvesting application coupled to a two-degrees-of-freedom portal frame. The two-degrees-of-freedom portal frame consists of the same physical model of Rocha *et al.* (2017a,b), which is a supported beam pinned in two columns clamped in their base. The beam and columns are considered as a concentrated mass, whose masses possesses nodal displacements in vertical and horizontal motion direction. The generalized coordinates of the portal frame are considered by the mid-span mass of the beam which possess a 2:1

internal resonance between vertical and horizontal coordinates. In addition, the system is base excited by a harmonic force in resonance to the vertical coordinate. With those conditions, saturation phenomenon occurs.

The piezoelectric material is considered as a linear device proposed by Erturk (2009). The PZT is coupled to a column of the structure to harvest the surplus energy after saturation phenomenon.

Next section, the mathematical modelling is carried out and discussed.

2. MATHEMATICAL MODELLING

The two-degrees-of-freedom portal frame model with the piezoelectric material coupled to a column is represented in Fig. 1. The two-degrees-of-freedom portal frame platform consists in the same of (Rocha, 2016) which is a supported beam of length L pinned to two columns with height h that are clamped in a base with mass m_0. The beam and columns are considered as a concentrated mass, which are M and m, respectively, whose masses are of two-degrees-of-freedom that is the vertical and horizontal motion. The coordinate x_1 is related to the horizontal displacement in the sway mode, with natural frequency ω_1, and x_2 to the mid-span vertical displacement of the beam in the symmetric mode, with natural frequency ω_2. The linear stiffness of the columns and the beam can be evaluated by a Rayleigh-Ritz procedure using cubic trial functions. Geometric nonlinearity is introduced by considering the shortening due to bending of the columns and of the beam.

The novelty of this work is the linear piezoelectric model proposed by Erturk (2009). The model consists of a RC circuit of voltage output v. The piezoelectric material is coupled to a column in order to convert the high amplitudes of column vibration energy into electric energy.

The external excitation is represented by a harmonic force that is $F_{ext} = E_0 cos\Omega\tau$, where E_0 is de amplitude of the external excitation and Ω is the frequency of the external excitation. The external frequency is set near resonance with the second mode, which is the twice of the frequency of the first mode, i.e., $\Omega \approx \omega_2 + \sigma_2$ and $\omega_2 \approx 2\omega_1 + \sigma_1$. Such resonant conditions are set to saturation phenomenon occurs.

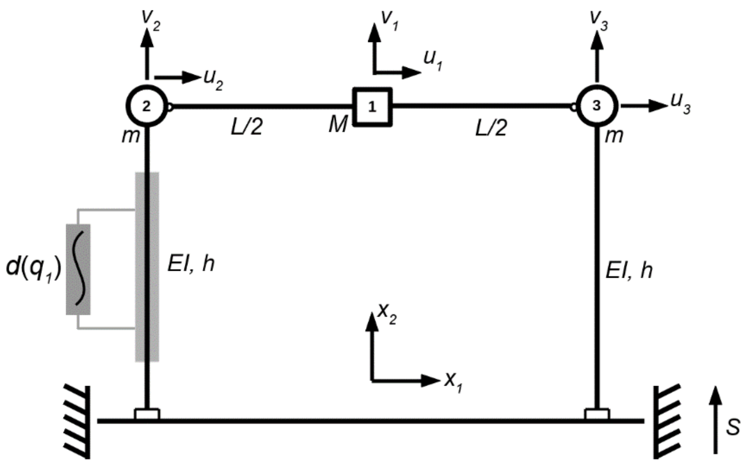


Figure 1. Portal frame physical model

The equations of motion of the system in dimensionless form is given by Eq. (1), where x_1 is the dimensionless horizontal displacement of the mid-span of the beam, x_2 is the dimensionless vertical displacement of the mid-span of the beam and v is the dimensionless voltage of the piezoelectric material.

