X CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA 2018
20 A 24 DE MAIO DE 2018 SALVADOR – BA – BRASIL

DYNAMIC ANALYSIS AND CONTROL FOR A FOUR-DIMENSIONAL LORENZ SYSTEM

Angelo Marcelo Tusset, tusset@utfpr.edu.br[1]
Hilson Henrique Daum, hilson.henrique@gmail.com[2]
Rodrigo Tumolin Rocha, digao.rocha@gmail.com[1]
Frederic Conrad Janzen, fcjanzen@utfpr.edu.br[1]
José Manoel Balthazar, jmbaltha@gmail.com[1]

[1] Federal University of Technology – Paraná (UTFPR), PR, Brazil
[2] São Paulo State University – (UNESP) Bauru, SP, Brazil

Abstract: *Chaos and control of chaotic systems have applications in aeronautics, communications, chemical process and even in biological systems. One of major interests in the study of chaos is that real systems usually are controlled taking into account the simplified hypothesis that cut out nonlinearities, delays, dimensions, thermal errors and effects. In this work are presented dynamical analysis and control for a four-dimensional Lorenz system. Numerical simulations have shown that for certain parameters, the system has a chaotic behavior, and that the SDRE control is efficient in taking the original system coordinate to the desired one.*

Key-words: *Lorenz system, SDRE control, Chaos, Dynamics.*

1. INTRODUCTION

According to Vaidyanathan and Volos (2016a, 2016b) chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. For the motion of a dynamical system to be chaotic, the system variables should contain nonlinear terms and it must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions.

According to Ge *et al*. (2000), Wang et al. (2006) and Sun *et al*. (2007), the control of chaotic systems designs state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. In addition, the active control technique is used when the system parameters are known, and adaptive control technique is used when the system parameters are unknown.

The SDRE technique has become very popular within the control community over the last decade. This method, first proposed in Pearson (1962) and later expanded in Wernli and Cook (1975), was independently studied in Mracek, and Cloutier (1998) and alluded in Friedland (1996). The method entails factorization (that is, parameterization) of the nonlinear dynamics into the state vector and the product of a matrix valued function that depends on the state itself Tusset *et al*. (2012).

According to Macek and Strumik (2010), the four-dimensional Lorenz system can be written as follows:

$$\begin{aligned} \dot{x} &= -\sigma x + \sigma y - \sigma_0 w \\ \dot{y} &= -xy + rx - y \\ \dot{z} &= xy - bz \\ \dot{w} &= \sigma_0 x - \sigma_m w \end{aligned} \tag{1}$$

The system represented by Eq. (1) can be represented in state-space form as:

$$\dot{\mathbf{X}} = \mathbf{A}(\mathbf{X})\mathbf{X} \tag{2}$$

where:

$$\mathbf{X} = [x \quad y \quad z \quad w]^T = [x_1 \quad x_2 \quad x_3 \quad x_4]^T \text{ and } \mathbf{A}(\mathbf{X}) = \begin{bmatrix} -\sigma & \sigma & 0 & \sigma_0 \\ -y+r & -x-1 & 0 & 0 \\ y & x & -b & 0 \\ \sigma_0 & 0 & 0 & \sigma_m \end{bmatrix} \tag{3}$$

2. NUMERICAL SIMULATION

In Fig. 1, it is observed the variation of x considering the following parameters: $x_0 = 1$, $y_0 = 1$, $z_0 = 15$, $w_0 = 1$, $r = 28$, $\sigma = 10$, $b = 8/3$, $\sigma_m = 2$ and $0 \le \sigma_0 \le 10$.

