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SIZING OPTIMIZATION OF TRUSS STRUCTURES UNDER TIME-DEPENDENT CONSTRAINTS USING THE EQUIVALENT STATIC LOADS METHOD

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Abstract. *It is well known that structural optimization subjected to transient loads is a very challenging problem in many aspects. First, because of the numerous time-dependent constraints involved in the dynamic response optimization which need to be imposed at all instants in the time domain. Second, the evaluation of the sensitivities of these constraints is very computational expensive and requires a large amount of storage. In the literature, it is shown a lot of effort on the treatment of such several constraints. In most cases a single equivalent functional constraint or the worst case is considered on computing the constraints. In this paper the Equivalent Static Load method (ESL) is considered to solve the dynamic structural response optimization problem. The ESL method transforms the original dynamic problem in a sequence of linear static optimization sub–problems. The use of such method is very interesting because it can be applied also in nonlinear dynamic optimization problems, avoiding the calculations of expensive sensitivities. Some classical planar truss-member structure problems subjected to transient loads are solved in order to validate the ESL method and comparing results in both traditional and ESL optimization approaches.*

Keywords: *equivalent static loads, structural optimization, dynamic analysis*

1. INTRODUCTION

Structural optimization is a field of study in engineering that has been developed for many decades now, and it continues to be a relevant and important area of research for the academic community in the present. Optimization has, for its primary purpose, finding the best possible configuration of a given system that minimizes or maximizes one or more of properties of interest (called objective functions) and, concerning about engineering problems, almost any problem of interest imposes limitations that must be respected (called constraints), in general because of physical properties or operation limitations of the system. The structural optimization field of research is then an area that focuses on such problems presents in engineer, such as civil structures, aerospace engineering and offshore engineering for example.

When structural optimization problems started to take place in the academic community, the major studies focused on static structural response problems. As researches followed, that group of structural optimization problems has become very well-known and there are several commercials software available on market that provide very reliable solutions to large-scale static response problems.

As most of the real world problems engineers face in their routines, the nature of the applied loads is not static, they have time dependency. Simplifications have been made by adopting dynamic factors to multiply the dynamic loads leading into equivalent loads, but the criteria to choose the factors are in general not very reliable (Kang et al, 2000).

Park and Choi (2000) developed and proposed through various articles a new method called Equivalent Static Loads (ESL) in which the static loads are defined under the condition that they must generate the same displacement field for each time interval as the actual dynamic load.

For the present paper the validation of the ESL method will be conducted in two linear truss-type member structures subject to dynamic loads and under time-dependent stress and displacement constraints, as referenced in (Silva, 1992).

2. THE DYNAMIC RESPONSE PROBLEM

2.1. FINITE ELEMENTS ANALYSIS

The dynamic governing equation for a system is described as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{F}(t) \tag{1}$$

Where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ and $\boldsymbol{F}$ represent the mass matrix, damping matrix, stiffness matrix and external loads vector respectively, $\boldsymbol{u}$, $\dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ are the displacement, velocity and acceleration responses for the dynamic load. From finite elements analysis theory, matrices and vectors for a structure discretized into small elements are generated in order to solve the dynamic response problem. This paper focuses in planar truss elements. For such member, with length L, the required matrices and vectors are (Weaver Jr., 1987):

$$\boldsymbol{N}(x) = \begin{bmatrix} 1 - \frac{x}{L} & \frac{x}{L} \end{bmatrix} \tag{2}$$

where $\boldsymbol{N}(x)$ contains the shape functions that relates the actual translation of the element coordinates $\boldsymbol{u}(t)$ with nodal displacements $\boldsymbol{u}_e(t)$:

$$\boldsymbol{u}(\boldsymbol{t}) = \boldsymbol{N}(x)\boldsymbol{u}_e(t) \tag{3}$$

The strain-displacement matrix is given by differentiation of $\boldsymbol{N}$. In a truss member, there is no stiffness but in the axial direction, therefore the strain-displacement matrix can be written as in Equation (4).

