X CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA CONEM 2018
ABCM
20 A 24 DE MAIO DE 2018 SALVADOR – BA – BRASIL

Experimental investigation of the wave propagation in beams undergoing flexural vibration

Daniely A. Neves, d.amorim.neves2@gmail.com[1]
Pedro Casado, pedrohcasado94@gmail.com[1]
Adriano T. Fabro, fabro@unb.com[1]

[1] Department of Mechanical Engineering, University of Brasilia Campus Darcy Ribeiro, CEP 70.910-900, Brasilia, DF, Brazil

Abstract: The wave approach in structural dynamics focus on local properties such as the dispersion relations, phase and group velocities, waves modes and energy transmission. This is in contrast to the modal approach, which typically focus in global properties such as the natural frequencies and modes shapes. Experimental technics for modal analysis are well stablished and have been successfully applied in the past decades unlike the experimental approaches for wave propagation. This paper aims to investigate some experimental approaches for the identification of the wavenumber in beams undergoing flexural vibration. The underlying experimental setup consists of measuring frequency response functions by an impact hammer test from a free-free beams at evenly spaced points. The influence of measurement points distance, positioning along the beam are investigated and the upper and lower limits of the frequency band for the analysis are given. Results are compared to an available analytical solution and showed good agreement.

Keywords: *wavenumber, wave propagation and experimental approach.*

1. INTRODUCTION

Wave propagation in structures has been immensely used to describe the dynamic response in structures such as beam, plates, shells, including structures that are nonhomogeneous as composites and vibro-accoustic metamaterials (McDaniel, et al., 2000). This method can describe structural vibrations of mechanical systems in terms of wave modes, dispersion relations, phase and group velocity and energy transmission. In special in metamaterials have relevant applications in identification of stopbanb behavior (Dong, et al., 2010). In this context, predicting the dynamic behaviour of the materials through experimental methods is very important for factors as stability, acoustic isolation, vibration attenuation and structural integrity in engeneering structures.

There are many experimental technics for this dynamical analysis succeeded in the literature including Discrete Fourier Transform (Bolton, et al., 1998), Inhomogeneous Wave Correlation (IWC) (Ferguson N.S., 2002; Ichchou, et al., 2008) and Prony's series (McDaniel J.G., 2000), and most concentrated on obtaining the dispersion curve for this analysis. By the dispersion curve, the wave number can easily be obtained and through its analysis the characteristics of the propagation wave, energy transfer and vicoelastic properties of the structure. The focus of this paper is the experimental investigation the flexural wavenumbers aand wave amplitudes in a beams from response measurements.

The present work provides analytical and experimental demostrations, present in sections. 3 and 4, where the experimental wavenumber is estimated thorough the an adjustment of analytical model. And dynamic response is obtained by transductores equaly spaced in beam for to measure lateral acceleration while modal hammer excited him (Kalkowski, et al., 2017). Application can be found in literature with sandwich beams (Hinke, et al., 2002),periodically undulated beams (Trainiti, et al., 2015), metamaterial beams (Casadei, et al., 2014) and locally resonant acoustic metamaterials (Lewinskaa, et al., 2017) using wave propagation for to describe local properties, mostly using analytical and numerical models.

There are many problems in applying experimental methods, mostly related to noise, nonlinearities, equipment calibration, evanescent waves, truncation, frequency and space resolutions causing distortion in acquired signals. The acquired data will be treated by classical FRF (Frequency Response Function) estimation approaches (H_1 and H_2) and the ordinary coherence function indicating the quality of the estimation. Since the beam has finite length, there will be wave overlap, especially at low frequency and in the contour regions, but at such frequency there will be a dominant response where the numbers will be relatively easy to measure. And especially in metamaterials the periodic locally resonant structures contribute to the rise of unique wave dispersive behavior.

The main purpose of the present work is to investigate an experimental approach for the the identification of wavenumbers in beams undergoing flexural vibration, including metallic in 3D printed, which consists of obtaining the Frequency Response Function (FRF) in points distributed in a structure. Where, fistly is present the analytical model for beam and derive the equations for the estimation of the wavenumbers from de three measured dynamic responses. For them to show the experimental setup and discuss the results comparing them with analytical prediction.

2. WAVENUMBER MEASUREMENT

An analytical solution of free waves in structure in a Euler-Bernoulli beam can be found, assuming a homogenous distribution of the geometrical and material properties, and its governing equation of motion is given by

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{1}$$

where E is the Young´s modulus, I is the inertia moment of area, ρ is the mass density and A is the are of the cross-section. Assuming time and space harmonic motion, i.e. $w(x,t) = a\, e^{-ikx}e^{i\omega t}$, then Eq.(1) (HinKe, et al, 2004) can be used to derive the caracteristic equation,

$$k^4 = \frac{\rho A\omega^2}{EI} \tag{2}$$

which has two pair of imaginary and real roots. The real roots give rise to the positive and negative propagating waves while the imaginary roots give rise to positive and negative going evanescent waves, i.e. non-propagating waves. This relation can be used to generate the dispersion curve.

Considering only propagating bending waves, i.e., only real wavenumbers, the displacement field can be written considering only the positive-going a^+ and negative-going a^- waves (Muggleton, et al., 2004)

$$a(x,t) = a^+ e^{i(\omega t - kx)} + a^- e^{i(\omega t + kx)} \tag{3}$$

Considering the ilustration of Fig. 1, where $a_1(t)$, $a_2(t)$ and $a_3(t)$ are displacement, *f(t)* is the exitacion, and *d* is the distance between the displacement transducers and *x* the distance between the excitation and the central transducers.