$$\begin{aligned} &x_1'' + \mu_1 x_1' + x_1 + \alpha_1 x_1 x_2 = \delta_1 \theta v \\ &x_2'' + \mu_2 x_2' + \omega_2^2 x_2 + \alpha_2 x_1^2 + G_2 = \theta \delta_2 v x_2 + E_0 \cos \Omega \tau \\ &v' + \delta_5 v - \theta\left(\delta_3 x_1 + \delta_4 x_2^2\right) = 0 \end{aligned} \quad (1)$$

The energy harvesting of the system is computed by Eq. (2).

$$P = \frac{v^2}{R} \quad (2)$$

where the constant R is the load resistance of the piezoelectric material.

In the next section, some numerical simulations involving the governing equations of motion of the system and the harvested power will be presented and discussed.

3. MATHEMATICAL MODELLING

The numerical simulations were performed using the method of Runge Kutta of 4th and 5th order with the parameters $\mu_1 = 0.007$, $\mu_2 = 0.0212$, $\alpha_1 = 4.6103$, $\alpha_2 = 1.6573$, $\alpha 4 = 0.1499$, $\omega_2 = 1.9974$, $G_2 = 0.0034$, $\theta = 0.3$, $E_0 = 0.007$, $\Omega = 2.0$, $\delta_1 = 1.6909\text{x}10^{-4}$, $\delta_2 = 4.2141\text{x}10^{-4}$, $\delta_3 = 486.48$, $\delta_4 = 843.24$, $\delta_5 = 0.1351$, R = 1kΩ.

Here, the parameters were set to satisfy the conditions of saturation phenomenon, which is the 2:1 internal resonance and external resonance. In addition, dampings have to be small, i.e., a low structural damping. Moreover, all the results provided in this work are dimensionless, including the average power.

Figure 2a shows the time history of displacement of horizontal (red) and vertical (black) displacement of the beam without the piezoelectric material coupling. It is possible to observe that saturation phenomenon occurred when the vertical displacement reached to a critical amplitude (near 100seconds), after that there was energy exchange and almost all the vibration energy was transferred to the horizontal direction.

With the coupling of piezoelectric material, the same time history of displacements, illustrated in Fig. 2b, shows that a great amount of vibration was suppressed. It is due to the piezoelectric material that converted the vibration energy into electric energy. The amount of average power harvested through the piezoelectric material was 0.0880.

In addition, the system keeps its behavior periodic, as shown in Figs. 2c and 2d, which is very important to maintain the energy harvesting through the piezoelectric material.

