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SEISMIC POUNDING MODEL USING HERTZIAN CONTACT

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Abstract: *Depending on the separation of adjacent structures, they may be subjected to impacts when driven by earthquakes. The seismic pounding, as this impact is referred to, can cause serious damages in the structures, which include wall damage, column shear failure and possible collapse. Considering the importance of the matter, this paper is aimed at presenting a continuous vibration model for seismic pounding, where the contact is modeled using the Hertz contact theory.*
Keywords: *Seismic pounding, continuous vibration model, Hertz contact theory, nonlinear impact model.*

1. INTRODUCTION

The vibrations induced by earthquakes, depending on the at-rest separation of two adjacent structures, can generate high magnitude and short duration acceleration pulses on them, which in some cases causes great damage in the structures. Seismic pounding occurs when the separation distance between adjacent buildings is not large enough to accommodate the relative motion during earthquake events (Raheem, 2006; Cole, 2012). The contact between the structures occurs mainly due to their different dynamic behavior when subjected to ground motion, this happens because of the different stiffness and mass properties of the structures, which generate an out-of-phase vibration. Building pounding occurs more often in large amplitudes earthquakes, which are strong enough to excite the structures to a larger amplitude than of their at rest distance. Some observed earthquakes that caused building pounding are shown in Cole *et al.* (2010), Wang and Chau (2008), EERI (1990), and Otsuka *et al.* (1996).

Since there is not much experimental data to be based on, the numerical modeling is commonly used to analyze the phenomenon. However, the modeling of the seismic pounding is a challenging task due to its highly nonlinear characteristic. In most cases, the modeling of the pounding has been done by means of stereomechanical approach and contact element approach (Muthukumar and DesRoches, 2006). In the former, the impact is considered instantaneous and the velocity after the impacts is obtained by means of the principle of momentum and the coefficient of restitution (COR). Although this approach is simple and easily implemented numerically, it does not consider the deformations that the bodies may acquire during the contact and is not applicable in cases where there is a large contact time. Some works that used stereomechanical models are presented in Papadrakakis *et al.* (1991), Athanassiadou *et al.* (1994), Malhotra (1998) and DesRoches and Muthukumar (2002). As for the contact element approach, the contact force is modeled by a continuous function in time, thus the model gives information of the impact duration. This function has generally a stiffness and a damping parameter, which may be proportional to the deformation and velocity, respectively. An important contact force function is based on the Hertz contact law, which, as discussed in Půst and Peterka (2003), models with more accuracy the impact phenomenon than the commonly used Kelvin model. Some works that used the contact element approach to model building pounding are Muthukumar and DesRoches (2006), Muthukumar and Desroches (2004), Maison and Kasai (1990), Maison and Kasai (1992), Kasai (1990) and Jankowski (2005).

Considering the aforementioned, this paper presents a continuous vibration model of two adjacent structures subjected to seismic pounding. The structures are modeled using the EulerBernoulli beam theory and the contact force given by the Hertzian contact.

2. VIBRATION MODEL

The vibration model is depicted in Figure 1 and consist in two structures modeled as cantilever beams at a distance d from each other. Both structures are subjected to a sinusoidal base excitation $y(t)$. The absolute displacement of the beams at a point x is given by $w_1(x,t)$ and $w_2(x,t)$. The equations of motion for the free vibration of the structures, considering viscous air and strain-rate damping, are given by (Erturk and Inman, 2008),

$$EI\frac{\partial^4 w_i(x,t)}{\partial x^4} + c_s I\frac{\partial^5 w_i(x,t)}{\partial x^4 \partial t} + c_a\frac{\partial w_i(x,t)}{\partial t} + m\frac{\partial^2 w_i(x,t)}{\partial t^2} = 0 \text{ for } i = 1,2. \tag{1}$$

where E is the modulus of elasticity, I is the cross-sectional area moment of inertia, c_s is the strain-rate damping coefficient, c_a is the viscous air damping coefficient, m is the mass per unit length. According to Weaver Jr *et al.* (1990), the absolute transverse motion of the beams can be written as,

