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NONLINEAR DYNAMICS AND LINEAR CONTROL APPLIED TO A FIGHTER AIRCRAFT IN LONGITUDINAL FLIGHT AT HIGH ANGLES OF ATTACK

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Resumo: *Fighter aircrafts operate regularly in-flight regimes where nonlinearities influence directly into their dynamics. This paper considers the study of a fighter aircraft which operates in high angles of attack of the wing. The mathematical modelling and numerical simulations were developed, becoming a system of nonlinear differential equations that represent the dynamics of an aircraft F-8 "Crusader" in longitudinal flight, considering the effect of wind speed constant in the dynamical response of the aircraft. It is performed the 0-1 test to determine whether the system is chaotic or periodic, in relation to the angle of attack of the aircraft. The control is proposed considering, as a control parameter, the tail deflection angle, and is designed using the control method of Linear Quadratic Regulator (LQR) with the purpose of stabilizing oscillations of angle of attack of the wing, considering critical regions of aircraft behavior. Numerical simulations demonstrated the efficiency of the proposed control strategy, where controller was able to respond quickly to retrieve the aircraft from a stall situation.*

Palavras-chave: *Fighter Aircraft, Longitudinal Flight, High Angles of Attack, Linear Control, Linear Quadratic Regulator (LQR).*

1. INTRODUCTION

The study of dynamic models of high performance aircraft have gained great relevance in the last years, due to high technological and scientific development that engineering in general is experiencing, different approaches are proposed to analyze the dynamics and control to guarantee the stability of these aircraft. Etkin (1982) comments that aircraft dynamics are inherently nonlinear as a consequence of the physical nature of lift and drag forces as well as air frame orientation relative to a desired reference. As an example, it is pointed the fighter aircraft that is submitted to operate at great angles of attack and at high speeds that can affect the dynamic response of the aircraft causing instabilities during the flight. Considering extreme situations like this, the response can be improved by using the mathematical model that represents the aircraft and its dynamic nonlinearities to perform out the design of a control. As in Dos Santos et al. (2017), that the authors consider the nonlinear dynamics and control in a stall situation, in addition a model of atmospheric turbulence on the speed of the aircraft is considered, to design the controller and ensure the stability of a high performance aircraft.

This paper aims to design and analyze a controller for the nonlinear dynamics of an aircraft in longitudinal flight at high angles of attack (above the stall condition), with constant speed. As object of studies, the F-8 "Crusader" aircraft is investigated via dynamical analyses, considering the tail deflection angle as a control parameter. The controller is designed using the technique of Linear Quadratic Regulator (LQR).

2. MATHEMATICAL MODELLING

2.1 Nonlinear dynamic model of the aircraft

The considered forces and the coordinates system of the F-8 "Crusader" aircraft used to represent its motion are presented in Fig. 1. The mathematical model presented is based on Dos Santos et al. (2017), Pereira (2007), and Gerrard and Jordan (1977). The drag force is disregarded in relation to the other parameters and the moment of inertia is taken to be proportional to the mass of the aircraft.

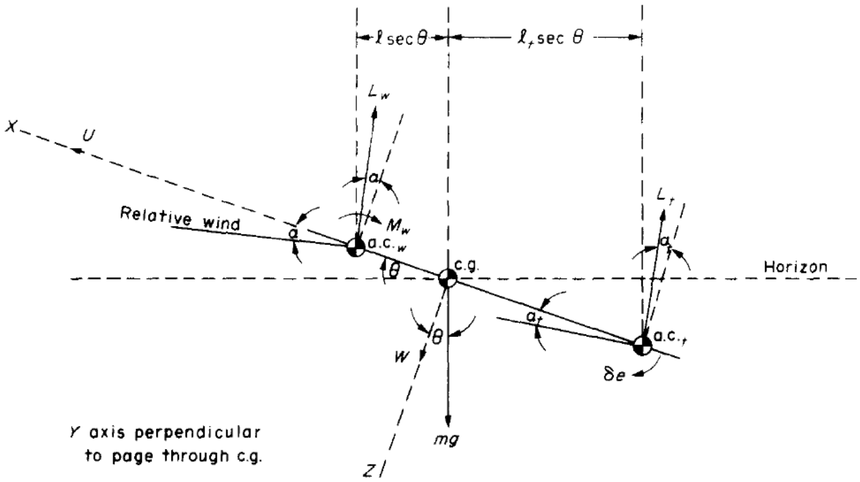


Figure 1. Dynamic aircraft model by Gerrard and Jordan (1977).