$$\boldsymbol{B} = 1/L[-1 \quad 1] \tag{4}$$

Now the stress-strain relation can be obtained by:

$$\boldsymbol{\sigma} = E\boldsymbol{B}\boldsymbol{u}_e(t) \tag{5}$$

The element stiffness and mass matrices, namely $\boldsymbol{k}$ and $\boldsymbol{m}$, can be computed using:

$$\boldsymbol{k} = \int_V \boldsymbol{B}^T E \boldsymbol{B} dV \tag{6}$$

$$\boldsymbol{m} = \int_V \rho \boldsymbol{N}^T \boldsymbol{N} dV \tag{7}$$

The rotation matrix $\boldsymbol{R}$ filled with the direction cosines is needed in order to assemble the elements matrices into the global matrices:

$$\widehat{\boldsymbol{R}} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R} \end{bmatrix} \tag{8}$$

$$\boldsymbol{R} = \begin{bmatrix} c_x & c_y \\ -c_y & c_x \end{bmatrix} \tag{9}$$

The global stiffness matrix $\boldsymbol{K}$ and $\boldsymbol{M}$ can be obtained by assembling the elementary stiffness and mass matrices, respectively, as follows:

$$\boldsymbol{K} = \sum_{e=1}^{N_{el}} \widehat{\boldsymbol{R}}^T \boldsymbol{k} \widehat{\boldsymbol{R}} \tag{10}$$

$$\boldsymbol{M} = \sum_{e=1}^{N_{el}} \widehat{\boldsymbol{R}}^T \boldsymbol{m} \widehat{\boldsymbol{R}} \tag{11}$$

2.2. NUMERICAL INTEGRATION

In order to solve Equation (1), a direct integration method is implemented. The advantages of this family of methods is that they are usually more efficient than normal mode decomposition methods, and furthermore, they can be applied to either linear and nonlinear systems. The present paper uses the Newmark's method to perform the direct numerical integration. In Newmark's method, the parameters γ and β are adjustable so we can integrate the dynamic equation by considering constant acceleration ($\beta = 0$), average acceleration, ($\beta = \frac{1}{4}$) or linear acceleration ($\beta = \frac{1}{6}$). In order to avoiding numerical damping, we set $\gamma = 1/2$. If γ is set greater than $1/2$, positive artificial damping occurs to the system, and, on the other hand, if γ is less then $1/2$, the system produces negative damping.

The Newmark numeric integration equations are expressed in Equations (12) through (16):

$$\bar{\boldsymbol{v}} = \boldsymbol{v}_0 + \Delta t(1-\gamma)\boldsymbol{a}_0 \tag{12}$$

$$\bar{\boldsymbol{u}} = \boldsymbol{u}_0 + \Delta t \boldsymbol{v}_0 + \Delta t(1-2\beta)\boldsymbol{a}_0 \tag{13}$$

$$[\boldsymbol{M} + \boldsymbol{C}\Delta t\gamma + \boldsymbol{K}\Delta t^2\beta]\boldsymbol{a}_1 = \boldsymbol{P}_1 - \boldsymbol{C}\bar{\boldsymbol{v}} - \boldsymbol{K}\bar{\boldsymbol{u}} \tag{14}$$

$$\boldsymbol{v}_1 = \bar{\boldsymbol{v}} + \Delta t\gamma \boldsymbol{a}_1 \tag{15}$$

$$\boldsymbol{u}_1 = \bar{\boldsymbol{u}} + \Delta t^2\beta \boldsymbol{a}_1 \tag{16}$$

3. THE EQUIVALENT STATIC LOAD METHOD

3.1. GENERATION OF EQUIVALENT STATIC LOADS

The general formulation for dynamic response structural optimization is given by:

$$\begin{aligned}
&\min && f(\boldsymbol{b}) \\
&\textit{s.t.} && c_{jt}\big(\boldsymbol{b}, \boldsymbol{u}(\boldsymbol{b})\big) \le 0, j = 1..m, t = 1..n \\
& && \boldsymbol{b}_l \le \boldsymbol{b} \le \boldsymbol{b}_u \\
&\textit{with} && \boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{F}(t)
\end{aligned}$$

Where $\boldsymbol{b}$ is the design vector, f is the objective function to be minimized, c_{jt} is the j-th inequality constraint in time step t, and the design vector is limited by lower and upper bounds, $\boldsymbol{b}_l$and $\boldsymbol{b}_u$. In dynamic structural optimization, the time-dependent constraints demand to be attended in every time intervals during optimization.