$$w_i(x,t) = w_{i,rel}(x,t) + w_{i,b}(x,t) \text{ for } i = 1,2. \tag{2}$$

Where $w_{i,b}(x,t)$ refers to the base motion and $w_{i,rel}(x,t)$ refers to the relative motion between the clamped end of the beam. In this work, the base motion is given as follows,

$$w_{i,b}(x,t) = y(t) = A\sin(\omega t) \text{ for } i = 1,2. \tag{3}$$

where A is the amplitude and ω the frequency of the excitation. By substituting Eq. (2) into Eq. (1), one may have,

$$\begin{aligned} EI\frac{\partial^4 w_{i,rel}(x,t)}{\partial x^4} + c_s I\frac{\partial^5 w_{i,rel}(x,t)}{\partial x^4 \partial t} + c_a\frac{\partial w_{i,rel}(x,t)}{\partial t} + m\frac{\partial^2 w_{i,rel}(x,t)}{\partial t^2} = \\ = -c_a\frac{\partial w_{i,b}(x,t)}{\partial t} - m\frac{\partial^2 w_{i,b}(x,t)}{\partial t^2} \text{ for } i = 1,2. \end{aligned} \tag{4}$$

According to the Hertz contact model, the contact force is given by (Muthukumar and DesRoches, 2006),

$$F = k_h \delta^{3/2} \tag{5}$$

where k_h is the Hertz stiffness coefficient, which depends on the bodies mechanical properties, and δ is the indentation or the relative penetration. The relative penetration for the structures will be $\delta = w_{1,rel}(h,t) - w_{2,rel}(h,t) - d$, thus the contact force is given as,

$$F = k_h[w_{1,rel}(h,t) - w_{2,rel}(h,t) - d]^{3/2}. \tag{6}$$

Please note that the contact force can assume only positive values, thus when $F < 0$ it is assumed $F = 0$, which means that there is no contact between the structures. By introducing the contact force in Eq. (4), one may have,

$$\begin{aligned} EI\frac{\partial^4 w_{i,rel}(x,t)}{\partial x^4} + c_s I\frac{\partial^5 w_{i,rel}(x,t)}{\partial x^4 \partial t} + c_a\frac{\partial w_{i,rel}(x,t)}{\partial t} + m\frac{\partial^2 w_{i,rel}(x,t)}{\partial t^2} = \\ = -c_a\frac{\partial w_{i,b}(x,t)}{\partial t} - m\frac{\partial^2 w_{i,b}(x,t)}{\partial t^2} + H(\delta)\delta_d(x-h)F(\delta) \text{ for } i = 1,2. \end{aligned} \tag{7}$$

where H is the Heaviside function, which is used to make $F = 0$ if $F < 0$; δ_d is the Dirac delta function, and h is the contact height. Equation (7) can be solved by means of the modal expansion method, which expresses the displacement of the beam as two functions of only x and t, that is,

$$w_{i,rel}(x,t) = \sum_{n=1}^{\infty} \phi_{i,n}(x)\eta_{i,n}(t) \text{ for } i = 1,2. \tag{8}$$

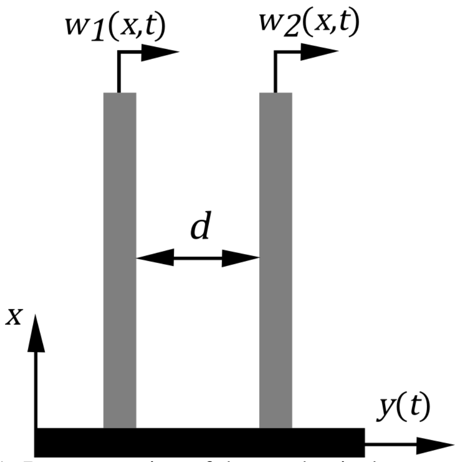


Figure 1: Representation of the mechanical system studied

where $\phi_{i,n}(x)$ and $\eta_{i,n}(t)$ are the mass normalized eigenfunction and the modal coordinate of the clamped-free beam for the nth mode, respectively. The mass normalized eigenfunction for a cantilever beam of the nth mode is given by (Erturk and Inman (2008)),