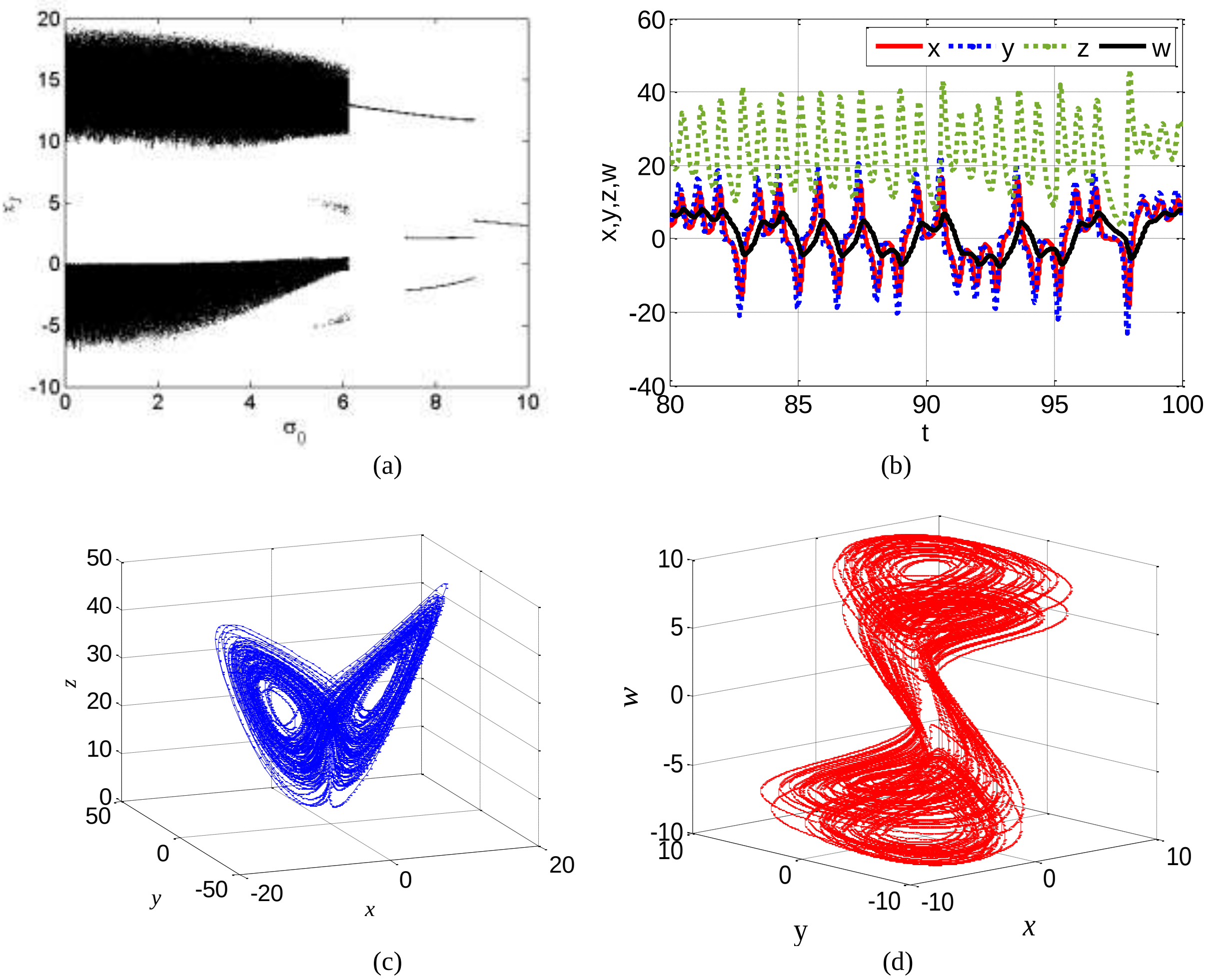
