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CONTROL OF A MEMS OSCILLATOR WITH CHAOTIC BEHAVIOR

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Abstract: *This work considers the dynamics and control of a micro-electromechanical system "MEMS", represented by an oscillator. Numerical simulations have shown that for certain parameters, the system has a chaotic behavior. The Sliding Mode Control is presented with the purpose of controlling the trajectory of the system in a periodic orbit. The numerical results showed that the control is efficient in lead the system to a desired orbit.*

Key-words: *MEMs, Sliding Mode control, Chaos, Dynamics.*

1. INTRODUCTION

Currently, a large number of researches have been performed to report nonlinear dynamic phenomena on MEMS resonators (Mestrom *et al*., 2007), as well as application of control techniques to obtain a desired behavior (Tusset *et al*., 2012). The nonlinearities in MEMS include nonlinear springs and damping mechanisms (Roukes, 2001), nonlinear resistive, inductive and capacitive circuit elements (Xie and Fedder, 2002)[4] and nonlinear surface, fluid, electric and magnetic forces (Younis and Nayfeh, 2003). In Pandey *et al*. (2008) is considered the Mathieu–van der Pol–Duffing equation for present a MEMS model, and the authors investigated the dynamics of the system considering forced both parametrically and nonparametrically. In Pandey *et al*. (2004), a thermomechanical model of a MEMS was developed, and its predictions were explored to explain and predict the entrainment phenomenon. In Blocher *et al*. (2013a), the nonlinear dynamics of micromechanical oscillators are explored experimentally, and a model was built to explain why high-order entrainment is seen only in doubly supported beams, and by its analysis, it is suggested that the strong amplitude–frequency relationship in doubly supported beams enables hysteresis, wide regions of primary entrainment, and high orders of sub- and super harmonic entrainment. In Blocher *et al*. (2013a) is considered a model for oscillations from the continuum description of the temperature and displacement field. A bifurcation analysis of the model is performed, allowing estimating the threshold power for self-oscillation as a function of geometric and optical constants of the beam.

In Fig. 1 is observed a MEMs oscillator composed of a DC source and an alternating current (AC) with a movable micro beam.

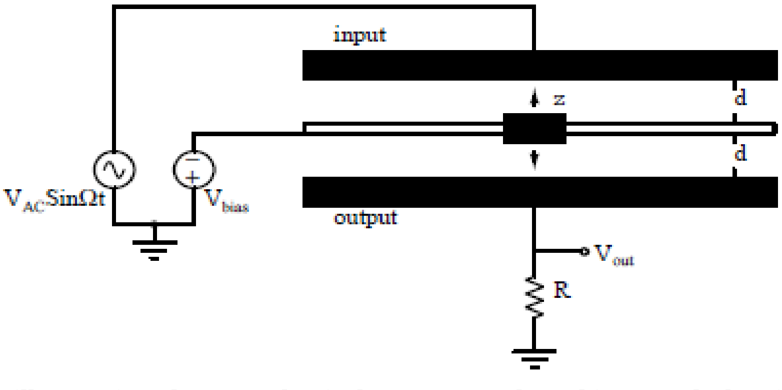


Figure 1. Oscillator Microelectromechanical (MEMS) (Sabarathinam and Thamilmaran, 2017)

According to Sabarathinam and Thamilmaran (2017), the mathematical model of MEMS oscillator with cubic nonlinearity term is represented by Fig. 1, and can be written as follows:

$$\dot{x} = -\mu\dot{x} - k_l x - k_{nl}x^3 + \alpha\left(1 + 2x + 3x^2 + 4x^3\right)\sin(wt) \tag{1}$$

