MAGNETORHEOLOGICAL DAMPER IN SEMI-ACTIVE VEHICLE SUSPENSION SYSTEM USING SDRE CONTROL FOR A QUARTER-CAR MODEL

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Abstract: *This paper presents a quarter-car model assuming a nonlinear model structure in the parameters with linear states, obtaining a more realistic model allowing a rigorous analysis of the efficiency and stability of the proposed control through computational simulations. It is proposed a control for the vehicle suspension system using a SDRE controller applied to the magnetorheological damper, resulting in a semi-active system. In this study, the dissipative element is replaced by a magnetorheological damper whose behavior is adapted to the disturbances to which it is submitted. The efficiency of the proposed control can be evidenced through computational simulations using a quarter-car nonlinear mathematical model. The analysis of the controller performance is performed considering the excitations caused by irregularities of the road represented by a step input. Computational simulations were performed using Matlab®. The simulation results show that the proposed control improves the vehicle dirigibility by reducing the vertical displacement of the wheel and also contributes to the passengers' comfort for it reduces the oscillations in the vehicle body. In addition, simulations of parametric variations were carried out in order to verify the behavior of the proposed control in face of uncertainties. Parametric studies demonstrate that the control remains stable, even when subjected to parametric variations.*

Key-words: *SDRE Control, Magnetorheological Damper, Nonlinear Vehicle Quarter-car Model.*

1. INTRODUCTION

The quarter-car model has been extensively used in the literature to capture a majority of the essential characteristics of an actual suspension system (Yildiz *et al*., 2013). This model represents the body and wheel displacement, promoting the effective analysis of comfort, safety and the proposed control strategy (Chantranuwathana and Peng, 2004). Figure 1 depicts a suspension model of two-degrees-of-freedom in the linear form for a quarter of a vehicle.

Considering as initial condition the static equilibrium and applying Newton's second law to the system shown in Fig. 1, the quarter-car model forced system can be expressed as:

$$
\begin{aligned}
m_s\ddot{x}_c &= b_s(\dot{x}_w-\dot{x}_c)+k_s(x_w-x_c)-F \\
m_u\ddot{x}_w &= -b_s(\dot{x}_w-\dot{x}_c)-k_s(x_w-x_c)-k_t(x_w-x_r)+F
\end{aligned}
\tag{1}
$$

In real applications, the spring stiffness coefficient increases exponentially as it moves away from the static equilibrium point to avoid shocks. In the work of Tusset (2008), the authors proposed to incorporate this nonlinear characteristic to the model as follows:

$$k_s(x_w-x_c)=k_s^l\left(x_w-x_c\right)+k_s^{nl}(x_w-x_c)^3 \tag{2}$$

where:

- k_s^l : Spring linear actuation range;

- k_s^{nl} : Spring nonlinear actuation range;

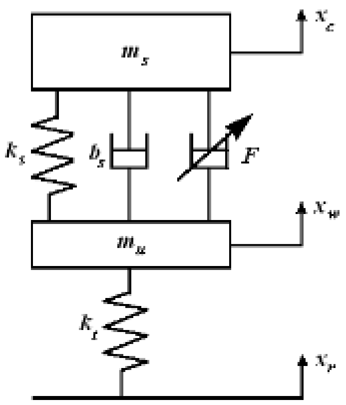

Figure 1. Quarter-car model of two-degrees-of-freedom

The road profile causes vertical oscillations of the shock absorbing in descending movement of the wheel which generates a small impact on the body. When considering the displacement direction of the damper, it is possible to identify the nonlinearity of the damper force relating to this characteristic to positive or negative speed. This nonlinear characteristic of the damper will also be incorporated into the model as follows (Szászi *et al*., 2002):

$$b_s(\dot{x}_w - \dot{x}_c) = b_s^l(\dot{x}_w - \dot{x}_c) - b_s^y|\dot{x}_w - \dot{x}_c| + b_s^{nl}\sqrt{|\dot{x}_w - \dot{x}_c|}sgn(\dot{x}_w - \dot{x}_c) \tag{3}$$

where

- b_s^l : Coefficient that affects damping force linearly;
- b_s^{nl} : Coefficient that acts nonlinearly on the damper;
- b_s^y : Coefficient that represents the characteristics of the asymmetric behavior of the damper.