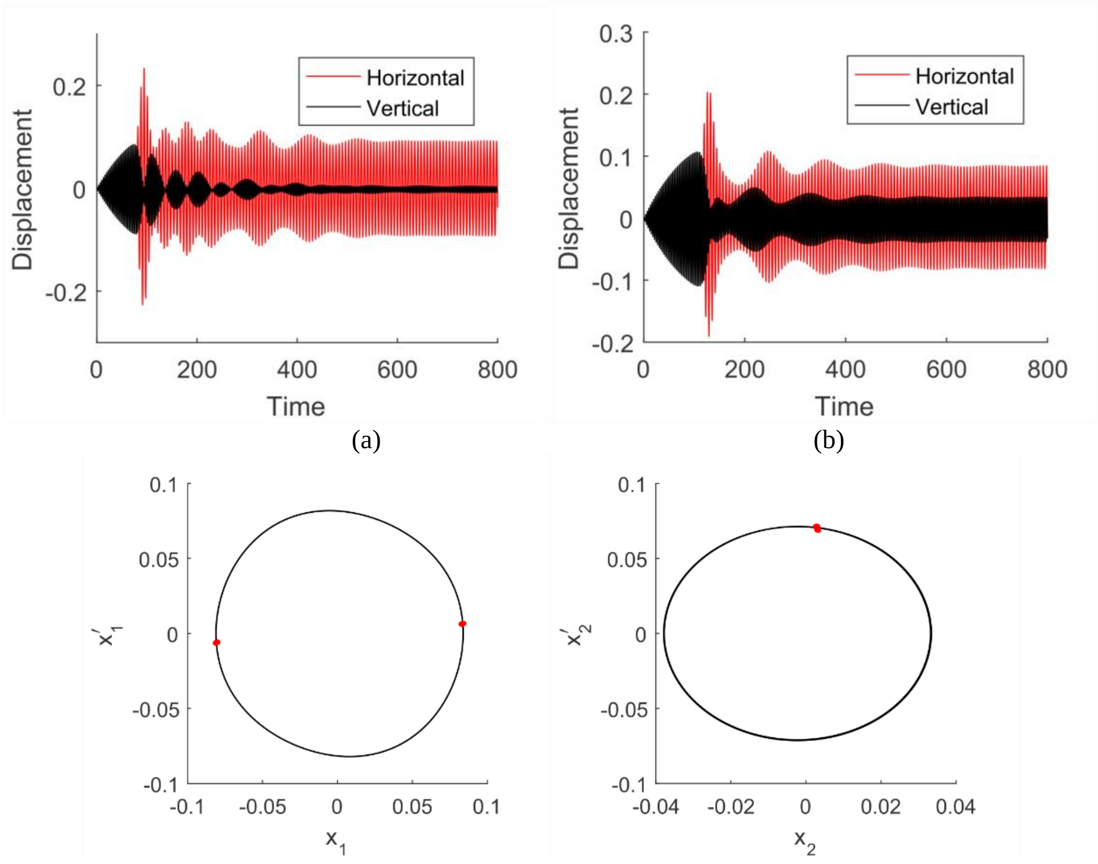


Figure 2. Time histories of displacement of vertical (in black) and horizontal (in red) motions when (a) $\theta = 0$; (b) $\theta = 0.3$; and Phase planes (black) and Poincare maps (red dots) of (c) horizontal motion, (d) vertical motion with $\theta = 0.3$

Some of the parameters of the system are very important to the analysis of energy harvesting and structural integrity, which are the amplitude E_0 and frequency Ω of the external excitation, and the piezoelectric coupling constant θ. The frequency is important due to saturation phenomenon conditions, and the amplitude and piezoelectric coupling constant are important to the average harvested power. Thus, the intervals of those parameters were set as $0.01 \leq E_0 \leq 0.09$, $1.4 \leq \Omega \leq 2.4$ and $0 \leq \theta \leq 0.5$.

Figures 3 show the analysis of the amplitude of horizontal displacement, in Figs. 3a and 3b, and the average harvested power, in Fig 3c, of the variation of E_0 and Ω. It is possible to see that saturation occurs when $1.8 \leq \Omega \leq 2.2$, approximately. In Fig. 3a, the blue lines show the increasing of the amplitude of the horizontal coordinate. When saturation occurred, the horizontal motion becomes higher and the vertical one decreases, as it is possible to observe in such Figures 3a and 3b. However, in the region delimited by the red line, which is the black region of Fig. 3b, the horizontal amplitude becomes such higher that the system becomes unstable (unsafe area), i.e., the system could collapse or be damaged.

Figure 3c shows that the highest average harvested power is near the unsafe area, that is $E_0 = 0.087$ and $\Omega = 1.807$, whose power increased to 8.189 amount of power, approximately.