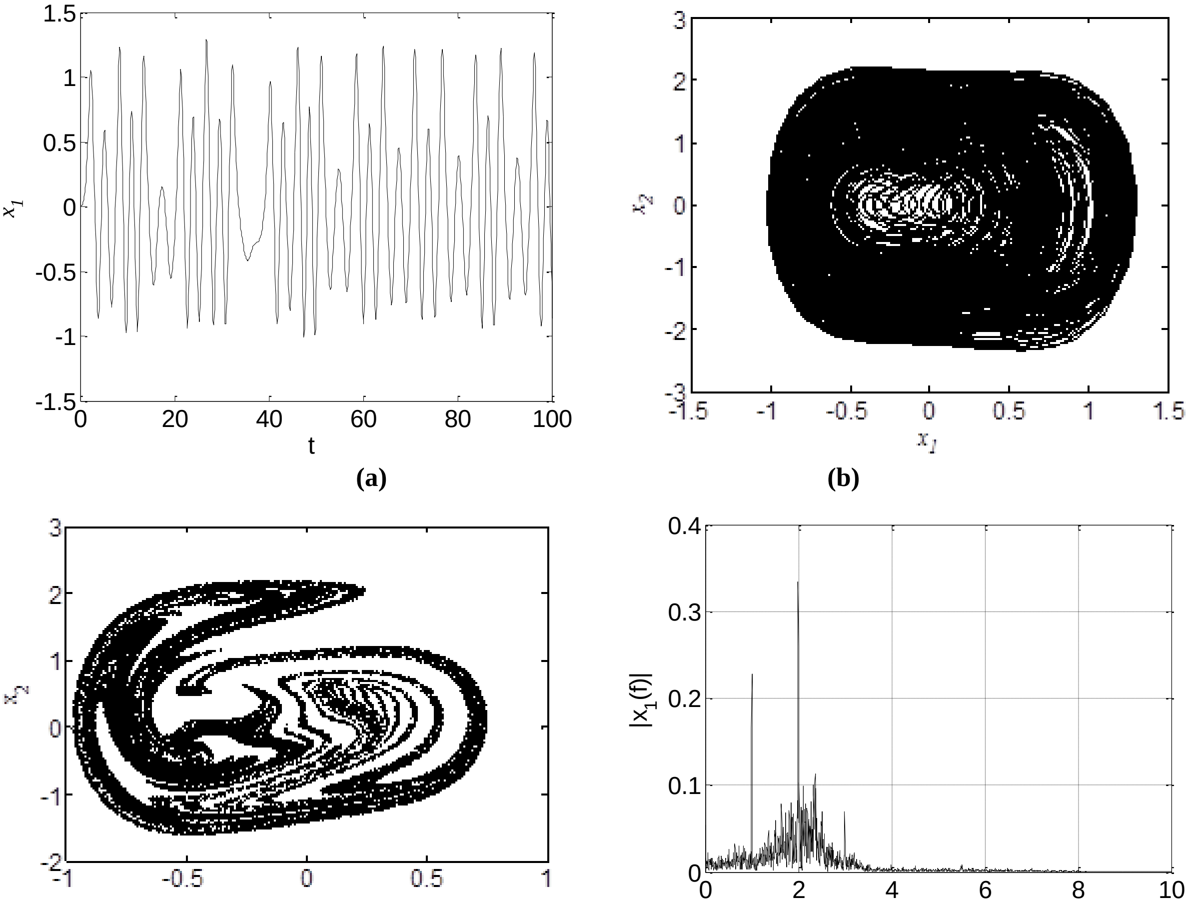
where k_l, k_{nl} represent the linear and nonlinear restoration coefficients, respectively, μ is a damping coefficient obtained from the combination of DC and AC current bias, and α and *w* are the amplitude and frequency of actuation, respectively.

2. NUMERICAL SIMULATION

Considering Eq. (1) in the form of state-space:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\mu x_2 - k_l x_1 - k_{nl}x_1^{\,3} + \alpha\left(1 + 2x_1 + 3x_1^{\,2} + 4x_1^{\,3}\right)\sin(wt) \end{aligned} \tag{2}$$

Figure 2 shows the dynamic behavior of the system (2). The parameters in which were considered to carry out the numerical simulations are the following: $x_1(0)=0.0001$, $x_2(0)=0.0006$, $\alpha = 0.64$, $\mu = 0.03$, $k_l = -0352$, $k_{nl} = 9.296$, $\alpha=0.64$ and $w=1$ [10].



(a) (b)

(c) (d)

Figure 2. Dynamics of the microelectromechanical oscillator (MEMS). (a) Displacement in time. (b) Phase Diagram. (c) Poincare Map. (e) FFT.

As can be observed in Fig. 2, the system (2) for the parameters used in the simulation has a chaotic behavior.