$$\phi_n(x) = \sqrt{\frac{1}{mL}}\left[\cosh\beta_n x - \cos\beta_n x - \left(\frac{\sinh\beta_n L - \sin\beta_n L}{\cosh\beta_n L + \cos\beta_n L}\right)(\sinh\beta_n x - \sin\beta_n x)\right] \quad (9)$$

where L is the length of the beam and β_n is the dimensionless natural frequency of the nth mode and can be obtained through the characteristic equation, which in this case is,

$$\cosh\beta_n L\cos\beta_n L + 1 = 0. \quad (10)$$

The function $\phi_n(x)$ given by Eq. (9) satisfies the following orthogonality conditions,

$$\int_0^L m\phi_r(x)\phi_n(x)dx = \delta_{rn}, \quad \int_0^L EI\phi_r(x)\frac{d^4\phi_n(x)}{dx^4}dx = \omega_n^2\delta_{rn} \quad (11)$$

where δ_{rn} is the Kronecker delta, which gives unity when $r = n$ and is null if $r \neq n$; and ω_n is the undamped natural frequency of the nth mode and it's given by,

$$\omega_n = (\beta_n L)^2\sqrt{\frac{EI}{mL^4}}. \quad (12)$$

By substituting Eq. (8) in the equation of motion (Equation (7)), multiplying by ϕ_r and integrating from 0 to L, and using the orthogonality conditions of Eq. (11), one may have,

$$\frac{d^2\eta_{i,n}(t)}{dt^2} + 2\zeta_n\omega_n\frac{d\eta_{i,n}(t)}{dt} + \omega_n^2\eta_{i,n} = Q_{i,n}(t) \text{ for } i = 1,2 \quad (13)$$

being,

$$\zeta_n = \frac{1}{2\omega_n}\left(\frac{c_s\omega_n^2}{E} + \frac{c_a}{m}\right) \quad (14)$$

and

$$Q_{i,n}(t) = B_{i,n}\sin(\omega t + \theta_{i,n}) + \phi_{i,n}(h)H(\delta)F(\delta) \text{ for } i = 1,2 \quad (15)$$

where B_n and θ_n are the amplitude and the phase of the base excitation, respectively, and they are given by,

$$B_{i,n} = \sqrt{(m\gamma_n\omega^2 A)^2 + (c_a\gamma_n\omega A)^2} \text{ for } i = 1,2 \quad (16)$$

$$\theta_{i,n} = \tan^{-1}\left(-\frac{c_a}{m\omega}\right) \text{ for } i = 1,2 \quad (17)$$

where

$$\gamma_{i,n} = \int_0^L \phi_{i,n}(x)dx \text{ for } i = 1,2. \quad (18)$$

It is seen from the ground excitation force, Equation (15), that the amplitude and phase of the harmonic excitation depend on the mass and damping properties of the structures. This is one of the main causes of building pounding, since it leads to an out-of-phase vibration, which can cause serious damage in the structures. In order to obtain the responses of the system, Equation (13) was solved numerically using the Runge-Kutta scheme. After obtaining the modal coordinate of the clamped-free beam, $\eta_n(t)$, for each structure, the relative displacement was obtained by,

$$w_{i,rel}(h,t) = \sum_{n=1}^{\infty}\phi_{i,n}(h)\eta_{i,n}(t) \text{ for } i = 1,2. \quad (19)$$

3. REQUIRED AT-REST DISTANCE BETWEEN STRUCTURES

In order to avoid seismic pounding between buildings, their separation must be large enough to admit their relative displacements when subjected to earthquakes. There are codes and regulations which specify the minimum separation between two adjacent structures. Two rules commonly present in regulations worldwide are the Absolute Sum (ABS) and the Square Root Sum of Squares (SRSS) (Raheem, 2006), which are given as follows, respectively,