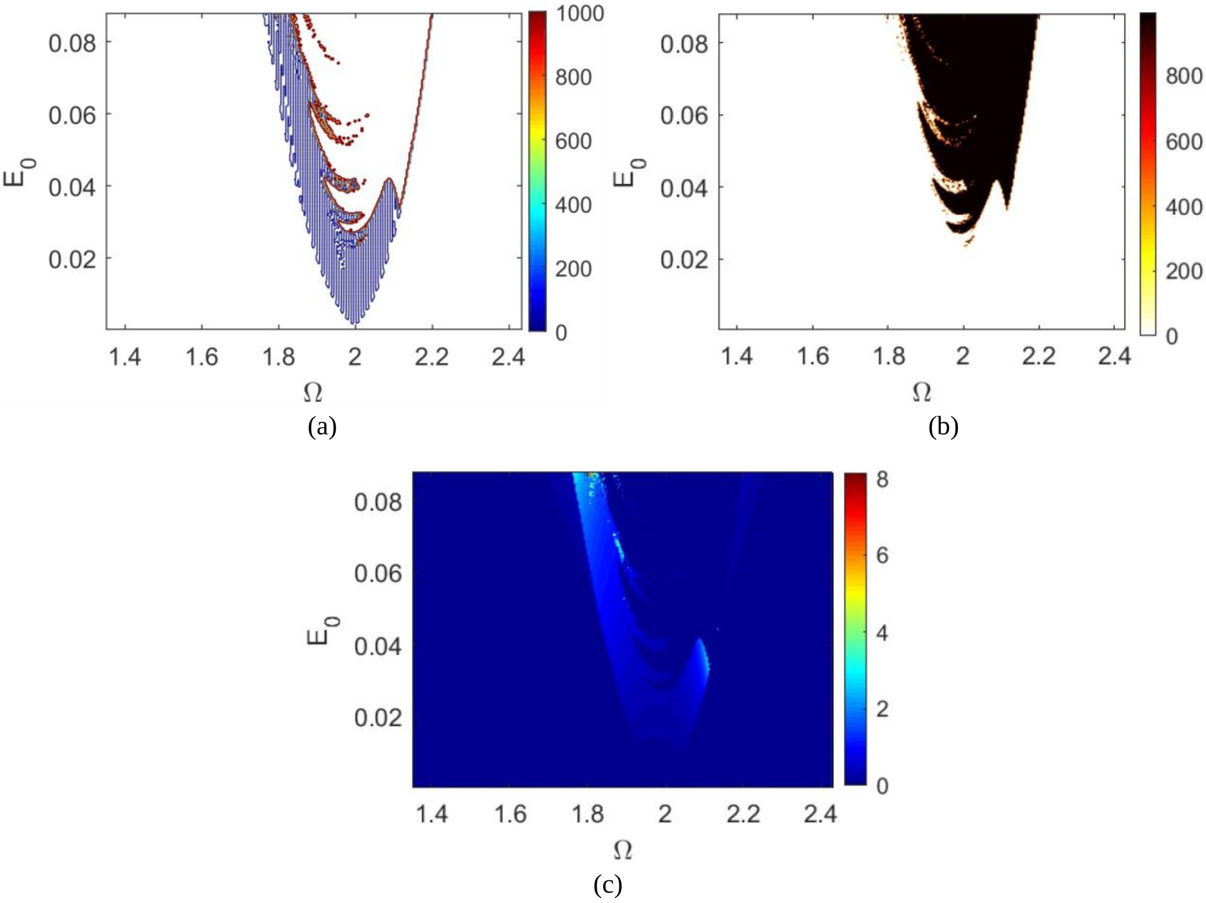
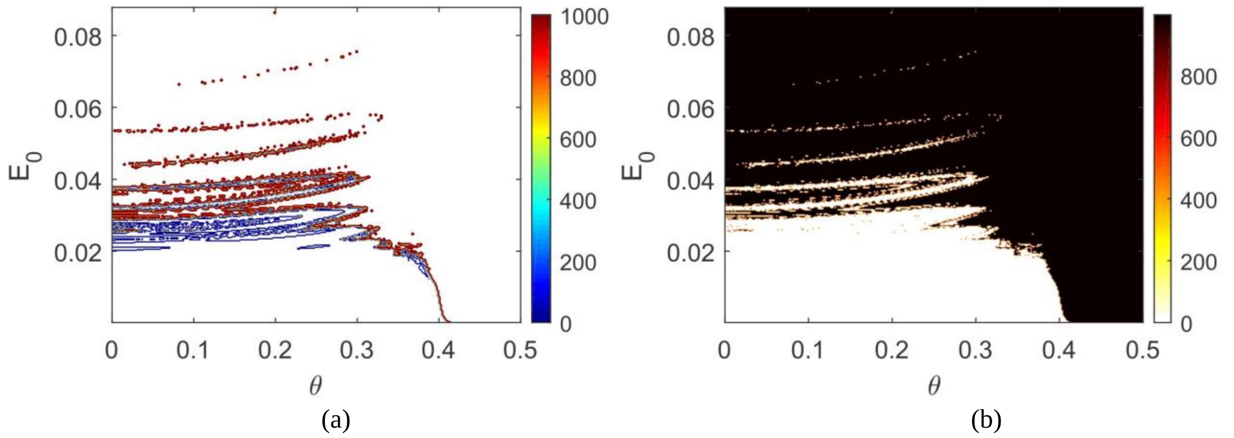
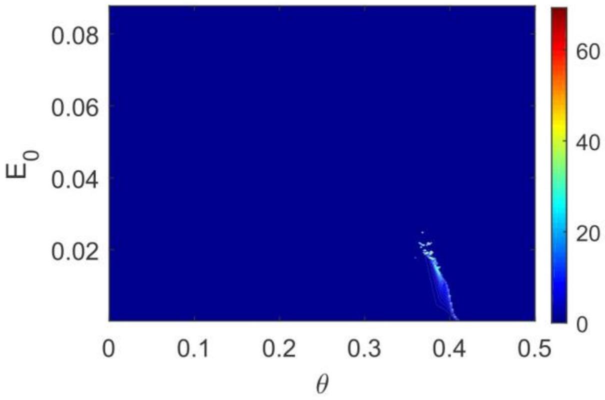