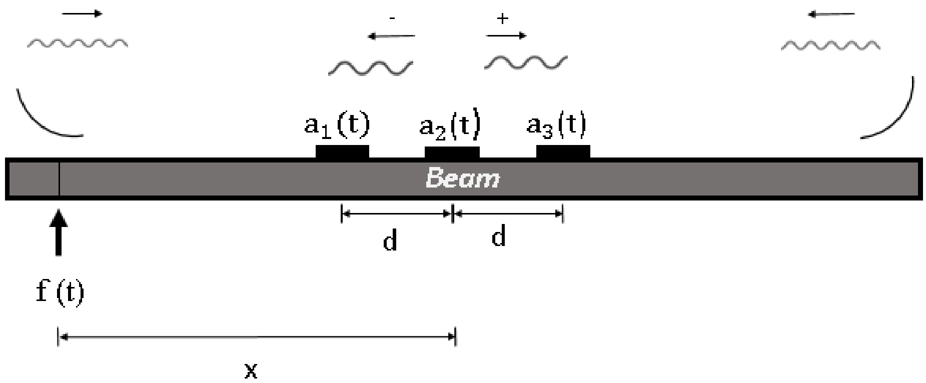


Figure 1. Schematic representation of the measurement setup for the method of three points.

Choosing three evenly spaced measurement points points $x_1 = -d$, $x_2 = 0$ and $x_3 = d$, as shown Fig. 1, where the positions of the acceleration measures $a_1(t)$, $a_2(t)$ and $a_3(t)$,

$$\begin{aligned} a_1(-d,t) &= -\omega^2(a^+ e^{ikd} + a^- e^{-ikd})e^{i\omega t} \\ a_2(0,t) &= -\omega^2(a^+ + a^-)e^{i\omega t} \\ a_3(d,t) &= -\omega^2(a^+ e^{-ikd} + a^- e^{ikd})e^{i\omega t} \end{aligned} \tag{4}$$

Combining the three expressions from Eq. (4) and omitting the time dependence, then (Muggleton, et al., 2004),

$$\frac{a_1(-d,t) + a_3(d,t)}{a_2(0,t)} = \frac{a^+ e^{ikx} + a^- e^{-ikx} + a^+ e^{-ikx} + a^- e^{ikx}}{a^+ + a^-}, \tag{5}$$

assuming $a_2(0,t) \neq 0$. Therefore, the beam is subjected to any forcing and estimates $A_1(\omega)$, $A_2(\omega)$ and $A_3(\omega)$ with from the FRF estimator H_1 and H_2. So, the equation (5) can then be rearranged such that the flexural wavenumber can be estimated by

$$k(\omega) = \frac{cos^{-1}\left(\dfrac{A_1(\omega) + A_3(\omega)}{2A_2(\omega)}\right)}{d} \quad (6)$$

It should be noted that wavenumber, can be complex. This approach can also be applied on the case of non-homogeneous waveguides (Kalkowski, et al., 2017). In this work, the dynamic response are obtained from a hammer test using accelerometers, as schematically shown in Fig. 2.

3. EXPERIMENTAL ESTIMATION TECHNIQUES

The method consistis of a standard hammer test in a beam with boundary condition free-free, using one modal hammer and hovering one accelerometer attached using bee wax, models specified in Tab. 1. The boundary condition is applied by resting the beam over a soft foam. Also, the data acquisition was done by a NI board (NI9205) and software Vibsolft 5.5. However, the number of frequency points were 7679 and 40000 for de steel beam and metamaterial beam respectively; and a frequency of acquisition were 6000 Hz and 4273 Hz, for the steel beam and the metamaterial beam respectively. A picture of the experiment setup is shown in Fig. 2.

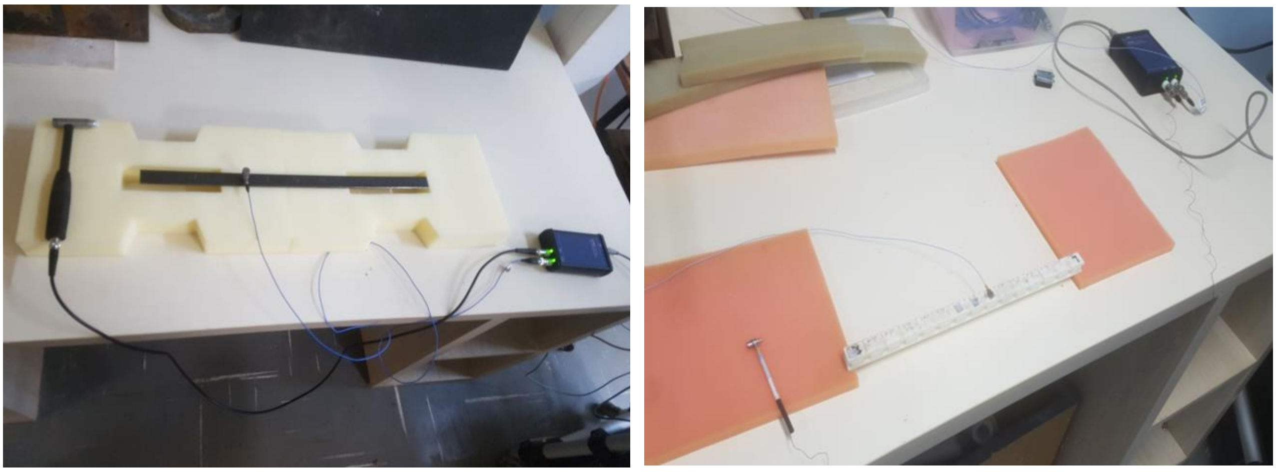


Figure 2. Experimental setup.

Table 1. Models of the equipment used in the experiment.

Equipment	Steel Beam	Metamaterial Beam
Modal Hammer	PCB086C01-Nylon	PCB084A17
Accelerometer	PCB352C33	352A21

The steel beam has a rectangular cross-section and its Young's modulus was estimated by using the lowest flexural natural frequencies and the mass density by weight (Travi, et al., 2016) and are sumarized in Tab. 2.

Table 2. Geometrical and material properties of the steel beam (Travi, et al., 2016).