$$R_{ABS} = w_{1,m} + w_{2,m} \quad (20)$$

$$R_{SRSS} = \sqrt{w_{1,m}^2 + w_{2,m}^2} \tag{21}$$

where R is the separation distance, $w_{1,m}$ is the maximum displacement of structure 1 and $w_{2,m}$ is the maximum displacement of structure 2. According to Raheem (2006), the values given by the ABS and SRSS are sometimes too exaggerated, specifically when the two structures have close natural frequencies. In such cases, the large distance required between buildings is not often possible due to economic or geometric reasons. Another rule used to estimate the at-rest distance between the structures is the Double Difference Combination (DDC) and is given by (Garcia, 2004),

$$R_{DDC} = w_{rel,m} = \sqrt{w_{1,m}^2 + w_{2,m}^2 - \rho_{12} w_{1,m} w_{2,m}} \tag{22}$$

where $w_{rel,m}$ is the maximum value for the relative displacement between the structures and ρ_{12} is the correlation value which depends on the structures natural frequencies and damping factors. The correlation factor is given by (Garcia, 2004),

$$\rho_{12} = \frac{2\sqrt{\zeta_1 \zeta_2}(\zeta_1 + r\zeta_2) r^{1.5}}{(1-r^2)^2 + 4r\zeta_1\zeta_2(1+r^2) + 4(\zeta_1^2+\zeta_2^2)r^2} \tag{23}$$

being $r = T_1/T_2$ the ratio between the structures natural periods and ζ_1 and ζ_2 the damping factors of the structures. The DDC rule give more suitable values than the ABS and SRSS for the values of T_1 and T_2 not much separated. In case when the periods have very different values, the rule gives conservative results (Raheem, 2006).

4. RESULTS AND DISCUSSION

The parameters of mass and stiffness were taken from Muthukumar and DesRoches (2006) and they are listed in Tab. 1. The parameters listed on the table refers to structure 1. In order to obtain different out-of-phase vibrations, the parameters of mass and stiffness of structure 2 were varied according to the parameters of structure 1. The contact height, h, was considered to be the length of the structures. Both structures were considered to have the same damping factor. In addition, a convergence study was made to choose the number of vibration modes and the time step used in the simulations. The results were three modes of vibration and a time step of 0.001.

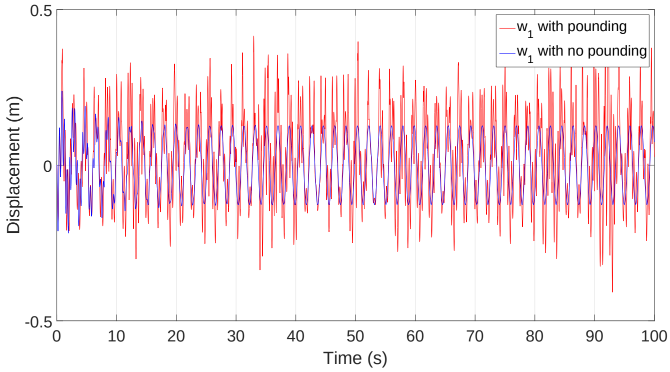
To study the dynamical characteristic of the system, two parameters were varied: the system period ratio (T_1/T_2 which are the periods corresponding to the first mode of vibration of each structure) and the ground motion effective period ratio (T_2/T_g where T_g is the ground period). In studies on seismic pounding, these parameters were recognized as critical (DesRoches and Muthukumar, 2002). The amplitudes of the ground excitation or peak ground acceleration (PGA) were taken from experimental data presented by Muthukumar and DesRoches (2006) and chosen based on the ground motion effective period ratio, for cases when $T_2/T_g < 1$, which are the most critical situations as shown by DesRoches and Muthukumar (2002). The ground motion records are classified into three levels, depending on the PGA intensity: low (0.1g≤PGA≤0.3g), moderate (0.4g≤PGA≤0.6g) and high (0.7≤PGA≤0.9g).