The objective here is to define a control law U that leads the system (1) from any initial condition $x(0) = x_0$ to a desired orbit $x(t) = x^*$, where x^* is previously due or obtained by the control law. Considering the introduction of the control signal *U* in Eq. (1), the control system is obtained as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\mu x_2 - k_l x_1 - k_{nl}x_1^{\,3} + \alpha\left(1 + 2x_1 + 3x_1^{\,2} + 4x_1^{\,3}\right)\sin(wt) \end{aligned} \tag{3}$$

In this work, the following orbits will be considered as desired:

$$[\sin(2t) \quad 2\cos(2t)]^T \tag{4}$$

Considering that U control is obtained through the sliding mode control strategy, the sliding surface is generally considered to be (Elahinia, and Ashrafiuon, 2002):

$$s = e_2 + \rho e_1 \tag{5}$$

where: $e = [x_1 - x_1^* \quad x_2 - x_2^*]^T$

The existence of the sliding mode requires that the following conditions have to be satisfied (Pandey et al., 2004):

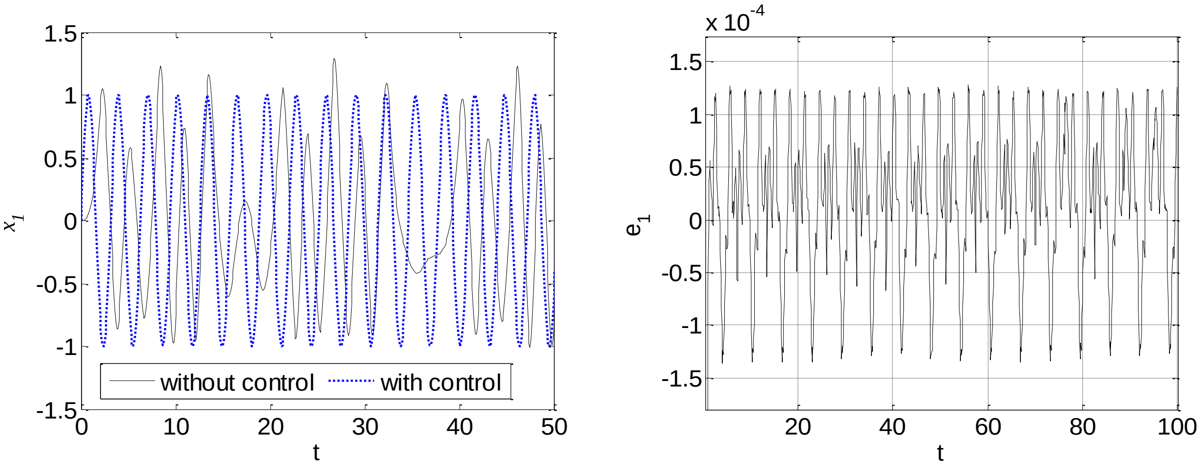
$$\begin{aligned} s &= e_2 + \rho e_1 = 0 \\ \dot{s} &= \dot{e}_2 + \rho \dot{e}_1 = 0 \end{aligned} \tag{5}$$

where, ρ represents a real number, and control U can be defined by (Elahinia, and Ashrafiuon, 2002):

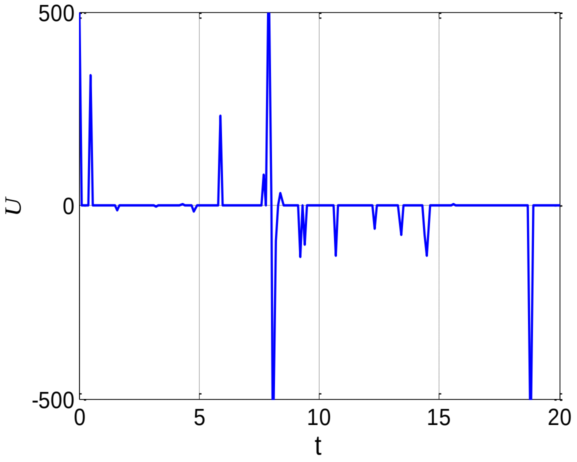
$$U = \begin{cases} U_{\max} \quad if \quad \dfrac{s}{\varphi} < -1 \\ -U_{\max} \quad if \quad \dfrac{s}{\varphi} < 1 \\ K_s \quad if \quad - < \dfrac{s}{\varphi} < 1 \end{cases} \tag{6}$$

where φ represents the thickness of the control layer, K is a proportional gain, and U_{max} is the saturation level of the control.