Young´s Modulus [GPa]	181.81
Density[kg/m³]	7452.8
Width [mm]	25.48
Height [mm]	4.90
Length [mm]	500.00

The metamaterial of beam is made of polyamide manufactured by a 3D printer with the method of Selective Laser Sintering. Its mechanical resonators are fixed symmetrically and periodically on both sides of the profile being a total of 15 units. The global geometry, with dimensions shown in Fig. 3 are the same as those used in (Beli, et al., 2016).

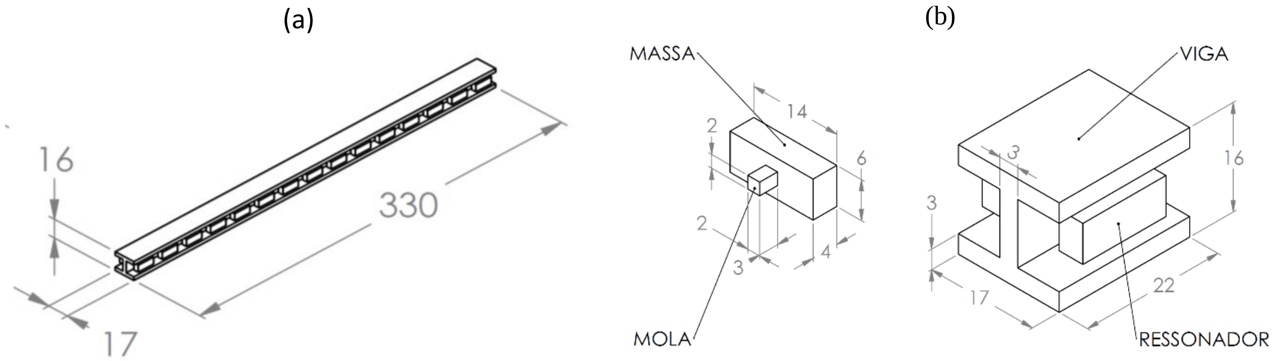


Figure 3. (a) beam geometry and (b) resonator geometry (Ampessan , 2016).

The metamaterial structure is a I-beam and whose material main properties are sumarized in Tab. 3.

Table 3. Geometrical and material properties of the metamaterial beam (Beli, et al., 2016).

Geometrical and properties	Beam	Resonator
Young´s Modulus [GPa]	0,86	0,96
Density[kg/m³]	700	1000
Length [mm]	330.00	14

For the output signal, an accelerometer was used an exponential window duo to the transient natural of the excitation. For the input signal from the modal hammer, a force window was used eliminating all the force signal before and after the impulse, which are only due to the instrumentation noise. As shown in Fig. 4, the excitation position was fixed at the point of 10 mm from the left boundary and the minimum spacing between measurements was 20 mm for the steel beam, while excitation position was 14 mm and the minimum spacing between measurements was 11 mm for the metamaterial beam. And that the displacement at the extremities were not analyzed due to the occurrence of evanescent waves. A total of 17 measurement point for the steel beam and 23 measurement points in the metamaterial beam were set.

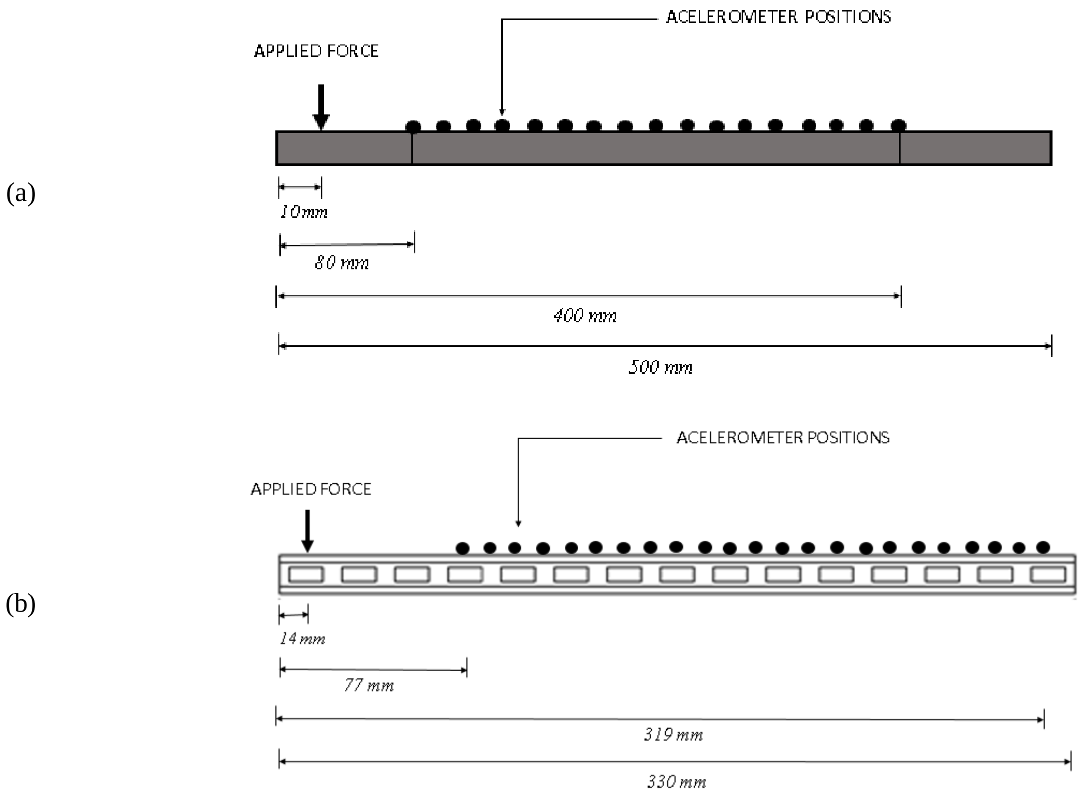


Figure 4. Schematic representation of the measurement points for (a) steel and (b) resonator beam (Ampessan , 2016).