Figure 2 presents the displacement with pounding and no pounding of the structures for the first case studied, which have a strong out-of-phase motion, with $T_1/T_2 = 0.3$ and $T_g = 1.9$ s. The amplitude of the oscillation was in the low intensity group, PGA = 0.21 g. It is seen from the figure that structure 1 had an increase in displacement while structure 2 had a decrease. This happened because the excitation frequency was close to the first natural frequency of the latter. In this case, structure 1 had an increase of 102.74 % while structure 2 had a decrease of 70.23 % in the displacement. Figure 3 shows the second case studied, where $T_1/T_2 = 0.5$, $T_g = 0.95$ s and PGA = 0.51 g, which correspond to a moderate earthquake amplitude level. The increase in the displacement of structure 1 was 114 % and the decrease in the displacement of structure 2 was 81.74 %. It is seen that, despite the structures in this case were less out-of-phase than in the last case (Figure 2), the pounding effect was stronger as noted in the higher displacement change. Thus, the high amplitude caused a higher influence in the pounding effect than of the out-of-phase characteristic. Figure 4 presents the last case studied, where $T_1/T_2 = 0.7$, $T_g = 0.65$ s and PGA = 0.6 g. The increase and reduction of the displacement of structures 1 and 2 were 167 % and 85.42%, respectively. Here, again the higher PGA showed a strong influence in the pounding effect.

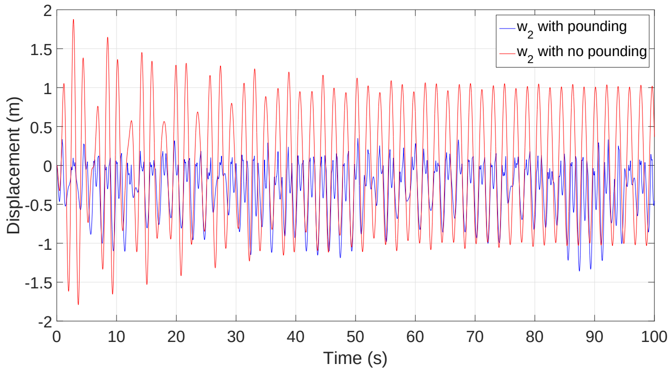
By comparing the displacements in the three cases analyzed (Figures 2, 3 e 4), one can note that when the natural period of the structures are more close together, for example in the case of Fig. 4, the system tends to reach a stationary

Table 1: **Parameters used in the simulations.**

Parameter	Value
Equivalent Stiffness (k_{eq})[MPa]	3.502×10^9
Mass (m_1)[kg]	2.273×10^6
Density (ρ)[kg/m^3]	7850.00
Hertz Stiffness (k_h)[kN/mm$^{3/2}$]	888.00
At-rest distance between the structures (d)[mm]	12.7
Damping Factor (ζ)	0.01

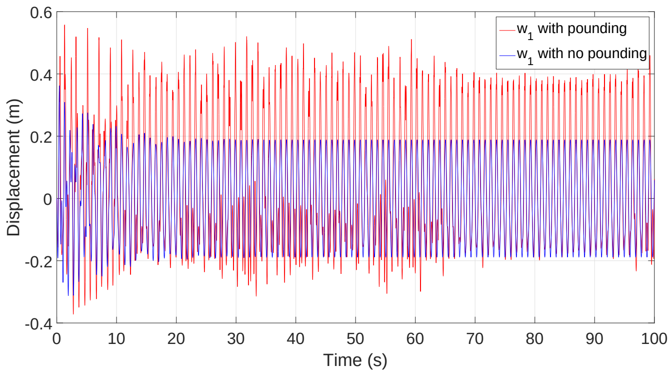


(a)

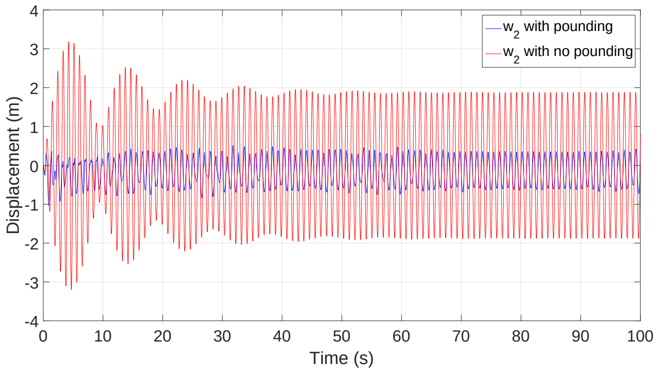


(b)

Figure 2: Responses of the system for $T_1/T_2 = 0.3$, $T_g = 1.9$ s and PGA $= 0.21$ g: (a) Displacement of Structure 1 and (b) Displacement of Structure 2.