Figure 3. Amplitude of horizontal displacement (a) in contours of safe area, (b) unsafe area; (c) average harvested power of the variation of E_0 and Ω

Figures 4 show the analysis of the amplitude of horizontal displacement, in Figs. 4a and 4b, and the average harvested power, in Fig 4c, of the variation of E_0 and θ. The analysis of the piezoelectric coupling is very important due to its direct link with the harvested power, the higher the piezoelectric coupling constant the higher is the harvested power. Thus, in this case, the region of stable system is smaller, as it can be seen by Figs. 4a and 4b, where the black area represents an unstable area (unsafe).

Figure 4c shows that the highest average harvested power is also near the unsafe area, whose point is $E_0 = 0.1893$ and $\theta = 0.3512$, with 43.75 amount of power. However, this value is very close to an unsafe point, and such power could be dangerous to be harvested.





(c)

Figure 4. Amplitude of horizontal displacement (a) in contours of safe area, (b) unsafe area; (c) average harvested power of the variation of E_0 and θ

4. CONCLUSIONS

This work showed the piezoelectric energy harvesting by using a two-degrees-of-freedom portal frame structure considering a linear piezoelectric model.

As predicted, saturation phenomenon occurred due to the conditions of the system, which are 2:1 internal resonance, the external resonance with the second mode and a quadratic coupling between the two coordinates of the system.

Still, parametrical analyses were developed considering the analysis of the average harvested power and the amplitude of horizontal motion at steady state, related to the combination of the amplitude of excitation versus external frequency and piezoelectric coupling. In such analyses, it was possible to observe that the average harvested power increased with saturation phenomenon, due to the coupling of piezoelectric on a column of the portal frame, and, in addition, the higher the piezoelectric coupling the higher is the average harvested power, however, the system became more sensible due to the piezoelectric parameter.

Moreover, one of the most important remark is that, when the system is stable, its motion is totally periodic, which is the recommended behavior to harvest energy using a PZT device.

5. ACKNOWLEDGMENTS

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