The lift force is separated into two components, wing and tail. With an approach similar to that of Gerrard and Jordan (1977), the equations of motion are written in terms of four variables $(x = (u, \alpha, \theta, q))$, where u is the longitudinal flight speed, the angle of attack is represented by α, the pitch angle is θ and the aircraft pitch rate is given by q. The basic equations of longitudinal motion are represented in the system of Eq. (1).

$$
\begin{aligned}
& m(\dot{u} + w\dot{\theta}) = -mg\sin\theta + L_w \sin\alpha + L_t \sin\alpha_t \\
& m(\dot{w} - u\dot{\theta}) = mg\cos\theta + L_w \cos\alpha + L_t \cos\alpha_t \\
& I_y\ddot{\theta} = M_w + lL_w\cos\alpha - lL_t\cos\alpha_t - c\dot{\theta}
\end{aligned}
\tag{1}
$$

Where: m= mass of aircraft, u= velocity of aircraft in X direction, w= velocity of aircraft in Z direction, θ= angular displacement about Y axis, measured clockwise from the horizon as shown in Fig. 1, I_r= moment of inertia of aircraft about Y axis, L_w= wing lift, L_t= tail lift, α= wing angle of attack, α_t= tail angle of attack, δ_e=deflection of elevator, M_w = wing moment, l= distance between wing aerodynamic center and aircraft center of gravity, l_t= distance between tail aerodynamic center and aircraft center of gravity, $c\dot{\theta}$= damping moment, C_L= coefficient of wing lift, C_{L_t}= coefficient of tail lift, $\bar{q}$= dynamic pressure, S= wing area and S_t= horizontal tail area.

A complete mathematical modelling can be found in Pereira (2007), then in this work there will be presented just some considerations for understanding the dynamical system. The lift force of the wing is $L_w = C_{L_w}\bar{q}S$ and the lift force of the tail is $L_t = C_{L_t}\bar{q}S_t$. The lift force of wing L_w is a function of α, and the lift of tail L_t is a function of α and δ_e. Two cubic polynomial functions were proposed in Gerrard and Jordan (1977) to approximate the coefficient of lift of the wing and the tail, as presented in Dos Santos et al. (2017), where ${C_{L_w}}^0$, ${C_{L_w}}^1$, ${C_{L_w}}^2$, ${C_{L_t}}^0$, ${C_{L_t}}^1$ and ${C_{L_t}}^2$ are constants and depend individually on the aircraft.

The model of longitudinal flight dynamics represented by the states $(u, \alpha, \theta, \dot{\theta})$ was presented in Pereira (2007) and can be applied to high performance aircraft, according to Eq. (2):

$$
\begin{aligned}
& \dot{u} = -uq\tan\alpha - g\sin\theta + \frac{\bar{q}}{m}\{SW\sin\alpha\left({C_{L_w}}^0 + {C_{L_w}}^1\alpha - {C_{L_w}}^2\alpha^3\right) + S_t\sin(0.25\alpha + \delta_e)\big[{C_{L_t}}^0 + \\
& \quad +{C_{L_t}}^1(0.25\alpha + \delta_e) - {C_{L_t}}^2(0.25\alpha + \delta_e)^3 + a_e\delta_e\big]\} \\
& \dot{\alpha} = q + \left(\frac{g}{u}\right)\cos\alpha\cos(\alpha - \theta) - \frac{\bar{q}}{mu}\cos\alpha\left\{SW\left({C_{L_w}}^0 + {C_{L_w}}^1\alpha - {C_{L_w}}^2\alpha^3\right) - S_t\cos(0.75\alpha + \delta_e)\left[{C_{L_t}}^0 + \right.\right. \\
& \quad \left.\left. +{C_{L_t}}^1(0.25\alpha + \delta_e) - {C_{L_t}}^2(0.25\alpha + \delta_e)^3 + a_e\delta_e\right]\right\} \\
& \dot{\theta} = q \\
& \dot{q} = M_w/I_y - (c/I_y)q + (\bar{q}/I_y)\{lS\cos\alpha\left({C_{L_w}}^0 + {C_{L_w}}^1\alpha - {C_{L_w}}^2\alpha^3\right)W - l_t\, S_t\cos(0.25\alpha + \delta_e)[{C_{L_t}}^0 + \\
& \quad +{C_{L_t}}^1(0.25\alpha + \delta_e) - {C_{L_t}}^2(0.25\alpha + \delta_e)^3 + a_e\delta_e]\}
\end{aligned}
\tag{2}
$$