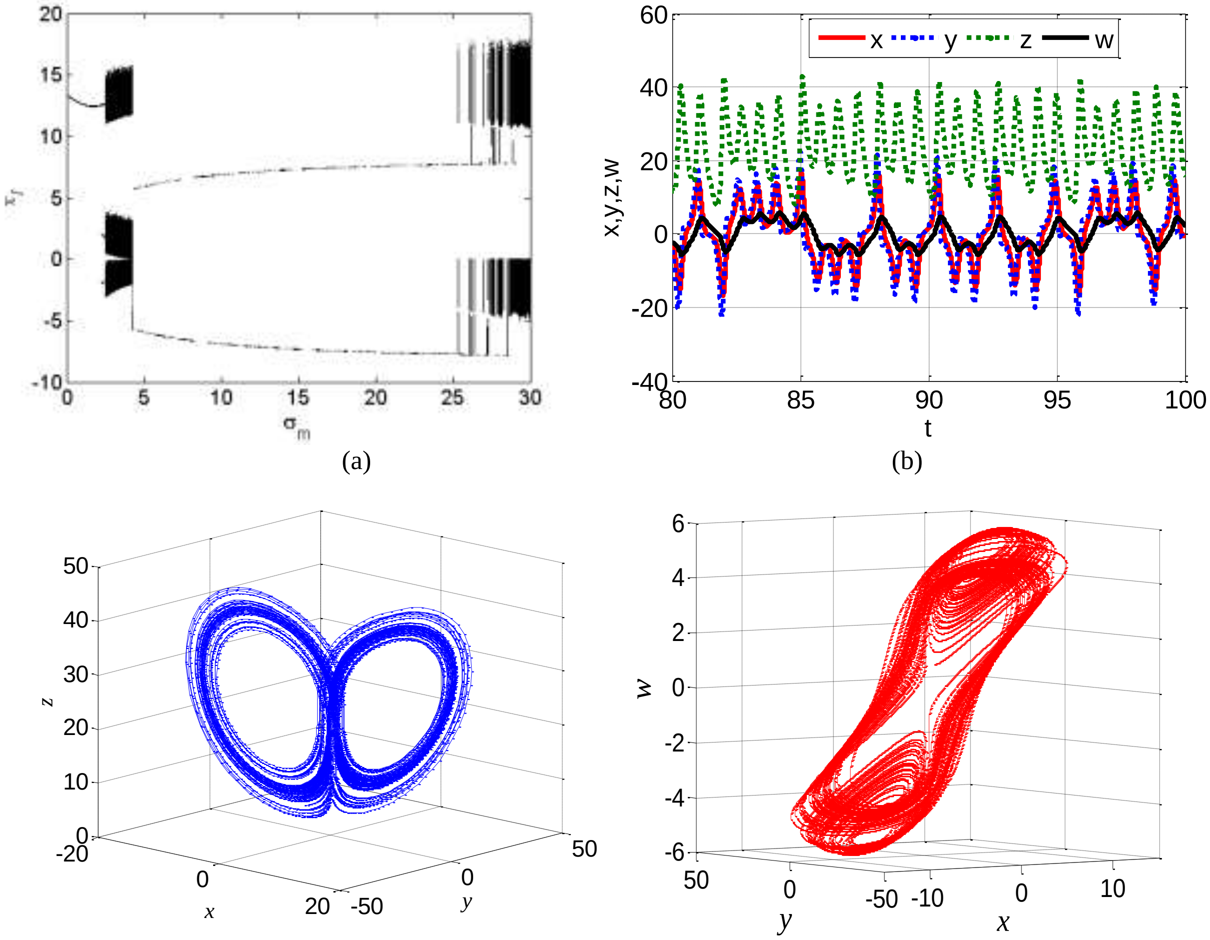


