A VARIATIONAL CONSTITUTIVE MODEL FOR SOFT BIOLOGICAL TISSUES CONSIDERING INDIVIDUAL PARAMETERS

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Abstract. *Soft biological tissues are mainly composed by collagen, elastin and ground substance. Due to the soft biological tissues constitutions, its mechanical behavior is nonlinear, viscoelastic and anisotropic. Moreover, tissue's mechanical behavior may be different for distinct individual, which represent a strong restriction to apply numerical simulations on real situations. Aiming to correlate the mechanical behavior of the biological tissue with individual parameters, the scope of the present work is to develop a material model capable to incorporate specific individual variables dependencies, such as physical, physiological and histological parameters. The constitutive model is based on a variational framework, in which a local minimization provides the internal variables updates for each load increment. The variational framework allows constructing suitable nonlinear potentials, with the expected individual dependence, in order to obtain the mechanical behavior observed experimentally. Numerical tests evidence the capability of the model to represent the tissue's mechanical behavior considering an individual parameter.*

Keywords: *Variational Constitutive Model. Biological Tissues. Individual Parameters.*

1. INTRODUCTION

Biological tissues present a structural functionality in animal bodies, influenced by the force systems in which the biological structures are subjected. The numerical simulation of these structures may be important in several applications, such as, artery/stent interaction, knee prosthesis/muscle skeletal system, control surgeries, among others. The prediction of internal forces and deformation may provide an important information in the choice of individualized medical procedures.

Biological tissues are composed by cellular and extracellular matrix (Junqueira *et al*., 2004). The extracellular matrix is formed by fibers (collagen and elastin) and ground substance which is composed by proteoglycans (Cowin and Doty, 2007). Arteries, tendons and ligaments, due to the presence of fibers highly oriented into a matrix represented by the ground substance, are usually modeled as composites (Holzapfel *et al*., 2000, Ehret *et al*., 2011, Vassoler *et al*., 2012). Their mechanical behavior is non-linear and anisotropic, being sensitive to strain velocity, humidity and temperature, and being susceptible to damage.

Despite this complex mechanical behavior, the major difficulty for the use of numerical simulation in real applications reside in to consider individual aspects. The same biological tissue may present a distinct mechanical behavior in different individuals or different positions of the same individual (Learoyd and Taylor, 1966, Maher *et al*., 2012). This aspect is not considered in most material models available in literature (Holzapfel *et al*., 2000, Ehret *et al*., 2011).

Then, the main objective of the present work is the study and development of a constitutive model capable to consider the individual aspects applying the variational framework.

2. CONSTITUTIVE MODEL

The mathematical description of biological tissues mechanical behavior is carried out through several assumptions and models. Variational approach is one of these forms.

The variational structure used herein is able to accommodate a wide range of constitutive models, which depend on the choice of certain potential functions. This family makes use of an incremental potential that satisfies the constraints on the evolution of internal variables through an extremity problem.

The general structure of this variational formulation was proposed by Radovitzky and Ortiz (1999) and Ortiz and Stainier (1999). Moreover, Fancello *et al*. (2006) extended the formulation proposing a spectral decomposition in the deformation and Vassoler *et al*. (2012) included the viscoelastic behavior reinforced by fibers.

The fibers behavior is included over an additive decomposition in the incremental potential (Vassoler *et al*., 2012),

$$\Psi = \Psi_{iso} + \Psi_f \tag{1}$$

where Ψ_{iso} is the isotropic incremental potential and Ψ_f is the fibers decoupled contribution. Both are represented by a Generalized Maxwell Model (Vassoler *et al*., 2012).

For the variational formulation to be sensitive to individual aspects, the energy potentials (φ_f and φ_f^e) and/or the dissipative pseudo potentials (ψ) must depend on individual parameters.

A way to introduce this dependency is incorporating a single parameter dependence (or parameters combinations) into the elastic energies φ_f and φ_f^e.

The energy potential proposed will depend on fiber stretch (λ_f) and on individual parameter (β), using a function $g(\beta)$. The individual parameter it is supposed to be parameter obtained from histological analyses of the tissue (Sokolis, 2010). Physical and physiological parameters can be used as well Dunkman *et al*. (2013) and Camhi *et al*. (2011). The proposal of the use of a function $g(\beta)$, which modifies a constitutive relation, is based on models found in literature (Avalle *et al*., 2007, Ehret *et al*., 2011), which use functions to incorporate certain characteristics into their phenomenological response. This proposal allows the use of classic fibers hyperelastic models in the potential φ_f. Then, starting from a generalized function, dependent on the individual parameters (or parameters combinations), the $g(\beta)$ function can be defined as

$$g(\beta) = \mu\beta^{\alpha} \tag{2}$$

where μ and α are material parameters.

Considering only the fibers contribution, following arguments like those used in Fancello *et al*. (2006) and Vassoler *et al*. (2012), the incremental potential of the variational model can be defined as

$$\Psi_f = \Delta\varphi_f\left(\lambda_{f_{n+1}}\right) + \min_{\lambda^v_{f_{n+1}}}\left\{\Delta\varphi_f^e\left(\lambda^e_{f_{n+1}}\left(\lambda^v_{f_{n+1}}\right)\right) + \Delta t\psi\left(d\left(\lambda^v_{f_{n+1}}\right)\right)\right\} \tag{3}$$

where the elastic potentials show the individual parameters dependency on the function $g(\beta)$ and where

$$\Delta\varphi_f = g(\beta)\bar{\varphi}_f\left(\lambda_{f_{n+1}}\right) - g(\beta)\bar{\varphi}_f\left(\lambda_{f_n}\right) \tag{4}$$

for the main spring and

$$\Delta\varphi_f^e = g(\beta)\bar{\varphi}_f^e\left(\lambda^e_{f_{n+1}}\right) - g(\beta)\bar{\varphi}_f^e\left(\lambda^e_{f_n}\right) \tag{5}$$

for the Maxwell spring.