Considering that it is not feasible to present the result of all the measurements obtained, only two points from each beam will be shown, on from the middle and one from extremity. Fig. 5 shows the amplitude and phase data from the steel (a,b) and metamaterial (c,d) beams estimated by classical FRF (Frequency Response Function) approaches (H_1 , H_2 and H_w). It is possible to notice resonance peaks occuring at the anti-resonance resonance frequencies of the system. Also is observed the occurrence of the bandgap phenomenon in the region 1500 to 1800 Hz due to the periodically attached resonators.

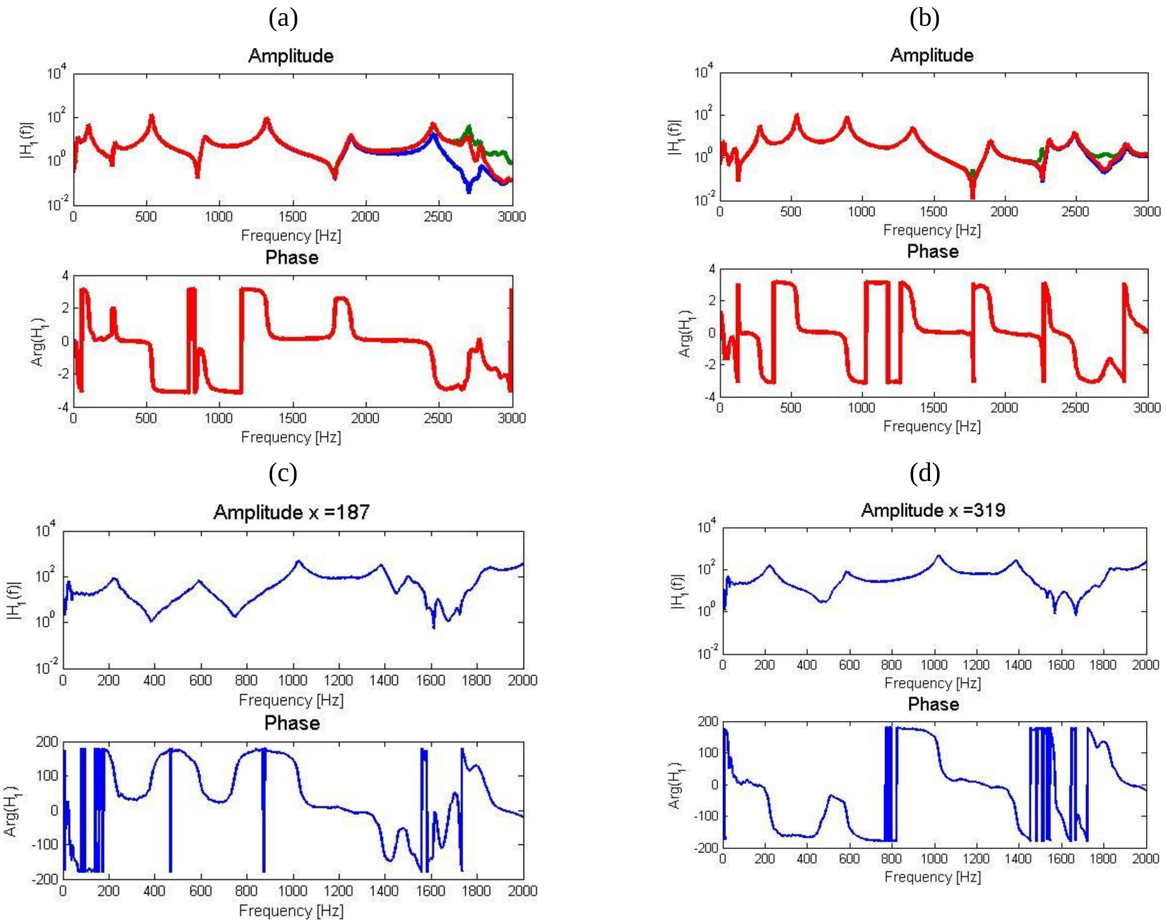


Figure 5. Amplitude and phase of the frequency response function at (a) 240 mm and (b) 400 mm in the steel beam and (c) 187 mm (d) 319 mm in the metamaterial beam.

The coherence measures the degree of “linear relation” between two signals, as for example the relation of input and output. So, the estimator of the coherence function measure the data quality, and it is calculate by using the auto and cross PSD (Power Spectral Density) estimators. In Fig. 6, it is shown the results obtained for the distances of 240 mm and 400 mm for steel beam. The sudden drops observed in the anti-resonance regions happen because of the large phase variations at these frequencies, this can be reduced by improving the resolution of the bandwidth estimator. Moreover, the other drops observed in the coherence graph above 2000 Hz are due to expectral content of the hammer, which is not large enough to excite all modes.

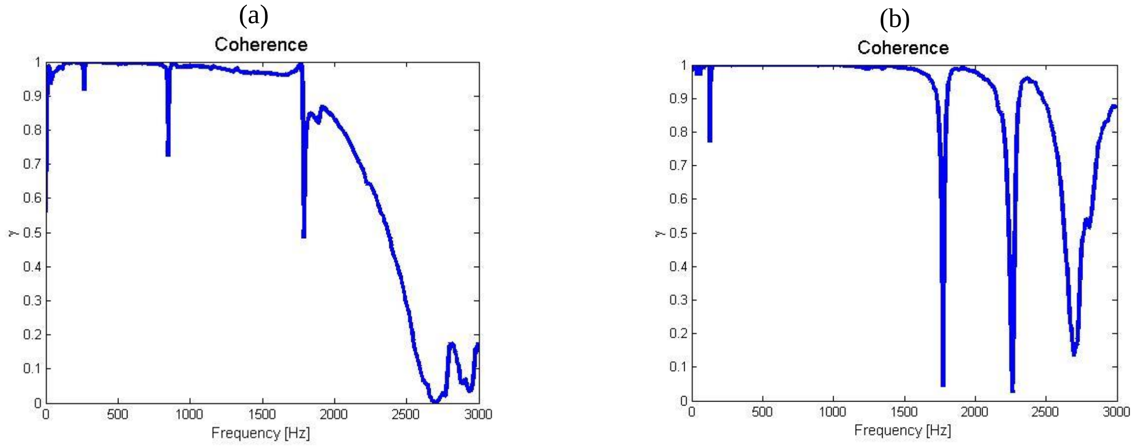


Figure 6. Coherence function for distance the (a) 240 mm and (b) 400 mm in a steel beams.