Replacing Eq. (2) and (3) into Eq. (1), according to Tusset (2008) and Szászi *et al*. (2002), the dynamic behavior of a quarter-car model shown in Fig.1 can be expressed as it follows:

$$\begin{aligned} m_s\ddot{x}_c &= k_s^l(x_w - x_c) + k_s^{nl}(x_w - x_c)^3 + b_s^l(\dot{x}_w - \dot{x}_c) - b_s^y|\dot{x}_w - \dot{x}_c| - b_s^{nl}\sqrt{|\dot{x}_w - \dot{x}_c|}sgn(\dot{x}_w - \dot{x}_c) - F \\ m_u\ddot{x}_w &= -k_s^l(x_w - x_c) - k_s^{nl}(x_w - x_c)^3 - b_s^l(\dot{x}_w - \dot{x}_c) + b_s^y|\dot{x}_w - \dot{x}_c| - b_s^{nl}\sqrt{|\dot{x}_w - \dot{x}_c|}sgn(\dot{x}_w - \dot{x}_c) - k_t(x_w - x_r) + F \end{aligned} \tag{4}$$

where

- m_S : Chassis weight;
- m_u : Mass of the wheel axle;
- k_t : Represents the tire as a bundle of springs;
- x_r : Vertical tire movements;
- x_w : Vertical movements of the wheel axis;
- x_C : Vertical chassis movements;
- F : Force applied by the magnetorheological damper.

The parameters of the vehicle suspension system treated in this work are presented in Tab. 1.

Table 1: Parameters to suspension system. Adapted from Tusset (2008).

m_s	m_u	b_s^l	b_s^{nl}
290 kg	40kg	700 Ns/m	200 Ns/m
b_s^y	k_s^l	k_s^{nl}	k_t
400 Ns/m	235.102 N/m	235.104 N/m	190.103 N/m

This paper applies a magnetorheological damper as the controllable actuator. This damper differs from others by the use of smart magnetic fluids. The peculiarity of MR fluids is the possibility of controlling their characteristics through the application of a magnetic field that allows a quick response when interfacing with electronic controllers and mechanical systems (Koo, 2003).

Many researchers have devoted their efforts to modeling nonlinear friction behavior that causes the hysteresis effect on MR dampers. An alternative way to solve this problem is the use of the LuGre friction model, which was originally developed to describe the phenomenon of nonlinear friction. The mechanism of friction is a phenomenon where two surfaces make contact in a certain number of roughness at microscopic level. In the LuGre model, this mechanism is expressed by relating the friction to the bristle behavior (Olsson *et al*., 1998).

According to the mechanical configuration shown in Fig. 1, the system damping force ***F*** is given by the LuGre model represented by Eq. **Erro! Fonte de referência não encontrada.**). For the computational simulations the parameters were adapted from Tusset *et al*. (2015).

$$F = \sigma_a z + \sigma_0 z v + \sigma_1 \dot{z} + \sigma_2 \dot{x} + \sigma_b \dot{x} v \tag{6}$$

in which

$$\dot{z} = \dot{x} - \sigma_0 a_0 |\dot{x}| z \tag{7}$$

where:

F: Force applied by the magnetorheological damper [N];
v: Electrical voltage applied to the magnetic field excitation coil of the magnetorheological damper [*V*];
$\dot{x}$: Piston speed [m/s];
σ_0 : Stiffness of [*z*] influenced by [*v*] equal to 8×10^5 N/m.V;
σ_1 : Damping coefficient of [*z*] equal to 1.6×10^3 N.s/m;
σ_2 : Viscous damping coefficient equal to 1.5×10^3 N.s/m;
σ_a : Stiffness of [*z*] equal to 4×10^5 N/m;
σ_b : Viscous damping coefficient influenced by *v* equal to 8×10^2 N.s/m.V;
a_0 : Constant value equal to 3×10^{-3} V/N;
z: Variable responsible for the representation of the hysteresis, being determined by Eq. **Erro! Fonte de referência não encontrada.**).