The concept of Equivalent Static Load is introduced as the external static load set that generates the same response field that the original dynamic load for all time steps. For a static response analysis, Equation (17) represents the structure static response:

$$\boldsymbol{K}\boldsymbol{x} = \boldsymbol{s} \tag{17}$$

Where $\boldsymbol{x}$ is the displacement field for an applied external static load $\boldsymbol{s}$. In order to the ESL and the dynamic response analysis generate the same displacements, the static analysis response must be the same obtained from Equation (1), i.e.:

$$\boldsymbol{r}_e = \boldsymbol{K}\boldsymbol{u} \tag{18}$$

The equivalent static load set $\boldsymbol{r}_e$ in Equation (18) generates exactly the same response field $\boldsymbol{d}$ as the dynamic load $\boldsymbol{F}(t)$ for all time intervals. Once the equivalent loads are static, this method generates multiple load cases, and the ESL set will have as much load cases as the number of time steps, corresponding to the original dynamic load in each time step.

3.2. STATIC SEQUENTIAL OPTIMIZATION BY ESL

The main idea behind the ESL algorithm is to perform sequential static response optimization sub-problems until the design variables vector converges under a pre-determined criteria. The algorithm starts with an initial design vector. For beginning, the dynamic analysis is performed and the dynamic response displacement field $\boldsymbol{d}$ is then used to calculate the ESL set for the current design variables set. After that, a static analysis is made considering multiple loading conditions contained in the ESL as external loads. Last, the static response optimization is made, a new design is generated and a new iteration begins.

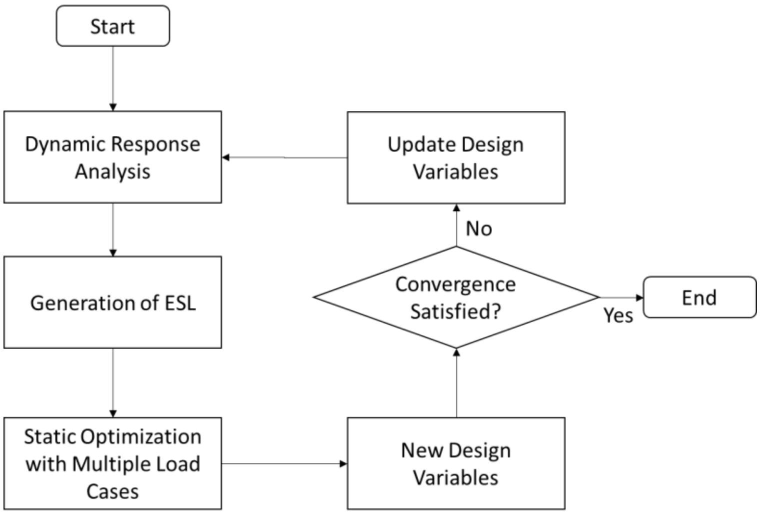

Figure 1. Flowchart for the ESL algorithm.

Every time a new ESL iteration starts, the convergence criteria must be verified. If the norm between the current and prior design vectors is under the criteria, the algorithm terminates. Otherwise, a new dynamic analysis is made in order to calculate a new set of equivalent loads and perform a new static response optimization. Figure 1 illustrates a flowchart of the ESL method in transformation from dynamic optimization to sequential static optimization.

4. EXAMPLES – TRUSS STRUCTURES

In the following section, the equivalent static method is applied in two truss examples found in Silva (1992). Also, dynamic optimization for the same examples is carried out for comparison and performance finalities. Those structures are examples of recurrent validation problems, once the dynamic behaviors are well known from vibration and finite elements models generate very accurate results.

For the two examples, no unit system in particular is determined. The values of Young's modulus and density are equal to 10^7 and 0.1 respectively. The dynamic differential equations of movement are solved from initial time 0 to final time 2, and the step time is constant and set to 0.2. The convergence tolerance set in ESL method for the design vector is 0.000001. Inside second sub-section, the evolution of each design variables vector through the ESL optimizations is described for a specific case.

4.1. FIVE-BAR TRUSS

The first example discussed in this paper is the five-bar truss problem solved in (Silva, 1992). The structure is discretized with five nodes and has 5 degrees of freedom, as depicted in Figure 2.