Figure 3 shows the system (3) with control by sliding modes (7), considering $\rho = 4$, $K = 1000$, $\varphi = 10^{-3}$ and $U_{max} = 500$.



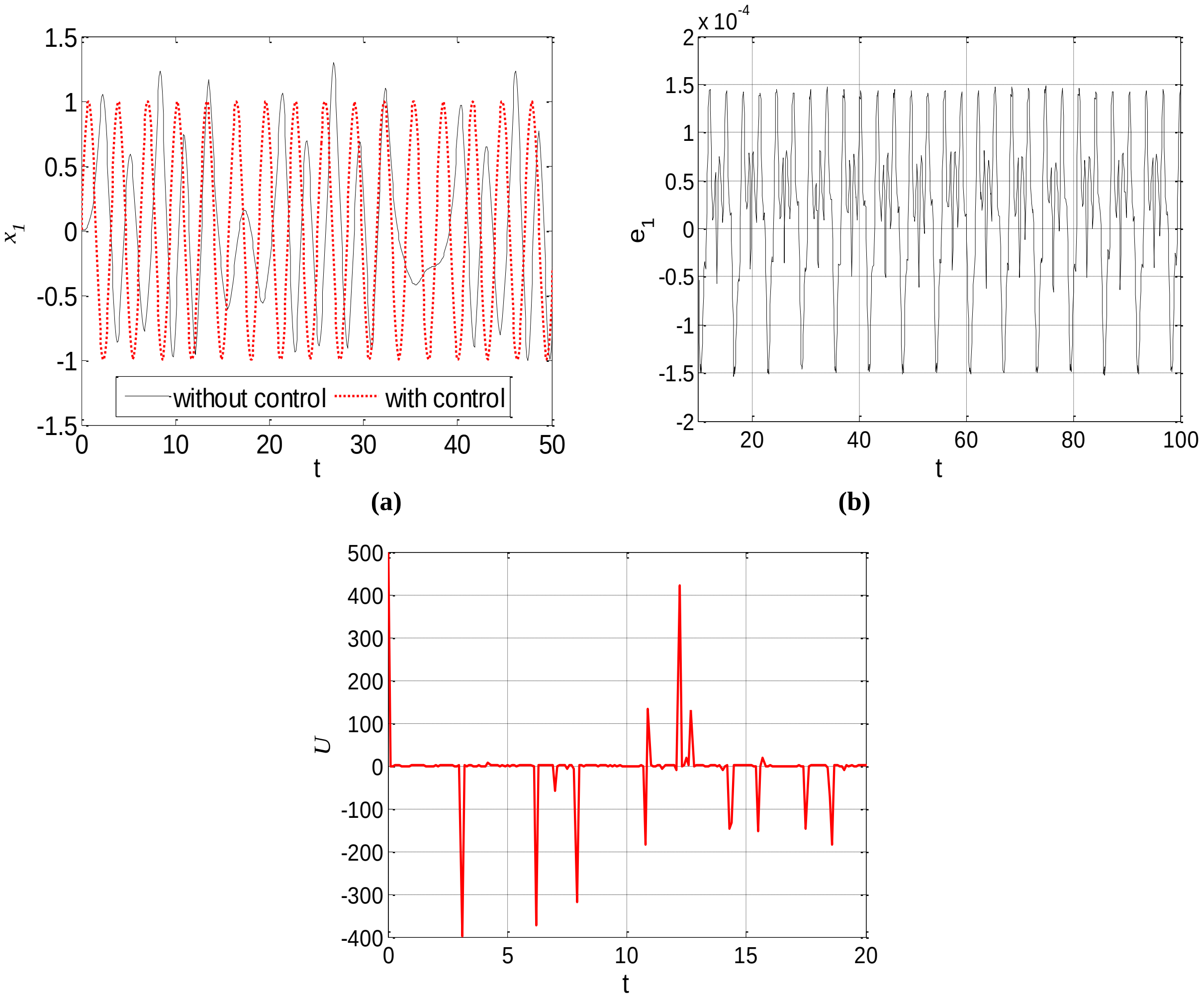
(a) **(b)**



(c)

Figure 3. (a) Displacement in time. (b) Variation of the error. (c) Control signal U.

Figure 4 shows the system (3) with control by sliding modes (7), considering $\rho = 3.2$, $K = 1000$, $\varphi = 10^{-3}$ and $U_{max} = 500$.