It can be observed in Eq. (6) and Eq. (7) that the LuGre model has a parameter, which represents the voltage applied to the coil of the damper. This parameter allows to control the strength of the MR damper through the voltage control, making the LuGre model the most suitable for the active control system.

2. COMPUTATIONAL PROCEDURE

There are four energy storage elements in the quarter-car model of Fig. 1: two springs and two masses. In this way, the suspension system can be approximated by a mathematical model of fourth-order, being represented in state-space form by four state variables. Therefore, in order to represent the system in the state-space form, the following equalities are assumed in Eq. (2) and (3):

$x_1 = x_c \qquad \dot{x}_4 = \ddot{x}_w$
$x_2 = \dot{x}_c \qquad \dot{x}_2 = \ddot{x}_c$
$x_3 = x_w \qquad w = x_r$
$x_4 = \dot{x}_w$ and $u = F$

then

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{k_s^l}{m_s}x_1 - \frac{b_s^l}{m_s}x_2 + \frac{k_s^l}{m_s}x_3 + \frac{b_s^l}{m_s}x_4 + \frac{k_s^{nl}}{m_s}(x_3 - x_1)^3 - \frac{b_s^y}{m_s}|x_4 - x_2| + \frac{b_s^{nl}}{m_s}\sqrt{|x_4} \\
&\quad - x_2|sgn(x_4 - x_2) - \frac{1}{m_s}u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{k_s^l}{m_u}x_1 + \frac{b_s^l}{m_u}x_2 - \frac{k_s^l + k_t}{m_u}x_3 - \frac{b_s^l}{m_u}x_4 - \frac{k_s^{nl}}{m_u}(x_3 - x_1)^3 + \frac{b_s^y}{m_u}|x_4 - x_2| - \frac{b_s^{nl}}{m_u}\sqrt{|x_4} \\
&\quad - x_2|sgn(x_4 - x_2) + \frac{k_t}{m_u}w + \frac{1}{m_u}u
\end{aligned} \tag{5}$$

Considering the model of Eqs. (5), the matrix form can be represented by the following configuration:

$$\dot{x} = A(x) + g(x) + BU \tag{9}$$

where:

$\dot{x}$: State vector (4x1);
$A(x)$: State-space matrix (4x4);
B: Control vector (4x1);
g(x): Nonlinear functions vector (4x1)

For the SDRE control, the following state-space control law suggested by Tusset *et al.*, (2013) was used:

$$\mathbf{U} = -\mathbf{R}^{-1}(x)\mathbf{B}^T(x)\mathbf{P}(x)\mathbf{x} \tag{10}$$

Where **P**(x) is the solution for the SDRE equation, suggested by Tusset *et al*., (2013):

$$\mathbf{A}^T(x)\mathbf{P}(x) + \mathbf{P}(x)\mathbf{A}(x) - \mathbf{P}(x)\mathbf{B}(x)\mathbf{R}^{-1}(x)\mathbf{B}^T(x)\mathbf{P}(x) + \mathbf{Q}(x) = 0 \tag{11}$$

The cost function for the regulator problem is given by Tusset *et al.* (2013):

$$\mathbf{J} = \frac{1}{2}\int_{t_0}^{\infty} [\mathbf{x}^{\mathrm{T}}\mathbf{Q}(\mathrm{x})\mathbf{x} + \mathbf{U}^{\mathrm{T}}\mathbf{R}(\mathrm{x})\mathbf{U}]\, \mathrm{d}t \tag{12}$$

where **Q**(x) and **R**(x) are positive definite and symmetrical matrices. Such matrices have to be carefully chosen for the stabilization and good system performance. The higher their values, the greater the control inputs and the lesser time is required to reduce system disturbances, as long as the matrix used for the control signal depends on its value. Increasing their values, the values of the feedback gain will reduce, this happens because the gain of the control depends on its inverse, as can be seen in Eq. (10).