where

$$M_w = l_t mg\cos\theta - (l - l_t)(\bar{q}S(C_{L_w}{}^0 + C_{L_w}{}^1\alpha - C_{L_w}{}^2\alpha^3)W)\cos\alpha \tag{3}$$

$$\bar{q} = \frac{1}{2}\rho V^2 \tag{4}$$

where ρ = atmospheric density. As the study proposes the control of a specific aircraft, due to the availability of data of other aircraft, the data of the F-8 “Crusader” aircraft is presented in Table 1.

Table 1. Data of the F-8 “Crusader” (Gerrard and Jordan, 1977).

$C_{L_w}{}^0 = C_{L_t}{}^0$	0
$C_{L_w}{}^1 = C_{L_t}{}^1$	4.0
$C_{L_w}{}^2 = C_{L_t}{}^2$	12
a_e	0.1
S	$33.75\ m^2$
S_t	$8.41\ m^2$
$C_{m_{a.c}}$	0
$\bar{c}$	$3.53\ m$
I_y	$127512\ Kgm^2$
l	$0.06\ m$
l_t	$5.01m$

As shown in Pereira (2007), considering Eq. (2) applied to the data in Tab. 1, it is obtained Eq. (5), which represents the equations of motion of the F-8 "Crusader " aircraft's longitudinal dynamic, used for the numerical simulations and for the control project.

$$\begin{aligned}
\dot{u} &= -uq\tan\alpha - 10\sin\theta + \frac{\bar{q}}{m}\{33.75W\sin\alpha\,(4\alpha - 12\alpha^3) + 8.41\sin(0.25\alpha + \delta_e)[4(0.25\alpha + \delta_e) - \\
&\quad -12(0.25\alpha + \delta_e)^3 + 0.1\delta_e]\} \\
\dot{\alpha} &= q + \left(\frac{10}{u}\right)\cos\alpha\cos(\alpha - \theta) - \frac{\bar{q}}{mu}\cos\alpha\,\{33.75W(4\alpha - 12\alpha^3) - 8.41\cos(0.75\alpha + \\
&\quad +\delta_e)\,[4(0.25\alpha + \delta_e) - 12(0.25\alpha + \delta_e)^3 + 0.1\delta_e]\} \\
\dot{\theta} &= q \\
\dot{q} &= \frac{50.1}{127512}m\cos\theta - \frac{171.1125(4\alpha - 12\alpha^3)}{127512}\bar{q}W\cos\alpha - \frac{50494.752}{127512}q + \frac{\bar{q}}{127512}\{2.025(4\alpha - 12\alpha^3)W\cos\alpha - \\
&\quad -42.1341\cos(0.25\alpha + \delta_e)[4(0.25\alpha + \delta_e) - 12(0.25\alpha + \delta_e)^3 + 0.1\delta_e]\}
\end{aligned} \tag{5}$$

2.2 Numerical simulations

The system of Eqs. (5) will be integrated by using the 4th order Runge-Kutta method. As presented in Dos Santos et al. (2017), Pereira (2007) and Gerrard and Jordan (1977), it is considered aircraft’s velocity $V = 277.7\ m/s$, the initial mass of the aircraft given by $m = 9773\ Kg$ and atmospheric density to 9144 meters of altitude $\rho = 0.4938$. The initial conditions are $u = 257.7\ m/s$, $\alpha = 0.24\ rad$, $\theta = 0.23\ rad$, $q = 0\ rad/s$ and $\delta_e = -0.1\ rad$. Figures 2 show the time histories of all coordinates of the system in the presented initial conditions.