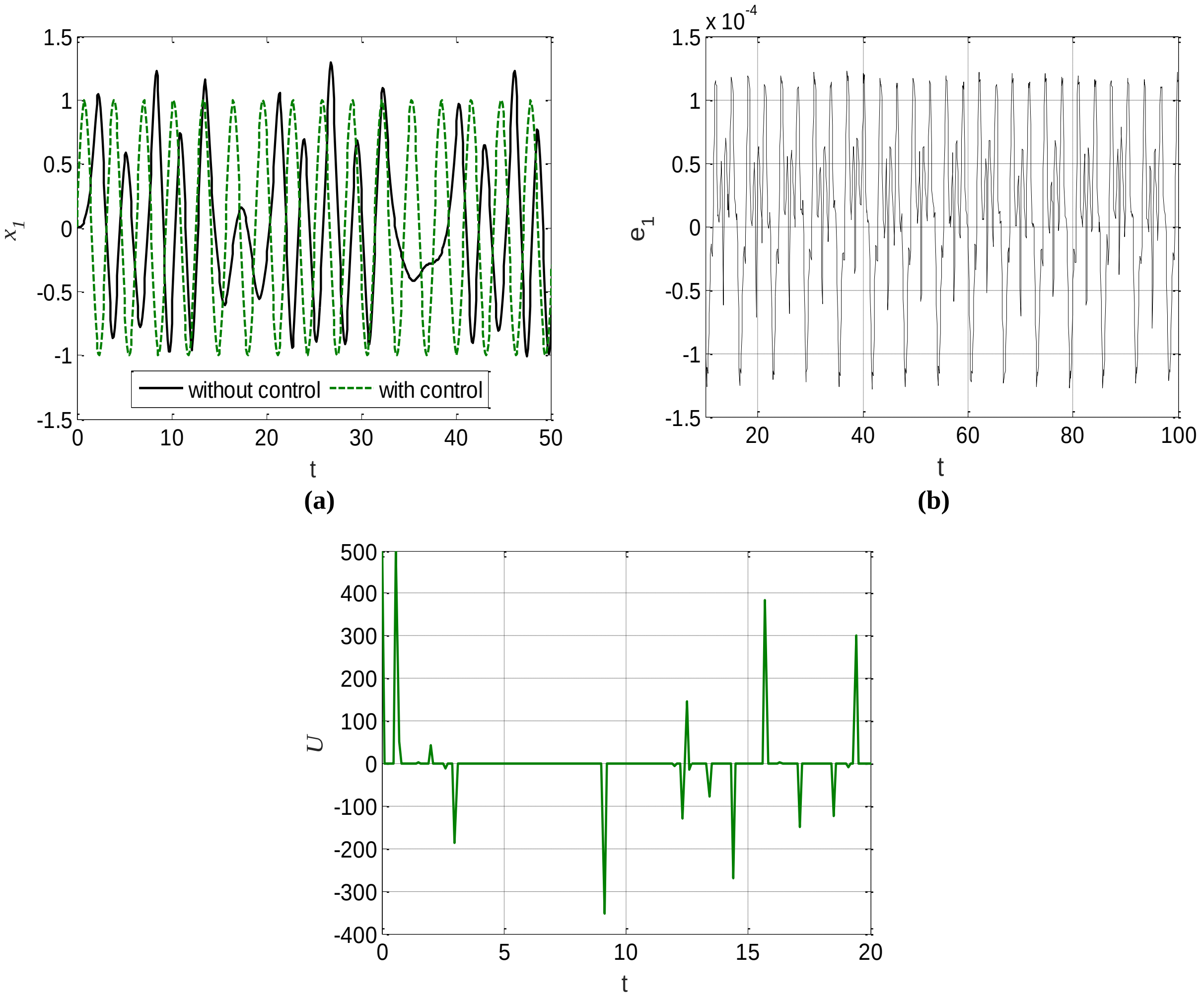


(a) (b)

(c)

Figure 4. (a) Displacement in time. (b) Variation of the error. (c) Control signal U.

Figure 5 shows the system (3) with control by sliding modes (7), considering $\rho = 4.4$, $K = 1000$, $\varphi = 10^{-3}$ and $U_{max} = 500$.