4. RESULTS AND DISCUSSIONS

For the three-point method, different arrangements will be explored in order to investigate influence of measurement points distance, positioning along the beam are investigated and the upper and lower limits of the frequency band for the analysis are given. Results are compared to an available analytical solution.

Wavenumbers were estimated at a few locations of the central sensor but four of which are discussed below: 80 mm, 220 mm, 240 mm, 400 mm with spacing of 20 mm for steel beam. The corresponding experimental estimates of the wavenumbers using of Eq. (6) and applying the method previously presented are shown in Fig.1. It may be noted that in all estimates, it followed the analytical prediction for the central sensor up to about 2 kHz, but for positions further away from the excitation source the estimate improves considerably.

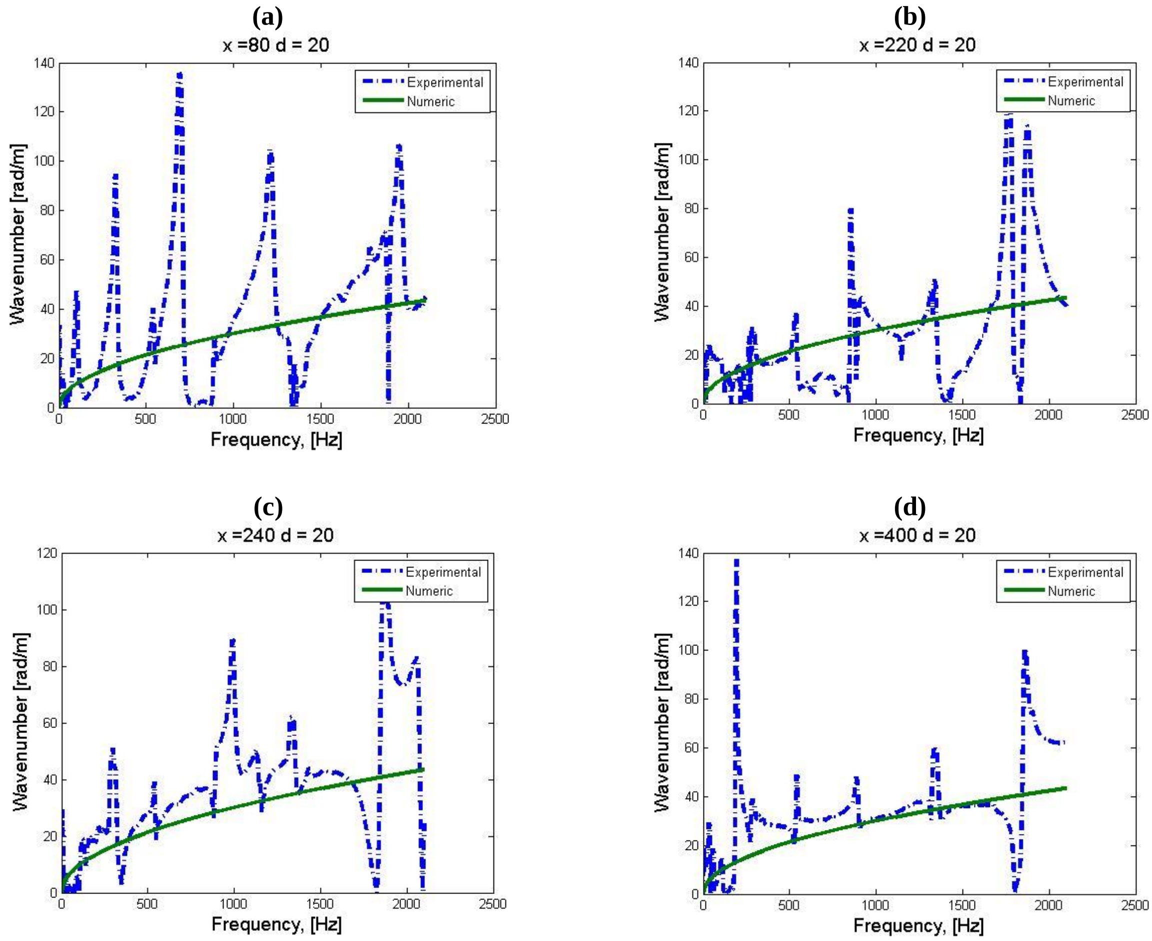


Figure 7. Comparation with wavenumber estimated the analytical (a) 80 mm, (b) 220 mm, (c) 240 mm and (d) 400 mm in the steel beam.

In Fig. 8, it is shown the wavenumbers estimated for the locations of the central sensor 77 mm, 187 mm, 198 mm and 297 mm with spacing of 11 mm for metamaterial beam. It may also be noted that in all estimates, it followed the analytical prediction. Another important point observed is the bandgap phenomenon in the region 1400 to 1800 Hz.