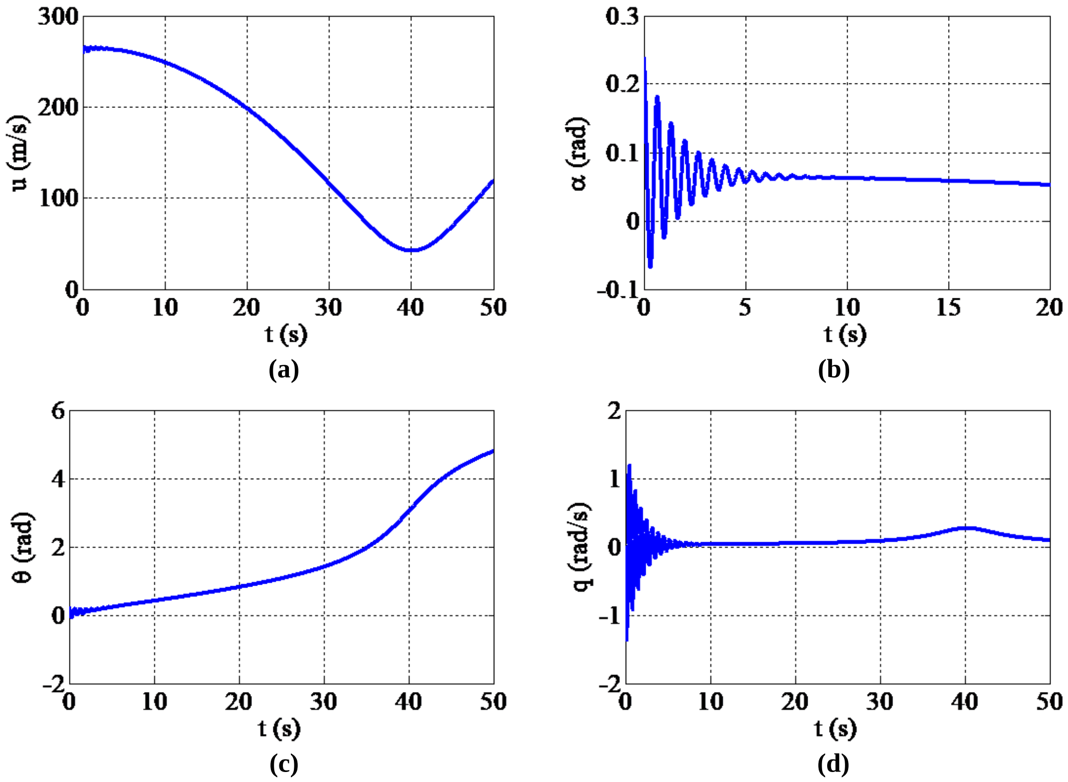


Figure 2. Time histories of (a) aircraft's speed, (b) angle of attack of the aircraft, (c) aircraft's pitch angle, (d) aircraft's rate of pitch.

To analyze the behavior of the system, it is chosen to apply the 0-1 test to determine whether the system is chaotic or periodic. The 0-1 test for chaos takes as input a time series of measurements and returns a single scalar value of either 0 for periodic attractors or 1 for chaotic attractors, as presented in Felix et al. (2014) and Tusset et al. (2015). Applying to the methodology presented in Gopal et al. (2013), the 0-1 test was applied in this work related to the coordinate α, resulting in values ranging from 0.78 to 0.85, depending on the simulation conditions, demonstrating that the system of equations of the longitudinal dynamics of the aircraft is characterized as chaotic in relation to the angle of attack. As the chaos situation is undesirable for the aircraft's angle of attack, which can hinder stability during flight or even damage the aircraft, a control project is proposed and discussed in the next section.