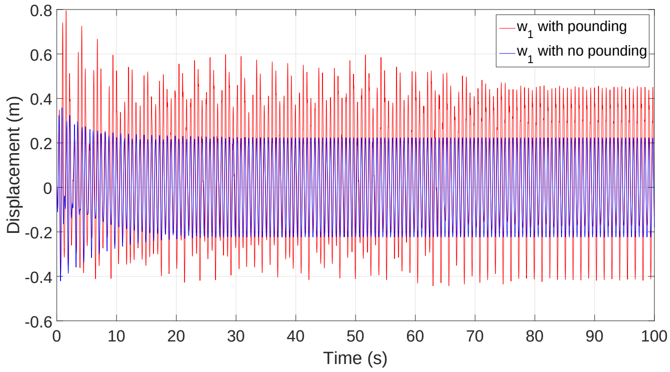


(a)

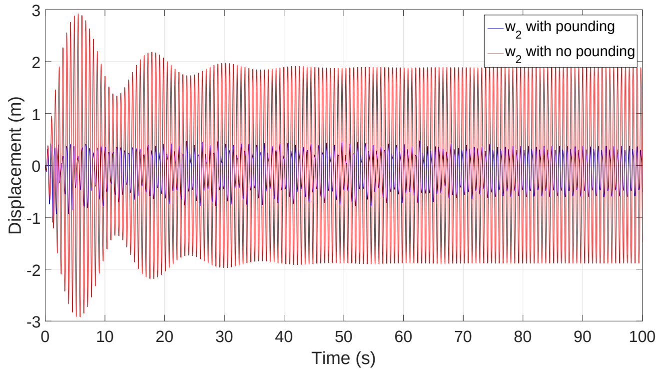


(b)

Figure 3: Responses of the system for $T_1/T_2 = 0.5$, $T_g = 0.95$ s and PGA $= 0.51$ g: (a) Displacement of Structure 1 and (b) Displacement of Structure 2.



(a)



(b)

Figure 4: Responses of the system for $T_1/T_2 = 0.7$, $T_g = 0.65$ s and PGA $= 0.65$ g: (a) Displacement of Structure 1 and (b) Displacement of Structure 2.

regime. However, for a more out-of-phase vibration, the response of the system gets rather complex as shown by the case of Figure 2. Comparing the responses with the ones presented in (Muthukumar and DesRoches, 2006), which was a two degree of freedom system, it is noted that the higher vibration nodes considered strongly influence the displacement of the structures.

The required at-rest separation between the structures according to the rules presented in Sec. 3. for the case with $T_1/T_2 = 0.7$ were,

$$R_{ABS} = 1.8936 \text{ m}$$

$$R_{SRSS} = 0.9286 \text{ m}$$

$$R_{DDC} = 0.9283 \text{ m}$$

which were sufficient for the pounding not to happen. It is noted, however, the great difference between the ABS and the other two rules in the required at-rest distance.

5. CONCLUSION

In this work, a continuous vibration model was presented with the purpose of modeling the seismic pounding phenomenon, which occurs when the distance between adjacent structures at rest is not sufficient to support the relative motion of them when driven by earthquakes. The structures were modeled as cantilever beams and the equation of motion was obtained using the Euler-Bernoulli theory. Analysis where performed varying the natural period of the structures with the purpose of seeing out-of-phase vibrations, which are the main causes to pounding. Also, the excitation period and amplitude were varied taking values from the literature, which correspond to experimental measurements. The criteria to chose the excitation period was so that the ratio $T_2/T_g < 1$, which correspond to the most critical situations in earthquakes. The responses obtained can be compared with the ones found in the literature, and it shows that there is a inlfuence on the higher modes of vibration considered in the displacement response.

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7. RESPONSIBILITY NOTE

The authors are the only responsible for the content of this work.