(a) **(b)**

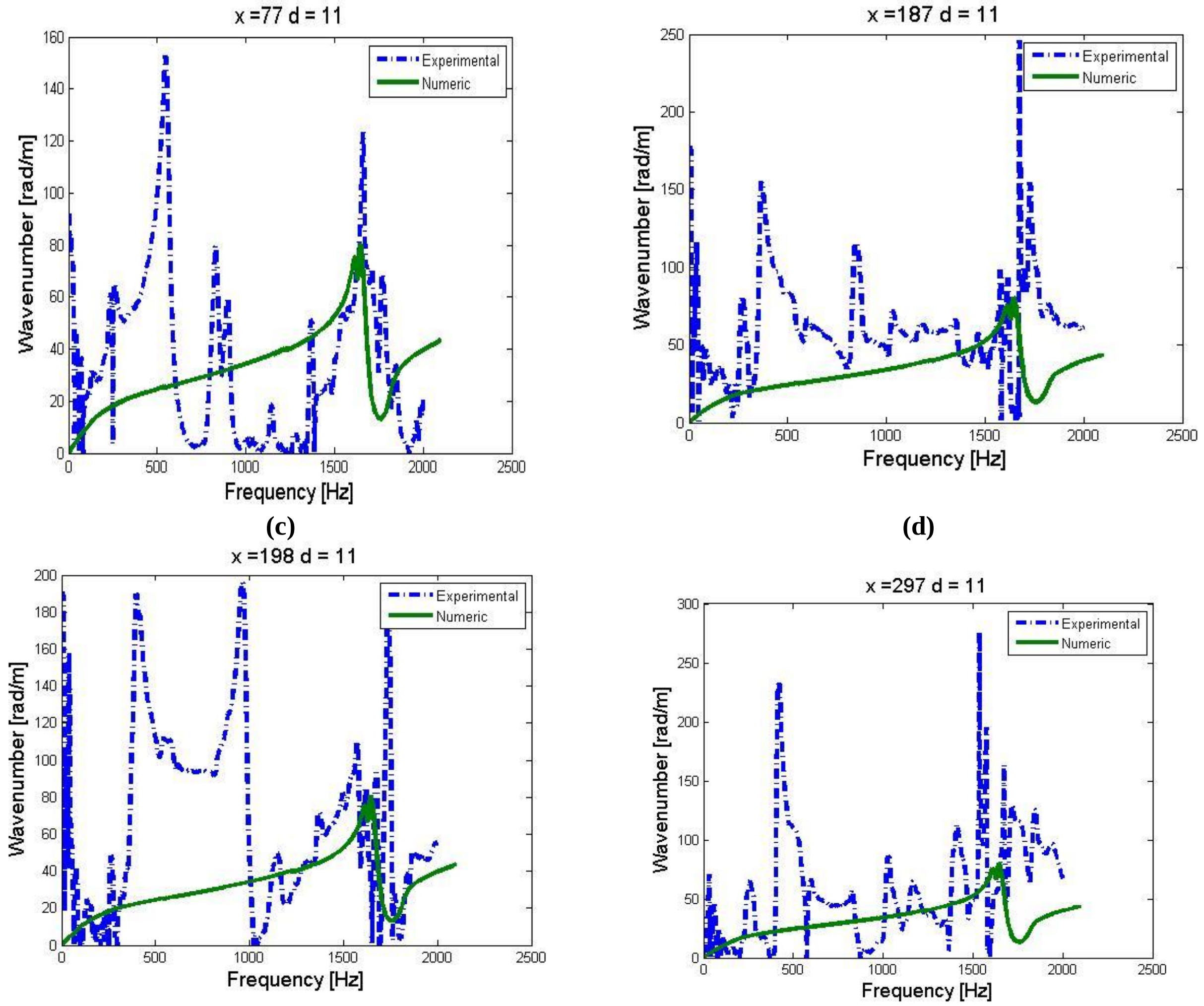


Figure 8. Comparation with wavenumber estimated the analytical (a) 77mm, (b) 187mm, (c) 198 mm and (d) 297 mm in metamaterial beam.

From in Eq. 4, it is necessary to choose a combination of points where the center point does not have amplitude close to zero for any frequency, for the relation does not tend to zero. The discretization of a signal in the space influences the maximum measurable wavenumber, given a spacing dx between two points of acquisition in the beam so the maximum number of wave is determined by

$$k_{max} = \frac{\pi}{dx} \tag{7}$$

In Fig. 7, it is observed a good approximation of the experimental and analytical curve up the wavenumber k = 15,7 rad/m for beam steel and k = 28,56 rad/m for metamaterial beam. The minimum wavenumber measurable is limited by the presence of evanescent waves in which they decay exponentially with position and also is wave propagation at low frequencies. This effect is observed in the experimental curve where only approaches the analytical curve from the number of waves around k = 100 rad /m.

In order to analyze the influence of the spacing between the sensor on the wavenumber of the curve always taken the most extreme location of the beam; according to the marking of Fig.3 because these were the best results as previously agreed. So, taken spacing between the sensor the 20 mm, 40 mm, 60 mm and 80 mm with locatin the central sensor the 400 mm, 360 mm, 320 mm and 280 mm respectively. The result can be seen in Fig. 9 where is made a comparison with the analytical curve.