3. PROPOSED CONTROL

The control technique Linear Quadratic Regulator (LQR) was applied, based on the control presented in Tusset et al. (2017). The dynamic system defined by Eq. (5) with a control signal (U) can be represented by Eq. (6) as shown in Molter et al. (2010):

$$\dot{x} = Ax + BU \tag{6}$$

As presented in Dos Santos et al. (2017), the deflection angle of the horizontal tail (δ_e) is used as a control actuator to recover the airplane from different flight situations that request a wing angle of attack higher than the stall angle of the aircraft. From the flight dynamics described in Eq. (5), some mathematical manipulations are considered, according to Eq. (7), to determine the control matrices.

$$\begin{aligned} &\tan\alpha \cong \alpha + \frac{\alpha^3}{3} \\ &\sin\alpha \cong \alpha - \frac{\alpha^3}{6}\ ,\ \sin\theta \cong \theta - \frac{\theta^3}{6} \text{ and } \sin\delta_e \cong \delta_e - \frac{{\delta_e}^3}{6} \\ &\cos\alpha \cong 1 - \frac{\alpha^2}{2}\ ,\cos\theta \cong 1 - \frac{\theta^2}{2} \text{ and } \cos\delta_e \cong 1 - \frac{{\delta_e}^2}{2} \end{aligned} \tag{7}$$

where the terms α^n, θ^n, ${\delta_e}^n$ with $n = 2,3,4, \ldots$ and $\alpha^n {\delta_e}^m$ with $n, m = 1,2,3,4, \ldots$ are eliminated, since these terms are small and they can be adopted by this simplification. As mathematical procedure, the Jacobian matrix was calculated to linearize the matrix, in this way, it has:

$$A(x) = \begin{bmatrix} 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -0.000331\bar{q} & 0 & -0.396 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 34.481 \\ 0 \\ -0.001354\bar{q} \end{bmatrix} \tag{8}$$

Where the control signal is given by:

$$U = -R^{-1}B^T P x \tag{9}$$

Considering a matrix Q being a semi-positive-definite matrix and R a positive definite matrix, the matrix P is obtained solving the Riccati equation given by:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{10}$$

so that the feedback of the signal output Eq. (9) is able to minimize the performance index J, where the non-quadratic cost function for the regulator problem is given by:

$$J = \frac{1}{2} \int_{t_0}^{\infty} [x^T Q x + U^T R U] dt \tag{11}$$

Therefore, the minimization of Eq. (11) implies the minimization of the states x and the signal (U) applied to deflection angle of the horizontal tail δ_e. The control signal U is determined using the matrices A and B. The positives definite matrices x^*, Q and R are defined as follows:

$$x^* = \begin{bmatrix} x_1 \\ 0.04 \\ 0.01 \\ x_4 \end{bmatrix}; Q = 1 \begin{bmatrix} 100 & 10 & 0 & 0 \\ 10 & 1000 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } R = [10000] \tag{12}$$

where x^* represents the desired states, Q and R are the matrices of Eq. (11) used in the calculation of the matrix of Riccati Eq. (10). By substituting the matrices R, B and P into Eq. (9), it is obtained:

$$U = -0.1x_1 - 0.2742(x_2 - 0.04) + 0.7477(x_3 - 0.01) + 0.2625x_4 \tag{13}$$

4. RESULTS AND DISCUSSION

As previously shown, the angle of attack is varied with values above the stopping angle to verify the response of the controller applied in the recovery and stabilization of the aircraft during flight. As presented in Dos Santos et al. (2017), Pereira (2007) and Gerrard and Jordan (1977), some parameters were set for the simulations, being: $V_0 = 277.7\ m/s$, the initial mass of the aircraft given by $m = 9773\ Kg$, atmospheric density to 9144 meters of altitude $\rho = 0.4938$ and the stall situation occurs in the F-8 "Crusader" with an angle of attack of $0.41\ rad$ ($23{,}5°\ deg$).

In addition, the critical initial conditions were considered for the system of Eq. (5) that are $u = 257.7\ m/s$, $\alpha = 0.64\ rad$, $\theta = 0.63\ rad$, $q = 0\ rad/s$, for the case without control $\delta_e = -0.1\ rad$, and $\delta_e = U$ for the case with control. Figures 3 show the time histories with control (With C. in red) and without the influence of the control (Without C. in blue).