Figure 1. Dynamics of the four-dimensional Lorenz system. (a) Bifurcation Diagram. (b) Phase Diagram for $\sigma_0 = 2$

As can be seen in Fig. 1a for $\sigma_0 < 6.132$ the system has chaotic behavior and for $\sigma_0 > 6.132$ the behavior becomes periodic. The phase diagram of Fig. 1b, with $\sigma_0 = 2$, proves the chaotic behavior.

In Fig. 2 is observed the variation of x_1 considering the following parameters: $x_0 = 1$, $y_0 = 1$, $z_0 = 15$, $w_0 = 1$, $r = 28$, $\sigma = 10$, $b = 8/3$, $\sigma_0 = 7$ and $0 \le \sigma_m \le 30$.

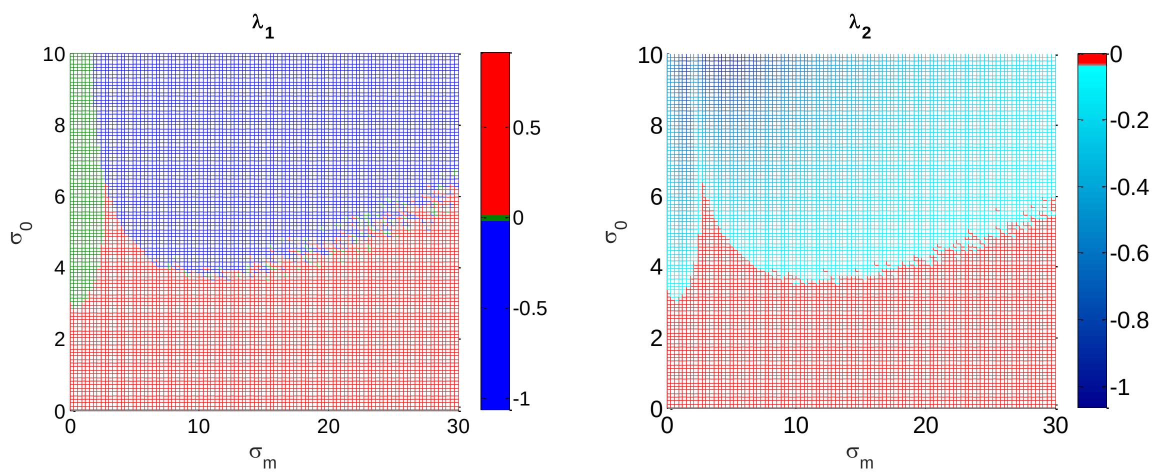


(a) (b) (c) (d)

Figure 2. Dynamics of the four-dimensional Lorenz system. (a) Bifurcation Diagram. (b) Displacement for $\sigma_m = 3$. (c) Phase Diagram (*x,y,z*) for $\sigma_m = 3$. (d) Phase Diagram (*x,y,w*) for $\sigma_m = 3$.

As can be seen in Fig. 2a, for the case when $\sigma_0 > 6.132$, it is possible to obtain chaotic behavior considering the interval $2.539 < \sigma_m < 4.25$. The phase diagram of Figs. 2b, 2c and 2d considering $\sigma_m = 3$ demonstrates the chaotic behavior.

Figure 3 shows the variation of the Lyapunov exponent, considering the two most significant ones, considering the variation of σ_m and σ_0 in the intervals $0 \le \sigma_0 \le 10$ and $0 \le \sigma_m \le 30$.



(a) (b)

Figure 3. Variation of Lyapunov exponent: (a) Lyapunov more significant. (b) Second most significant Lyapunov.

As it can be observed in Fig. 3a that the region in red represents for which parameters of σ_m and σ_0, the system (1) will have the chaotic behavior, and in Fig. 3b the region in red represents for which values of σ_m and σ_0 the system tending to have hyperchaos.

3. PROPOSED CONTROL

Considering the introduction of the control signal **U** in Eq (1), it is obtained the control system:

$$\begin{aligned} \dot{x} &= -\sigma x + \sigma y - \sigma_0 w + u_1 \\ \dot{y} &= -xy + rx - y + u_2 \\ \dot{z} &= xy - bz + u_3 \\ \dot{w} &= \sigma_0 x - \sigma_m w + u_4 \end{aligned} \tag{4}$$

The objective is to define a control law $\mathbf{U} = [u_1 \quad u_2 \quad u_3 \quad u_4]^T$ that takes the system (4) from any initial condition **X** (0) = $\mathbf{X}_0$ to a desired coordinate **X** (t) = $\mathbf{X}^*$, where $\mathbf{X}^*$ is previously due.

Considering the control system (4) in matrix form:

$$\dot{\mathbf{X}} = \mathbf{A}(\mathbf{X})\mathbf{X} + \mathbf{B}\mathbf{U} \tag{5}$$

The control **U** can be obtained by the following equation:

$$\mathbf{U} = -\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P}\,\mathbf{e} \tag{6}$$

where: $\mathbf{e} = [x_1 - x_1^* \quad x_2 - x_2^* \quad x_3 - x_3^* \quad x_4 - x_4^*]$ and $\mathbf{P}$ is a symmetric matrix and can be obtained from Riccati algebraic equation:

$$\mathbf{PA} + \mathbf{A}^{\mathbf{T}}\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{7}$$

where ***Q*** is a definite symmetric or semi-definite positive matrix and ***R*** is a symmetric positive definite matrix.

Considering the following parameters: $x_0 = 1$, $y_0 = 1$, $z_0 = 15$, $w_0 = 1$, $r = 28$, $\sigma = 10$, $b = 8/3$, $\sigma_0 = 2$ and $\sigma_m = 2$. With matrices **A(X)** and **B** given by:

$$\mathbf{A}(\mathbf{X}) = \begin{bmatrix} -\sigma & \sigma & 0 & \sigma_0 \\ -y+r & -x-1 & 0 & 0 \\ y & x & -b & 0 \\ \sigma_0 & 0 & 0 & \sigma_m \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

and defining: $X^* = [2 \quad 2 \quad 10 \quad 2]$, and the matrices ***Q*** and ***R*** as:

$$\mathbf{Q} = 10^4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{R} = 10^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

In Fig. 4 is possible to observe the system (5) with the introduction of the SDRE control (6), obtained considering the matrices given by Eqs. (8) and (9).