3. RESULTS AND DISCUSSION

To demonstrate the applicability of the $g(\beta)$ in a viscoelastic variational formulation and clarify the models' individual parameters influence, some tests were performed. For potentials ψ, φ_f (elastic) and φ_f^e where used Hencky and Holzapfel models, its parameter values were obtained from Vassoler *et al*. (2012).

The $g(\beta)$ parameter function, except β, present generic values (μ=1 and α=3), only to visualize the model behavior. It was stipulated that the parameter β varies from 0 to 1, which could represent the tendon fibers percentage, where 0 corresponds to the tendon fibers absence and 1 corresponds to a tendon formed only by fibers, obtained from a histological analysis of the tissue.

The Fig. (1) and Fig. (2) show the results only in the elastic arm (spring) of the Generalized Maxwell Model together with the function $g(\beta)$ and the Fig. (3) and Fig. (4) demonstrate the results only in the Maxwell arm of the Generalized Maxwell Model together with the function $g(\beta)$.

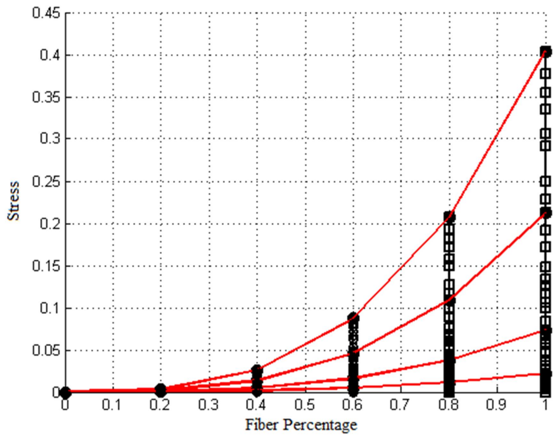

Figura 1. Stress – Fiber percentage in the elastic arm.

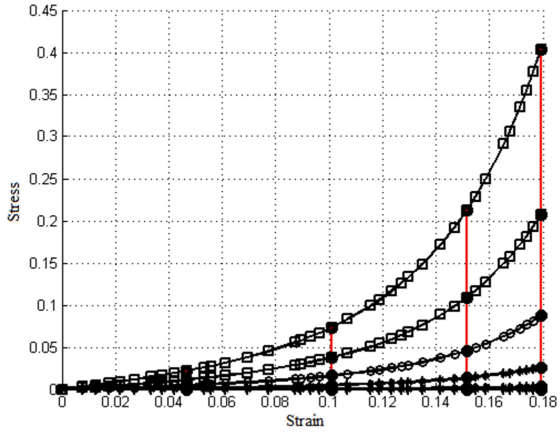

Figura 2. Stress – Strain in the elastic arm.

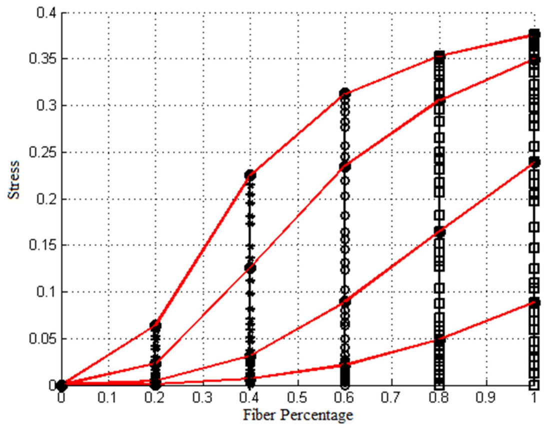

Figura 3. Stress – Fiber percentage in the Maxwell arm.

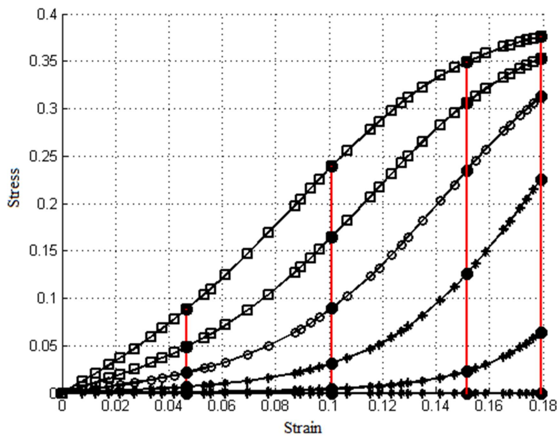

Figura 4. Stress – Strain in the Maxwell arm.

The results presented in Fig. (1), Fig. (2), Fig. (3) and Fig. (4) demonstrate that stress response changes with the fibers proportion. In addition, one can obtain great flexibility in the dependence representation of individual parameters. Depending on the choices of the other $g(\beta)$ material parameters, one can further improve the representativeness of the model.

4. CONCLUSIONS

This proposal incorporates a dependence of individual aspects into a variational constitutive model presented in literature. Through the numerical tests the capability of the model was evidenced, demonstrated that this model, together with the function $g(\beta)$, may represent the tissue's mechanical behavior considering an individual parameter. In this proposal, the individual parameters of a tissue must to be previously known, which require an experimental characterization of the tissue.

The model can improve its representativeness through a parameter identification procedure of experimental results.

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7. RESPONSIBILITY FOR INFORMATION

The authors are the solely responsible for the information included in this work.