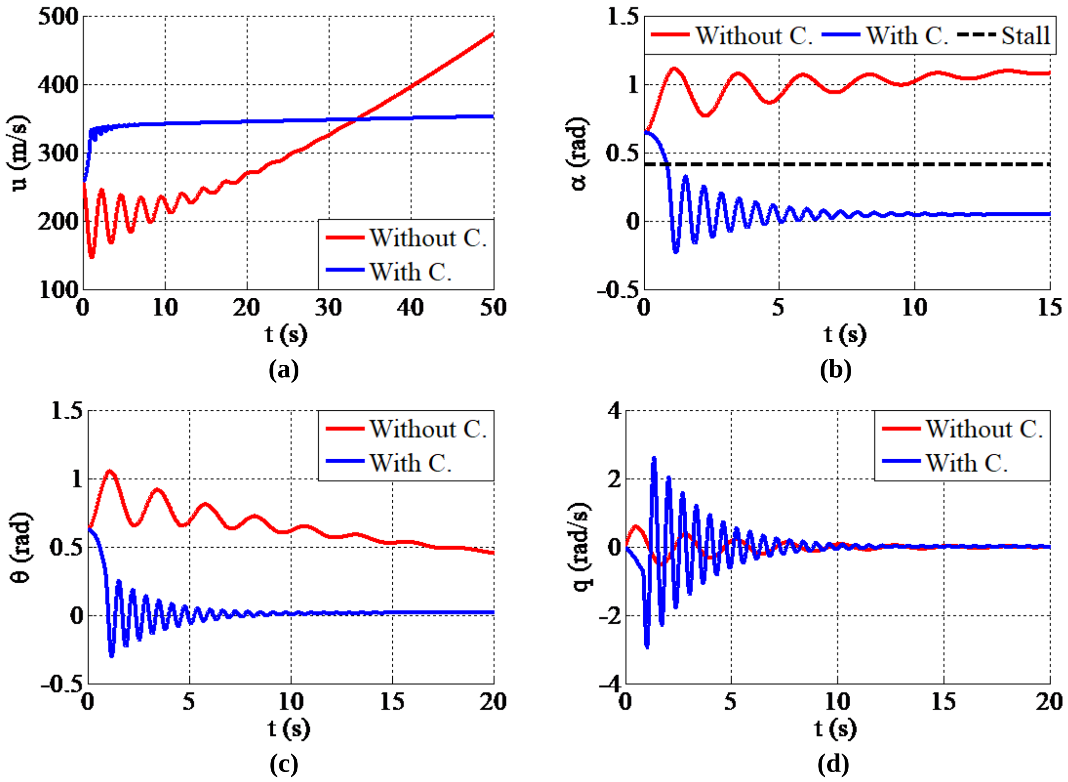
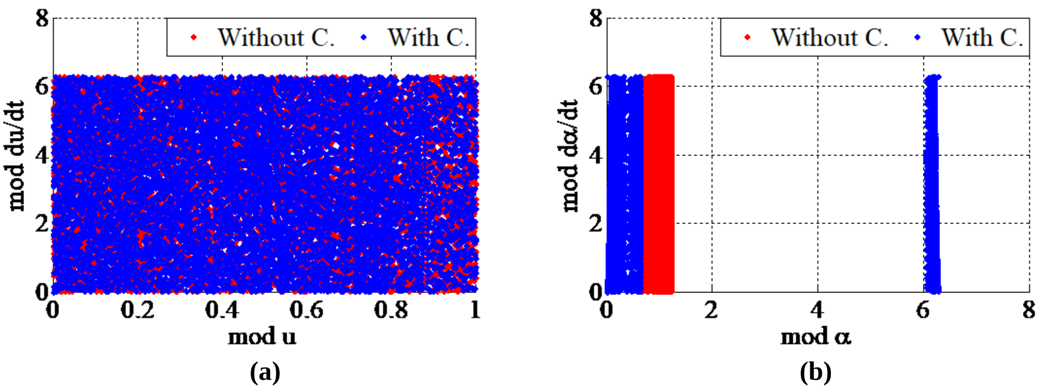


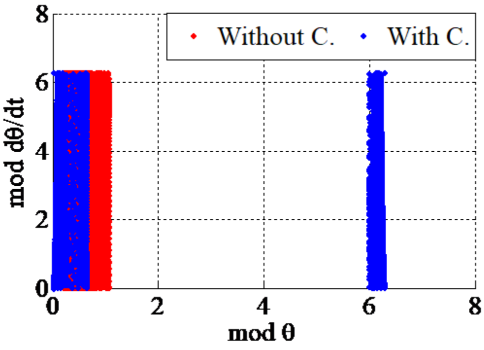
Figure 3. Time histories of (a) aircraft's speed, (b) angle of attack of the aircraft, (c) aircraft's pitch angle, (d) aircraft's rate of pitch.