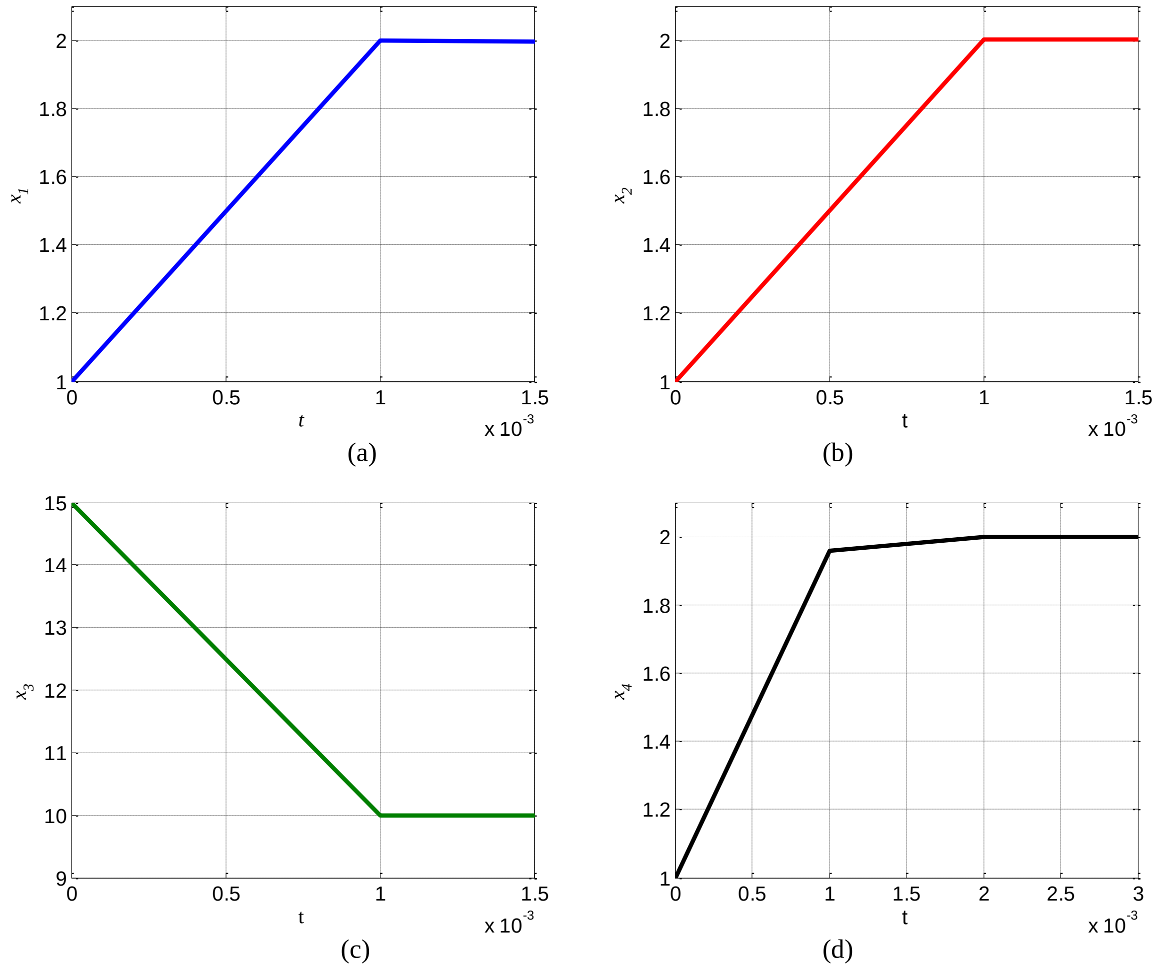


Figure 4. Dynamics of the four-dimensional Lorenz system with control. (a) Position of x_1. (b) Position of x_2. (c) Position of x_3. (a) Position of x_4.