(a) **(b)**

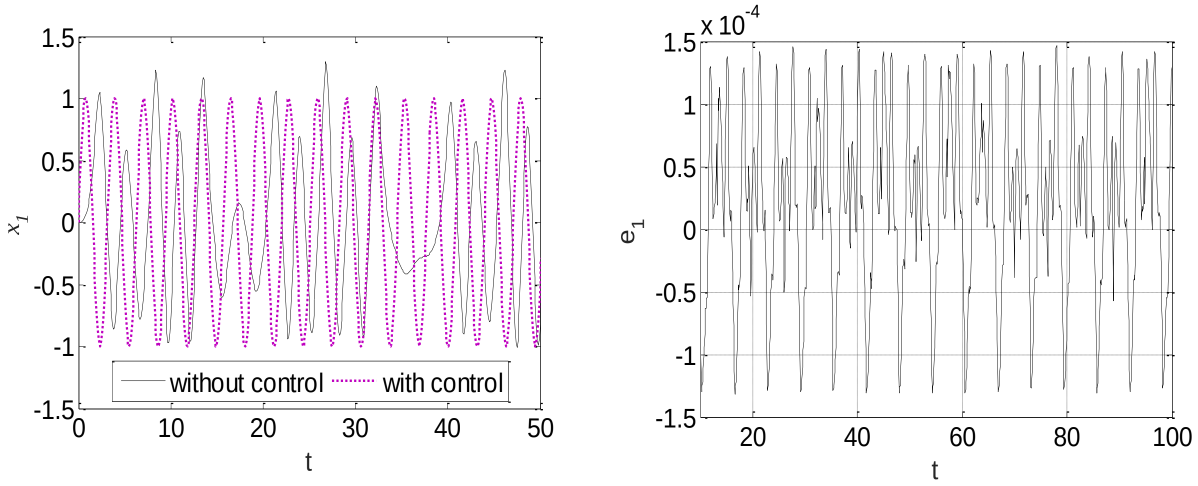
(c)

Figure 5. (a) Displacement in time. (b) Variation of the error. (c) Control signal U.

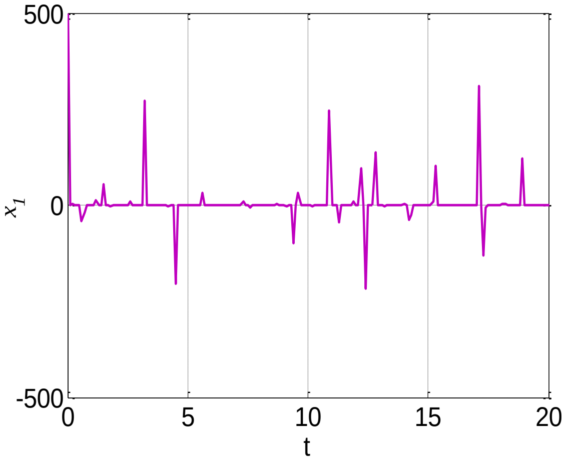
As can be seen in Figs. 3, 4 and 5, the control by SDRE is effective in maintain the system in the desired coordinate.

In order to determine the effects of uncertainties on the performance of the proposed controller, it is estimated that an error of $\pm 20\%$ is encountered in the parameters ($\rho=4$), with a similar strategy used in Tusset *et al*. (2017).

In Fig. 5, the robustness of the control to maintain the system in a desired coordinate is observed, considering the proposed control with variation in parameter $\rho = 4(0.8+0.4r(t))$, where $r(t)$ is a random number $r(t)=[0,1]$.



(a) **(b)**



(c)

Figure 4. (a) Displacement in time. (b) Variation of the error. (c) Control signal U.

In Fig. 5 is possible to observe the robustness of the control with an uncertainty in the parameter $\rho = 4$. This demonstrates that the control is robust to variations in the parameter $\rho=4$.

3. CONCLUSIONS

In this work, the SMC control strategy was presented in a MEMS, the aim of the technique was to eliminate the chaotic behavior of the system. As could be seen, the strategy was efficient to maintain the amplitude of the oscillator in the desired periodic orbit. The best configuration to reduce the error was considered to $\rho = 4$, and for lower control signal levels the best option is to use the variable ρ ($\rho = 4(0.8 + 0.4r(t))$, where $r(t)$ is a random number $r(t) = [0,1]$)

4. ACKNOWLEDGMENTS

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6. RESPONSIBILITY NOTICE

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