It can be observed in Fig. 3a that the control acts by minimizing the effects of the stall in relation to the aircraft speed, and in Fig. 3b is observed the efficiency of the control in the recovery of the aircraft from an angle of attack of $\alpha = 0.64\,rad\,(36{,}7°\,deg)$, above the aircraft stall angle, which is $0.41\,rad\,(23{,}5°\,deg)$, in a time of less than 3 seconds, allowing the aircraft to resume wing lift and maintain the balance of flight.

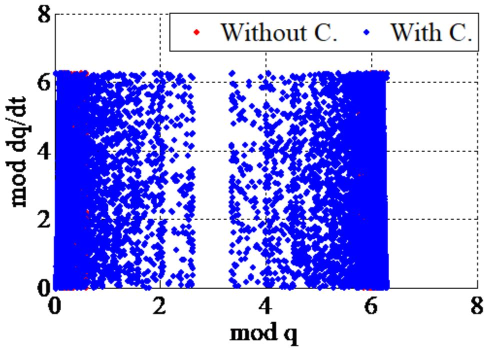
Figure 3c shows that the pitch angle of the aircraft stabilizes in a safe region in a time less than 10 seconds, which is also very important to ensure that the control can be applied in a real situation, since each second in a critical situation may be crucial for the pilot and for aircraft. Finally, as it is presented in Fig. 3d, the intensity of reaction of the aircraft increases with the action of the control. The displayed value of angle of attack was the maximum reached in which the controller was effective. Values above those presented for the angle of attack, the controller is not effective in controlling the system.

Another approach to the control analysis can be observed in Fig 4, which show phase planes of the system so that the behavior and action intervening for each system variable can be analyzed, taking into account the initial conditions presented in the analysis of time histories. It is analyzed the phase diagram shown in Fig. 4, which maintains the caption pattern of the previous figures, with control (With C. in red) and without the influence of the control (Without C. in blue).





(c)



(d)

Figure 4. (a) aircraft's speed phase plane, (b) phase plane of the angle of attack of the aircraft, (c) phase plane of the aircraft's pitch angle, (d) phase plane of the aircraft's rate of pitch.

It is observed that in Fig. 4a the system with influence of the control has a behavior similar to or equal to the uncontrolled behavior. In Fig. 4b, it can be seen that the angle of attack remains in regions less critical with the performance of the control, where the interval at which the angle of attack is submitted is reduced in relation to the system without the control. As shown in Fig. 4c the pitch angle also remains in regions less critical to the aircraft. Finally, for Fig. 4d, it can be observed that the system with influence of the control has a greater range of action, as it is a question of rate of pitch, this means that the system acts more quickly with the action of the control.

5. CONCLUSIONS

This work presented the study, analysis and control design for longitudinal dynamics of a F-8 "Crusader" aircraft, considering operating angles above the stall angle of the aircraft. The control was designed using the deflection angle of the horizontal tail as control actuator, whose control technique is the Linear Quadratic Regulator, designed to act automatically and to assist the pilot in different situations during the flight.

The results presented in this paper indicate that the designed control can lead to significant improvements in aircraft performance, ensuring stability automatically in situations of the angle of attack changes. When placed in flight conditions with angle of attack above the stall situation, it was demonstrated that the control was able to smooth and maintain the conditions of flight in equilibrium for an angle of attack up to 60% above of the stall angle. In relation to the control, it can be verified through the comparison, using the system time histories and the phase planes, that the designed control is effective and has an excellent behavior considering the situations in which it was submitted, recovering the aircraft of a stall situation in a maximum time of 3.5 seconds for the angle of attack and time less than 10 seconds for the pitch angle, as shown in Figs. 3 and 4.

For the future, a work with the objective of considering a model of wind gusts as a component of the speed of the aircraft is intended, to verify how the linear control would act under these conditions, and finally to perform an analysis of how each parameter can affect the control.

6. ACKNOWLEDGMENTS

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