As can be seen in Fig. 4, the control by SDRE is effective in maintain the systems in the desired coordinate. In order to determine the effects of uncertainties on the performance of the proposed controller, it is estimated that an error of $\pm 20\%$ is encountered in the parameters (r , α , b , α_0 and α_m), with a similar strategy used in Tusset *et al.* (2017).

In Fig. 5, the robustness of the control to maintain the system in a desired coordinate is observed, considering the proposed control with variation in parameters: $a_u = a(0.8 + 0.4r(t))$, $r_u = r(0.8 + 0.4r(t))$, $\alpha_u = \alpha(0.8 + 0.4r(t))$, $b_u = b(0.8 + 0.4r(t))$, $\alpha_{0u} = \alpha_0(0.8 + 0.4r(t))$, and $\alpha_{mu} = \alpha_m(0.8 + 0.4r(t))$, where $r(t)$ is a random number $r(t) = [0,1]$.

Considering that ‘ e ’ is the error obtained with the control without parametric uncertainties and ‘ e_p ’ is the error obtained with the control with parametric uncertainties, in Fig. 5 can be observed the robustness of the control considering the matrix: $\mathbf{A}(\mathbf{X}) = \begin{bmatrix} -\sigma_u & \sigma_u & 0 & \sigma_{0_u} \\ -y + r_u & -x - 1 & 0 & 0 \\ y & x & -b_u & 0 \\ \sigma_{0_u} & 0 & 0 & \sigma_{m_u} \end{bmatrix}$ of the control $\boldsymbol{U}$ (Eq. (6)).