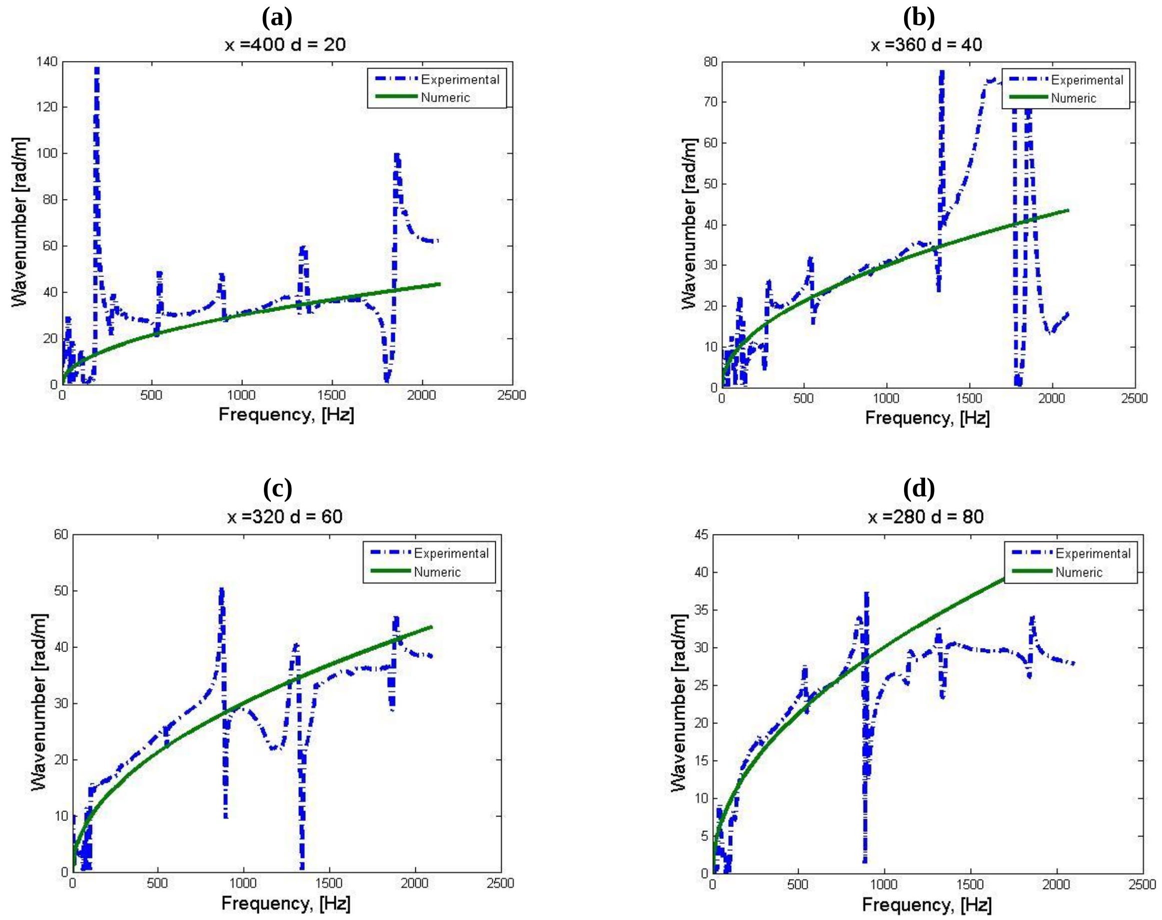
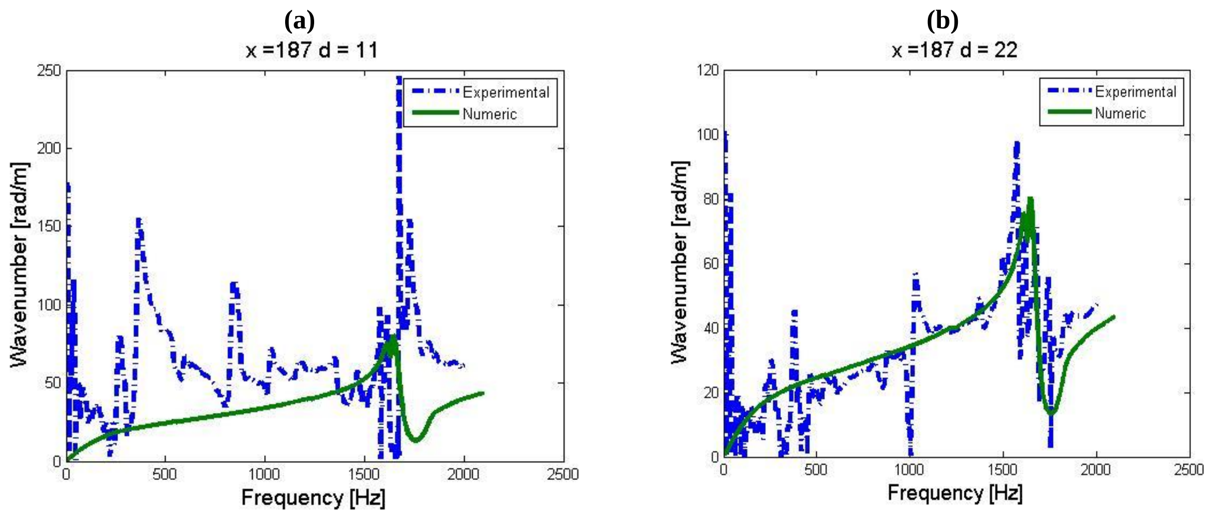


Figure 9. Comparation with wavenumber estimated the analytical for spacing between the sensor the (a) 20 mm, (b) 40 mm, (c) 60 mm and (d) 80 mm for steel beam.

The maximum wavenumber measurable are k = 157.07 rad/m , k = 78.54 rad/m, k = 52.37 rad/m and k = 39.27 rad/m for the 20 mm, 40 mm, 60 mm and 80 mm respectively. Also, it is shown in Fig. 10 the wavenumbers for different spacings for the metamaterial beam, 11 mm, 22 mm, 33 mm and 44 mm. All of them follow the numerical curve, but for the smallest spacing there are more discontinuities, this behavior is observed in both beams. Therefore it is observed that a small distance between the acquisition points can also generate noisy results, because when two near accelerometers overlap the wave; then the difference in amplitude between the points causes noise (Kalkowski, et al., 2017).