The control signal **U** is determined by matrices **A**(x) and **B**, it defines the defined positive and symmetrical matrices **Q** and **R**:

$$\mathbf{Q} = 10^4\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

R=0.1

To simplify the process, only constant values for **Q** and **R** matrices will be applied, a strategy similar to that proposed by Tusset et al. (2015). The **Q** matrix was selected to reduce perturbations in the states and sprung mass, penalizing the states x_3 and x_4 (wheel axis), and **R** was used to reduce the stabilization time. The performance of the controller obtained by the SDRE technique can be modified by adjusting **Q** and **R**.

According to Tusset and Balthazar (2013), the controller must numerically determine the voltage V to be applied to the coil of the MR damper which results in a MR damper force $\boldsymbol{F}$, described in Eq. (5) which is equivalent to the control signal $\boldsymbol{U}$ calculated by the SDRE control in the equation. The MR damper voltage varies from 0 V to 5 V, thus one needs to find the numerical value between 0 and 5 that satisfies the condition $\boldsymbol{F} = \boldsymbol{U}$, for this, the bisection method is applied. This method consists of finding by inspection two points and such that they have opposite signs (see Fig. 2).

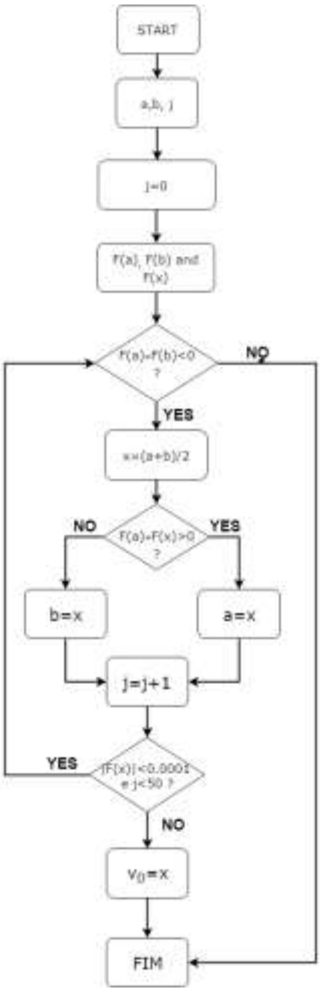

Figure 2. Bisection method flowchart

2.1. Numerical Simulations using SDRE control and MR damper on the suspension system of a quarter-car model

To analyze the system dynamics and compare the results between the SDRE control and the MR damper, it is considered the step type input with 10 cm of amplitude. The use of such a signal will enable a comparison between a passive system response and the semi-active system. According to Moura (2003), systems designed considering step input result in a satisfactory performance compared to real inputs.

The response of the passive system (black line), SDRE active control (blue line) and the semi-active control using the MR damper in parallel with the passive system (red line) are shown in Fig. 3.

The computational simulations of the quarter car model were carried out using the parameters shown in Tab. 1. For LuGre model the parameters were adapted from Tusset *et al*. (2015), expressed on Table 2 below:

Table 2. Values of the parameters used for the MR damper simulation using the LuGre model. Adapted from Tusset *et al*. (2015)

σ_0	σ_1	σ_2	σ_a	σ_b	a_0
8×10^5	1.6×10^3	1.5×10^2	4×10^5	8×10^2	3×10^{-3}

The suspension system parameters were chosen in order to obtain a natural frequency around 2 Hz for m_s and 12 Hz for m_u, that are frequencies close to those used for automobile suspension simulations (Pinheiro, 2004).