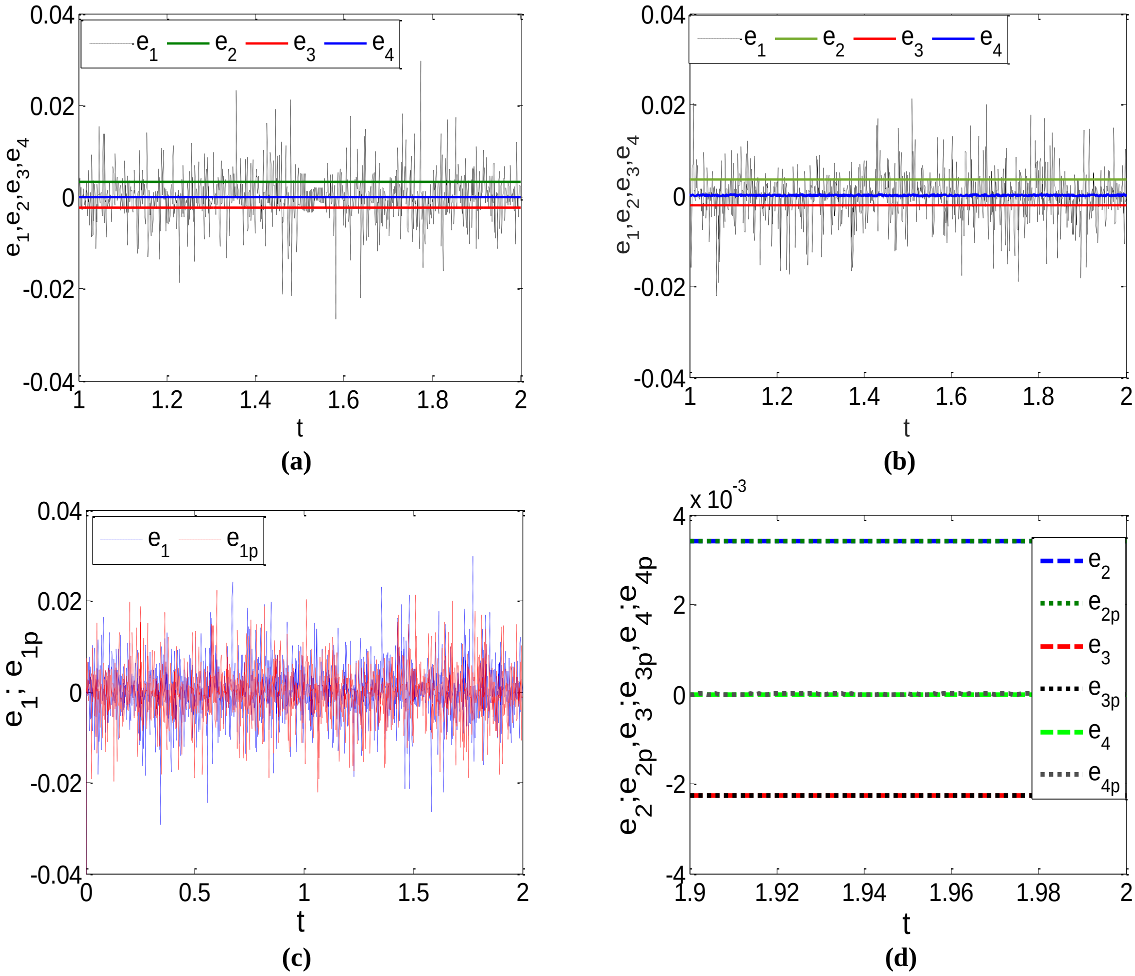


Figure 5. (a) Error obtained with the control without parametric uncertainties. (b) Error obtained with the control with parametric uncertainties. (c) Comparison between ' e **' and '** e_p **' for position** x_1**. (d) Comparison between '** e **' and '** e_p **' for position** x_2**,** x_3 **and** x_4**.**

In Fig. 5 is possible to observe the robustness of the control with uncertainties in keep the same error obtained with the control without uncertainties. This demonstrates that the control is robust to parametric uncertainties.

4. CONCLUSIONS

In this work the SDRE control technique was presented in a four-dimensional Lorenz system. The objective of the technique was to eliminate the chaotic behavior of the system. As can be seen, the technique was efficient in maintain the system in a desired coordinate. Considering the variation of the parameters of the state matrix **A**(**X**), it was possible to demonstrate that the control is robust in the case of parametric variations.

5. ACKNOWLEDGMENTS

The authors acknowledge support by CNPq, CAPES, FAPESP and FA, all Brazilian research funding agencies.

6. REFERENCES

Friedland, B., 1996, "Advanced Control System Design", Prentice-Hall, Englewood Cliffs NJ, pp. 110-112.

Ge, S.S., Wang, C. and Lee, T.H., 2000, "Adaptive backstepping control of a class of chaotic systems", Internat. J. Bifur. Chaos, Vol. 10, pp. 1149-1156.

Macek, W.M. and Strumik, M., 2010, "Model for hydromagnetic convection in a magnetized fluid", Physical Review E, Vol. 82, pp. 027301:1-4.

Mracek, C.P. and Cloutier, J.R., 1998, "Control designs for the nonlinear benchmark problem via the state-dependent Riccati equation method", International Journal of Robust and Nonlinear Control, Vol. 8, pp. 401-433.

Pearson, J.D., 1962, "Approximation methods in optimal control", Journal of Electronics and Control, Vol. 13, pp. 453-469.

Sun, M., Tian, L., Jiang, S. and Xun, J., 2007, "Feedback control and adaptive control of theenergy resource chaotic system", Chaos, Solitons and Fractals, Vol. 32, pp. 168-180.

Tusset, A.M., Balthazar, J.M., Bassinello, D., Pontes Jr, B.R. and Felix, J.L.P., 2012, "Statements on chaos control designs, including a fractional order dynamical system, applied to a "MEMS" comb-drive actuator", Nonlinear Dynamics , Vol. 69, pp. 1837-1857.

Tusset, A.M, Janzen, F.C, Piccirillo, V., Rocha, R.T., Balthazar, J.M. and Litak, G., 2017, "On nonlinear dynamics of a parametrically excited pendulum using both active control and passive rotational (MR) damper" Journal of Vibration and Control, pp. 1-13.

Vaidyanathan, S. and Volos, C., 2016a, "Advances and Applications in Chaotic Systems", Berlin, Springer-Verlag,.

Vaidyanathan, S. and Volos, C., 2016b, "Advances and Applications in Nonlinear Control Systems", Berlin, Springer-Verlag

Wang, X., Tian, L. and Yu, L., 2006, "Adaptive control and slow manifold analysis of a new chaotic system", Internat. J. Nonlinear Science, Vol. 21, pp. 43-49.

Wernli, A. and Cook, G., 1975, "Suboptimal control for the nonlinear quadratic regulator problem", Automatica, Vol. 11, pp. 75-84.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the content of this work.