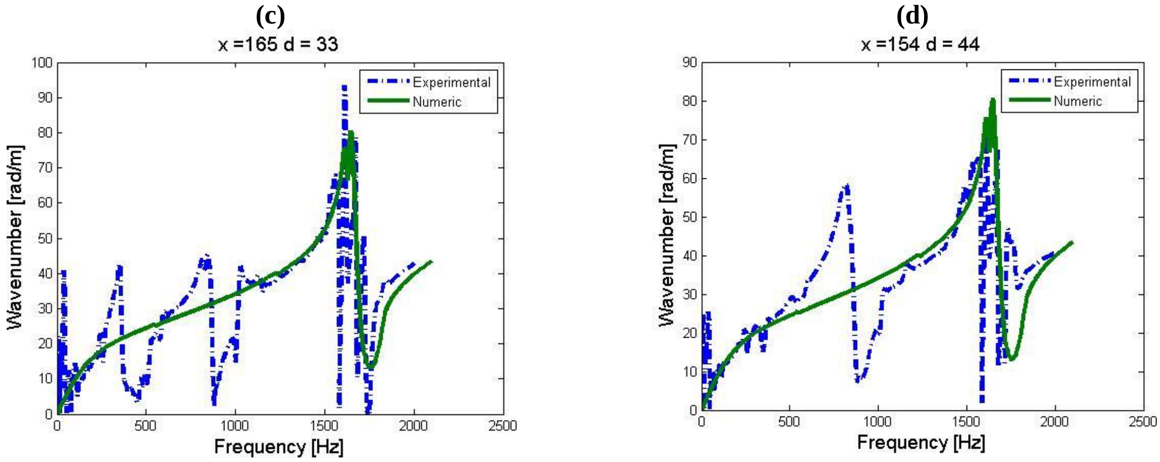


Figure 10.Comparation with wavenumber estimated the analytical for spacing between the sensor the (a) 11 mm, (b) 22 mm, (c) 33 mm and (d) 44 mm for metamaterial beam.

However, the method presented in this paper is predicting flexural wavenumbers in beams; and it has been contacted that their variation is strongly dependent on the position of the central sensor in the beam and spacing between them. All estimates follow the trends of the analytical predictions, but for the metamaterial beam it was shown a wavenumber more dispersive and it is possible to identify the BandGapbehavior.

5. CONCLUING REMARKS

In this work, the wavenumber of beams undergoing flexural vibration was estimated using a analytical expression and a small number of FRF measurements at evenly spaced locations. The experimentally obtained dispersion curves shown a good agreement with the analytical solution, provided and adequate spacing between the accelerometers measurement point is chosen. It was shown that the sensor spacing has a significant effect on the quality the estimates; distance between sensor was determining an upper and lower limit for the estimated wave number and position along the beam has to choose a combination of points where there is no charge in the effect if singularity. Therefore this method requires some a priori knowledge of the expected wavelengths.

6. ACKNOWLEGEMENTS

The authors would like to thank the financial suport of the Consellho Nacional de Pesquisa (CNPq), Fundação de Pesquisa do Destrito Federal (FAP-DF) and University of Brasília and the the Programa de Iniciação Científica (PIBIC).

7. REFERENCIAS

Ampessan , T.L., 2016, "Investigação Experimental dos Efeitos da Variabilidade de Fabricação em Metamaterias Acúticos", Brasilia, Brazil.

Beli, D. and Arruda, J.R.F., 2016, "Influence of Additive Manufacturing Variability in Elastic Bandgaps of Beams with Periodic Distributed Resonators", 3rd International Symposium on Uncertainty Qualification and Stochastic Modeling, Brazil.

Bolton, J., Kim, Y. and Song, H., 1998, "The Wavenumber Decomposition Approach to the Analysis of Tire Vibration", Publications of the Ray W. Herrick Laboratories.

Casadei, F. and Bertoldi, K., 2014, "Wave Propagation in Beams with Periodic Arrays of Airfoil-Shaped Resonating Units", Journal of Sound and Vibration, pp 6532–6547.

Dong, F., Hong, J.Z., Zhu, K. and Yu, Z.Y., 2010, "Numerical and Experimental Studies on Impact Dynamics of a Planar Flexible Multibody System", Acta Mechanica Sinica., Shanghai, China

Ewins, D.J., 2000, "Modal Testing: Theory, Practice and Application", John Wiley & Sons Ltda, p. 561, Bognor Regis, England.

Ferguson N.S., Halkyard C.R., Mace B.R., Heron K.H., 2002," The Estimation of Wavenumbers in Two-Dimensional Structures".

Hinke, L., Mace, B.R. and Brennan, M.J., 2004, "The Estimation of Wavenumbers in Two Dimensional Structures",Institute of sound and Vibration.

Ichchou, M. N., Berthaut, J. and Collet, M., 2008, "Multi-Mode Wave Propagation in Ribbed Plates. Part II: Predictions and Comparisons," International Journal of Solids and Structures, pp. 1179-95.

Kalkowski, M., Muggleton, J. and Rustighi, E., 2017, "An Experimental Approach for the Determination of Axial and Flexural Wavenumbers in Circular Exponentially Tapered Bars", Journal of Sound and Vibration, pp 67–85.

Lewinskaa, A. M., et al., 2017, "The Attenuation Performance of Locally Resonant Acoustic Metamaterials Based on Generalised Viscoelastic Modelling", International Journal of Solids and Structures.

McDaniel J.G., Shepard W.S., 2000, "Estimation of Structural Wavenumbers from Spatially Sparse Response Measurements", The Journal of the Acoustical Society of America, pp. 1674–82.

Muggleton, J. M., Brennan, M. J. and Linford, P. W., 2004, "Axisymmetric Wave Propagation in Fluid-Filled Pipes: Wavenumber Measurements in Vacuo and Buried Pipes", Journal of Sound and Vibration, pp. 171-190.

Sousa, K. and Fabro, A., 2016, "Wave Modelling of a Lightweight Aerospace Panel Using a Finite Element Approach", XXXVII Iberian latin america congress on computational methods in engineering.

Trainiti, G., Rimoli, J.J and Ruzzene, M., 2015, "Wave Propagation in Periodically Undulated Beams and Plates", International Journal of Solids and Structures, pp. 260-276.

Travi, M. S. and Fabro, A.T. 2016. "Experimental Study of the Dynamic Behavior of a Beam with Spatially Correlated Masses for Different Random Field Models".