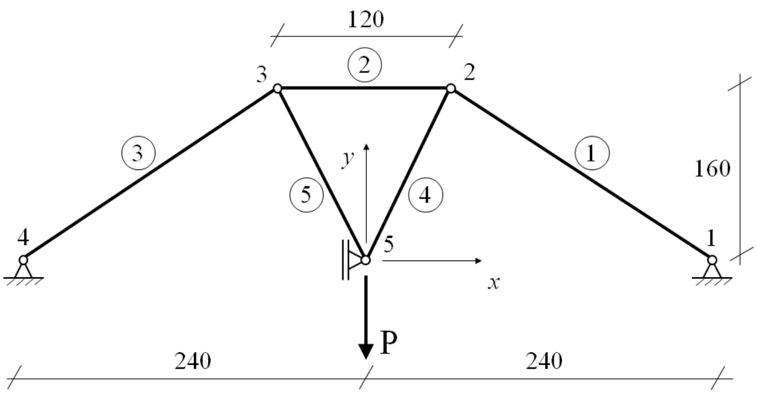

Figure 2. Five-member truss Subject to load P at node 5. Node 5 is fixed in the horizontal direction.

All the trusses in this paper are subjected to simple sine loadings. The nature of load P is dynamic and its formula is given by $P = Rsin(t)$.

For the five-member truss example, R assumes a value of 20000, whereas for the ten-bar structure, the amplitude of P is 100000. Optimization problem formulation with stress and displacement constraints is set as:

$$\begin{aligned} \min \quad & weight(\boldsymbol{b}) \\ s.t. \quad & |\sigma_i| \leq \sigma_{adm}, i = 1..n \\ & |u_j| \leq u_{adm}, j = 1..p \\ & \boldsymbol{b}_l \leq \boldsymbol{b} \leq \boldsymbol{b}_u \end{aligned}$$

Where b_i is the cross section area of the *ith* member, the allowable stress is 20000, the maximum displacement allowable is set to 1.2, p is the number of degrees of freedom subject to displacement constraint and m is the number of design variables. Because each truss member is assumed to be prismatic, the number of design variables and elements, n, is identical. The design vector is subject to lower and upper limits, $\boldsymbol{b}_l$ and $\boldsymbol{b}_u$ respectively.

The truss in Figure.1 is subject to load P and the optimization is performed in two distinct cases: Case 1 imposes stress constraint to all members and minimum area cross section allowable of 0.1 for all members. Case 2 comprehends case 1 and adds the displacement constraint for the vertical degree of freedom in node 5.

Table 1. Comparative for weight objective function.

	Initial Weight			Final Weight		
	Silva, 1992	ESL	Dynamic	Silva, 1992	ESL	Dynamic
Case 1	93.62	94.34	94.34	63.58	64.51	64.51
Case 2	93.62	94.34	94.34	67.28	67.93	67.89

Table 2. Iterations in ESL and dynamic optimizations and outer cycles of ESL algorithm.

	Iterations		
	Dynamic	ESL	ESL Cycles
Case 1	5	14	5
Case 2	13	72	25

Table 1 shows the comparison between the ESL method and Silva's work. The results obtained in the ESL method are very accurate compared to Silva (1992), and the dynamic optimization results validate the ESL method. Table 2 compares the iterations in dynamic and ESL optimizations. The total number of iterations for the ESL methods comprehends all iterations through all cycles of static optimizations performed until optimum design is found. The total iterations required in dynamic optimization is lesser than in ESL, but with respect of the number of cycles it took for the ESL to reach the final results, the values are more closely related.

4.2. TEN-BAR TRUSS

The last example is a ten-member truss, represented in Figure 3. The same optimization formulation is proposed as in previous sub-section. Now 3 cases are performed:

Case 1 - Allowable stress for tension or compression is 25000; Minimum area is 0.1.

Case 2 - Allowable stress for tension or compression is 30000. Minimum area is 0.1;

Case 3 - Allowable stress for tension or compression is 25000; Allowable positive or negative vertical displacement of nodes 1, 2, 3 and 4 is 2; Minimum area is 0.1;

The maximum amplitude of load P is 100000. Table 3 shows the results obtained in both standard dynamic optimization and ESL method for static optimization.