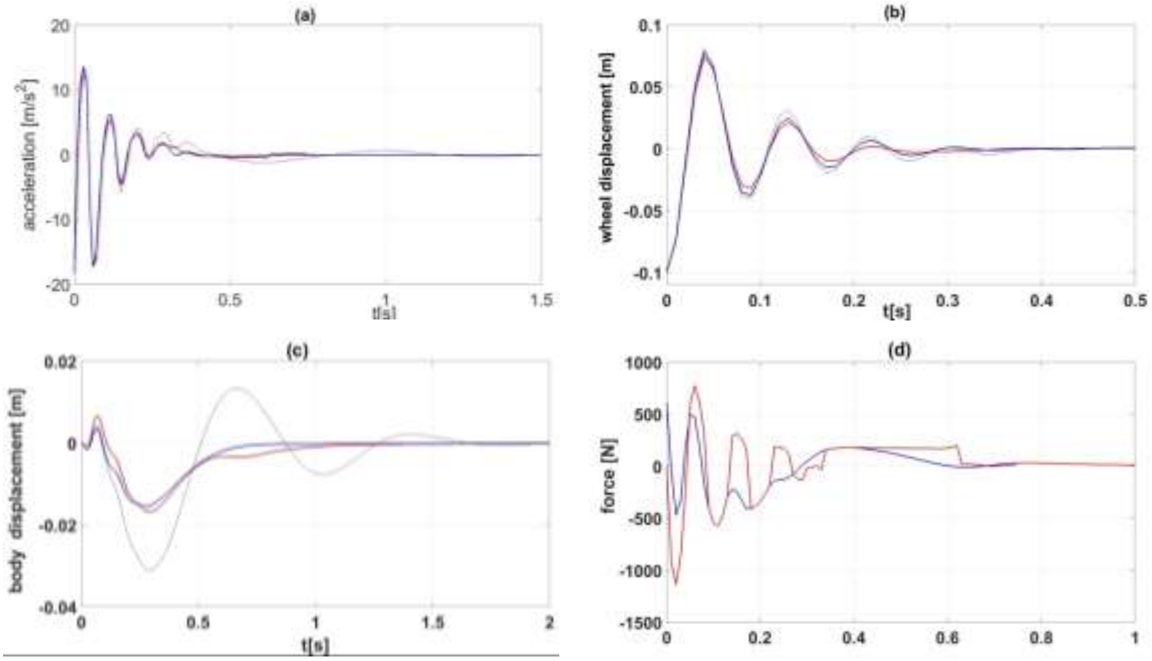

Figure 3. Comparison of the dynamics of the passive, semi-active and active system. (a) Sprung mass acceleration. (b) Wheel displacement. (c) Sprung mass displacement. (d) Signal force.

As can be seen in Fig. 3, the SDRE predictive control applied to the MR damper can act in a very similar way to the active control for the same suspension system. In general, oscillations were significantly reduced in the controlled systems in comparison to the passive system, evidenced by the amplitude value of the ploted curves. In addition, the accommodation time of the active and semi-active system is smaller for both controlled systems compared to the passive one. This means that the entire system, sprung and unsprung mass, has returned to its equilibrium point in a shorter time, which it is very important for comfort and safety. A smaller acommodation time also means that the tire loses its contact with the road for a very small period of time resulting in better dirigibility of the vehicle.

3. CONCLUSIONS

Considering the passengers‘ comfort, it was verified that the proposed semi-active control reduces the oscillations of the sprung mass stabilizing the system in a shorter time, compared to the passive one. Regarding to safety and handling, the semi-active control obtained good results by keeping the wheel oscillations in low magnitudes, ensuring the tire contact with the road.

The obtained results clearly demonstrate that the proposed semi-active control is more appropriate than the use of a passive suspension system. In addition, numerical and computational simulations demonstrated the feasibility of the use of magnetorheological dampers in parallel with the passive suspension as a semi-active control form in substitution for the active SDRE control.

The active suspension system is quite effective, however its cost is relatively high, thus it is feasible for use it only in luxury cars. When it is aimed at the best cost-benefit ratio in vehicles that require suspension systems that meet a variation of track and load the semi-active system becomes feasible and the studies carried out in this paper demonstrate that the proposed system is applicable and effective.

4. ACKNOWLEDGMENTS

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