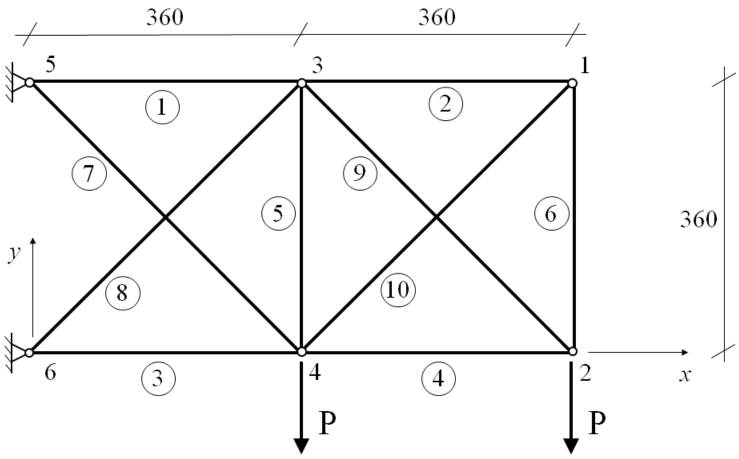

Figure 3. Ten-member truss subjected to two loads P at nodes 2 and 4.

Table 3. Initial and optimized values for the structure weight, Silva (1992), ESL and dynamic optimization for the ten-member rod truss.

	Initial Weight			Final Weight		
	Silva, 1992	ESL	Dynamic	Silva, 1992	ESL	Dynamic
Case 1	4196.4	4196.46	4196.46	1609.18	1563.51	1563.51
Case 2	4196.4	4196.46	4196.46	1342.55	1304.84	1304.84
Case 3	4196.4	4196.46	4196.46	5138.66	5056.27	5026.15

Table 4. Number of iterations in dynamic and ESL optimizations and outer cycles in ESL.

	Iterations		
	Dynamic	ESL	ESL Cycles
Case 1	10	32	6
Case 2	10	33	6
Case 3	45	118	9

Table 3 shows that the ESL method obtained better results than shown in Silva. Results from dynamic and ESL optimizations are very similar, identical in cases 1 and 2. Table 4 shows, just as in Table 2, that the dynamic optimization takes less iterations to converge. The total iterations required for ESL are bigger, but the number of optimization cycles (outer iterations) is less than the number required in the standard dynamic optimization. Table 5 contains the evolution of convergence for all design variables in case 3 by ESL method.

Table 5. History of design variables in ESL method for case 3.

	Cycles									
Areas	1	2	3	4	5	6	7	8	9	10
A1	10	37.741	30.691	30.280	30.901	30.774	30.752	30.749	30.748	30.748
A2	10	3.622	0.809	0.100	0.100	0.100	0.100	0.100	0.100	0.100
A3	10	32.358	24.363	22.786	22.970	22.913	22.901	22.901	22.901	22.901
A4	10	16.403	14.306	15.137	15.153	15.035	15.019	15.017	15.017	15.017
A5	10	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
A6	10	0.100	0.372	1.209	0.483	0.411	0.405	0.404	0.404	0.404
A7	10	14.938	9.603	7.565	7.598	7.569	7.565	7.564	7.564	7.564
A8	10	24.114	20.725	21.228	21.484	21.373	21.353	21.351	21.351	21.351
A9	10	23.026	20.238	21.199	21.500	21.339	21.317	21.316	21.316	21.316
A10	10	4.678	1.117	0.100	0.100	0.100	0.100	0.100	0.100	0.100

The analysis of Table 5 shows that the ESL method quickly approximates the design vector to the optimal solution, especially when the variable is set to minimum allowable bounds. Then, the following cycles take place in order to satisfy the criteria convergence set to the problem.

5. CONCLUSIONS

In this paper the Equivalent Static Load Method was applied to solve dynamic structural optimization problems of truss structures. The ESL was calculated using the governing equation of the finite element method for dynamic systems. The ESL method solves a sequence of static response optimization problems in which the cost for sensitivity analysis optimization is much smaller than that in dynamic response optimization. Two truss structures examples have been presented and the ESL could obtain very accurate results. The results from the proposed method were validated with results from standard dynamic optimization. Another advantage of the ESL method is that we can use commercial computer-aided engineering (CAE) tools as black boxes.

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